The Theory of the Evolution of Economic Relations as an Approach to Generalize the Theory of Games

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Abstract: - The Theory of Games in the classical sense uses the idea of a system of economic relations of one type, namely economic relations of a private nature when the goal of rational behavior of the subject of economic relations (ER) is to maximize the benefit (own profit). In the process of evolution, economic relations go through several stages (namely 16), and for each stage, the set of characteristics of the basic elements strictly differs from the set of other stages. The rational behavior of the subjects is largely determined by this particular set of characteristics. Thus, the corresponding Theory of Rational Behavior of Economic Subjects (TRES) is the generalization of the Theory of Games for application in economics (that is, it applies not only to one type of economic relations, aiming at private benefit). Another aspect of the generalization of the Theory of Games in conditions of global information and computer accessibility is the transition from money as a medium of exchange that was a "transferable numerical utility" to another medium of exchange - a full range of goods with the complete dynamically changing set of exchange coefficients as a "generalized medium of exchange" that organically corresponds to the economic content of the production process.

Key-Words: - economic relations, game theory, rational behavior, evolution, private nature, public nature.

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1 Introduction

The works of J. Von Neumann, including, [1], are fundamental in the field of game theory. His results are used in a variety of fields - from algebraic topology, biology, and meteorology to military affairs, [2], [3], [4], [5]. This widespread use is because the concept of competition or rivalry that underlies the Theory of games is typical for a huge number of similar applied areas.

The area of our study is economics in a broader sense than the "game" between a buyer and a seller, that is, one in which "rational" behavior does not aim to maximize one's benefit.

Therefore, all the results obtained in this study do not apply to other fields, such as, for example, military affairs.

To be more precise, in this article, we are not considering the Theory of Games itself, but the Theory of Rational Behavior of Economic Subjects (TRES). The Theory of Games is a special case of TRES.

Our goal is to formulate precisely the approach to generalizing the Theory of Games, but not to create the generalized theory itself.

2 Problem Formulation

The term "Generalization" in this work refers to the expansion of the Theory of Games for application at the stages of evolution of economic relations which differ from those considered by von Neumann's "competitive" economic relations, characterized by the desire for the subjects to maximize their profit.

To be precise, the generalization of the Game Theory should describe the rational behavior of a participant in economic relations, where each act of economic interaction does not aim at the profit maximization of the participating economic entities, but at the maximization of utility for the entire economic community as a whole.

At the same time, the focus of the Generalized Theory of Games shifts from describing the rational behavior of players to:

- description of the usefulness of the economic act for the community based on the knowledge of the utility for each of the subjects and the usefulness of the subject for the community,
- description of the rational behavior of the subjects in the sense of "social" utility.

Thus, it would be correct to talk about the generalization of the Theory of Games for evolving economic relations, which could be called the

Theory of "rational" behavior of economic subjects (TRES).

Von Neumann and Morgenstern developed the Theory of Games for use in economics based on a set of hypotheses about basic economic concepts.
This set of hypotheses and concepts characterizes one particular case in the series of successive stages in the evolution of economic relations.

Thereby the theories of "rational" behavior of the subjects of economic relations, based on the characteristics of economic categories for each stage of development of economic relations present the generalization of von Neumann's Theory of Games.
It should be noted that the basic hypothesis for both theories is the hypothesis of contradiction between the subjects of ER. In the Theory of Games, the opposite goals of economic entities present the form of such a contradiction. In TRES, we consider the existence of the property of independence between economic agents as a main contradiction.

3 Problem Solution

Throughout the history of mankind, economic relations have been developing in one way or another, each time going through the stages described, for example, in [6].

Relying on the laws of evolution of economic relations, specifically on the structure of removing the quality of independence of the subjects of economic relations, [7], we will indicate the stages of evolution of economic relations throughout the entire period of human history.

Since the generalization of the Theory can be carried out based on different grounds, the most important proof of the correctness of the approach to the generalization of the Theory is the precise formulation of the research methodology.

In this work, we use the following methodology for the Theory of the evolution of economic relations building:
I. Establishing the boundaries of the phenomenon under study - economic relations by identifying pre-economic relations from the whole set of "economically similar" phenomena.
II. Identification of the initial concept of pre-economic relations ("Impact and Appropriation" – the dialectical pair containing the developing contradiction).
III. Formulating a hypothesis about the essence of the contradiction that serves as the cause and driving force for the development of economic relations (independence of economic entities).
IV. Obtaining the exhaustive list of forms of PrER (4 forms in total) when carrying out the minimum possible acts of development of the initial concept of pre-economic relations. The highest form is one where the property of independence between Impact and Assignment is removed to the limit - to the point of their complete dependence.
V. We introduce into the scope of consideration the interaction between subjects and, thereby, move on to the study of economic relations. As a result, we obtain four forms of economic relations (Initial Economic Relations, IER) from four forms of pre-economic relations.
VI. In connection with the emergence in the field of research on relationships where more than one subject is involved, we introduce a new concept that is impossible in pre-economic relations, namely, the nature of the economic relationship (private or public). The private nature indicates the independence of the subjects in this regard. Public character indicates the dependence of subjects in the relationship.
VII. We carry out the minimum possible acts of development similar to the process of pre-economic relations development. Namely, we consider four IERs, in which the social character extends to increasingly higher forms of IERs. We receive the exhaustive list of 16 types of Systems of Economic Relations, which indicates all types of economic relations both in the past, and the present and future.
In this work, we solve the following problems:

a. It is necessary to consider economic relations throughout the history of mankind. From the emergence of economic relations in the past to the future forms of economic relations;
b. It is necessary to establish a criterion indicating precisely the development (qualitative leaps) of economic relations;
c. Using this universal criterion, it is necessary to compile the exhaustive list of main forms of economic relations development (stages, formations);
d. It is necessary to indicate for what forms of economic relations the Theory of Games is applicable and for what ones it is not;
e. We have to specify the main features of the TRES for the forms of economic relations for which the Theory of Games is not applicable.

3.1 Based Concepts

As a method for studying the development of economic relations, we use dialectics, which describes changes (leaps) in the forms of a certain phenomenon through its internal contradictions, which are overcome (dialectically eliminated) in

each of the subsequent forms of development of this phenomenon.

In this paper, we accept the following hypotheses:

- the most elementary form of the phenomena of “the Economic Relations" is the “Impact and Appropriation”;
- the cause for the Development of Economic Relations (contradiction developing ER) is the property of Independence of the parties in Economic Relations;
- the purpose of “the Development" is to eliminate (dialectically remove) the Property (Quality) of Independence of the parties. Thus, the parties (economic agents) become dependent.

The Theory of evolution of economic relations, [6], is based on the definition of pre-economic relations (PrER) and the analysis of their transformation into economic relations.

System-forming properties that distinguish pre-economic relations from economic ones are:

- exclusively individual (private) labor, and not the division of labor, as it occurs in economic relations (transition from singular to plural).

The transition of pre-economic relations into economic relations occurs according to this criterion of division of labor.

3.2 Developing a Theory of the Evolution of Economic Relations

To systematically obtain a complete variety of types of economic relations and identify all stages of their evolution, the principle of dialectics is used, [8].

According to this principle, any type of evolution includes pairs of fundamental concepts that are somewhat similar to each other, but also having a contradiction that is the cause of evolution. Another important dialectical principle is that each stage of evolution contains all its previous stages in some sublated form. The transition from singular to plural is also an important evolutionary principle.

Using this methodology, we highlight the following four basic elementary forms-stages of the evolution of pre-economic relations between a person and the objects of nature:

Using this methodology, we identify the following four basic elementary forms - the stages of the evolution of pre-economic relations between humans (subject) and nature (object):

- independent (not mutually dependent) Appropriation and Impact,
- Exchange “subject-object” as an interrelated (mutually dependent) Appropriation – Impact,
- Concentration - Distribution as a set of Exchanges of "one subject with many objects" or "many subjects with one object",
- Production as a network of all preceding forms including Interdependent Concentration – Distributions (the relations "many subjects – many objects").

Stage 1. Independent Impact and Appropriation. Our first hypothesis is that this is the most elementary form of the phenomenon being studied and this is depicted in Figure 1. A subject can appropriate an object of nature that is not the result of human impact. And vice versa, the Subject can exert an impact not for appropriation, but, for example, just out of curiosity.

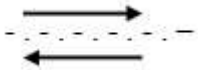

Fig. 1: Independent Impact and Appropriation

Stage 2. Exchange as an interdependent Appropriation – Impact. The simplest step in the evolution of the first stage is the transformation to the interdependent Impact/Appropriation which is shown in Figure 2. Stage 2 is the form of relationship when the result of the Impact causes the Appropriation (IA exchange), and the form of relationship when the result of the Appropriation causes the Impact (AI exchange).

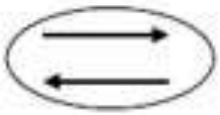

Fig. 2: Exchange as an interdependent Appropriation – Impact

Stage 3. Concentration and Distribution as a set of Exchanges as per Figure 3. We consider the Distribution as a bundle (set) of Impacts of one subject on several objects of nature, and the Concentration as a bundle (set) of Appropriations of several objects of nature by a subject.

The next elementary step in evolution is the transition from singular to plural relations (a cluster of exchanges).

These relations have forms of Concentration and Distribution.

Fig. 3: Concentration and Distribution

Stage 4. Production is a network of all preceding forms including interdependent Concentration - Distributions as provided in Figure 4. The simplest

step in the transformation of Stage 3 is the evolution from a single Distribution/ Concentration to multiple interdependent Distributions/ Concentrations in which a set of subjects are in relation with a set of objects.

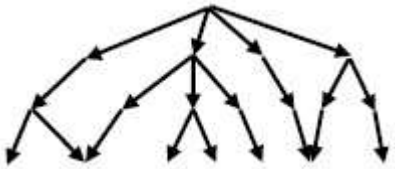

Fig. 4: Production

Briefly, the stages of the evolution of pre-economic relations and the corresponding pre-control systems are presented in Table 1.

Table 1. Evolution of the concepts of PrER and their equivalent concepts of pre-management systems

	Pre-economic relations	**Pre-Management System**
Stage 1	Independent Impact/ Appropriation	Accounting – Transmitting
Stage 2	Exchange	Comparison of Accounting/ Transmitting
Stage 3	Distribution/ Concentration	Search/ Choice
Stage 4	Production	Coordination of the Search / Choice network

3.3 Evolution of Pre-Economic Relations into Economic Relations

Pre-economic relations evolve into economic relations in the following way:

- When moving from the consideration of the subject-nature relationship to the consideration of the subject-subject relationship. At the same time, in addition to the private nature of relations in pre-economic relations, the social nature of relations also arises. For management systems, it is customary to talk about the emergence of democratic management systems in addition to authoritarian management systems.

It must be emphasized that in the evolution of pre-economic relations, there is another intermediate stage - this is the transition from relations in which only natural tools are used to relations that include tools produced with the help of other tools. The appearance of artificial tools is a sign of a high level of pre-economic relations, but not yet economic ones.

Moving from subject-nature relations to subject-subject relations, we get 4 types of relations, we call them initial economic relations (IER).

The most important difference between economic relations and pre-economic ones is that they (ER), with the advent of the second subject, acquired the opportunity to have both a private and public character. As a result, four types of IER turn into an exhaustive set of 16 evolutionary forms of economic relations (ER).

We note the essential features of the methodology for constructing the forms of evolution of ER:

- each stage of IER development includes all previous forms of IER as non-defining elements. It should be noted that the previous stages also contain, in a certain sense, more complex forms of IER, namely, those in the form of causes for their actual occurrence at subsequent stages (ER developing contradictions [6], [9]).
- forms of PrER, "acquiring" the second subject and turning into forms of ER, simultaneously get the opportunity to acquire one of two properties, namely the acquisition of either private or public nature of relations between two economic entities. It is this property that is the most important characteristic of the basic elementary forms of ER evolution and it is this property that gives rise to all 16 types of ER.
- for the Management Systems of Economic Relations, this most important characteristic of ER turns, respectively, into the authoritarian or democratic nature of the management system.

It should be emphasized that this characteristic of economic relations applies equally to both entities involved. Thus, economic relations are always of either private-private nature or public-public nature.

- Each subject of ER is described by a set of four IERs. Thus, we consider such relationships between ER subjects as those where each subject is represented by four IERs (that is, we consider the subsets of relationship pairs). In other words, because economic relations are intersubjective relations, economic relations are introduced in the form of pairs of such fours.

The main distinguishing feature of stages of the economic relations evolution is their differences in the set of characters (private or public) of the four IERs that form each given type of ER (formation of ER).

Likewise, the nature of the main primary forms of management systems (public/private) is their main distinguishing feature. It is this character (public or private) that determines the authoritarian

or democratic forms of systems for managing economic relations.

The law by which the transition from one quad-characteristic feature to another takes place is the dialectical law of withdrawal (negation of the negation).

The process of changing stages is a stepwise change in the form of economic relations when independence is removed (the acquisition of a public character) in a certain, higher type of IER with the simultaneous restoration of the private nature of one of the aspects of IER in the previous form.

Thus, with the transition from stage to stage, the dependence of the subjects moves to a higher and higher level (Table 2), accumulating the social nature of its aspects.

It should be noted that the transition from studying the relationship between humans and nature to considering the relationship as "a group of people – a group of people" was carried out in two steps. The first elementary step is the transition from the study of human-nature relationships to the study of relationships between people (person/person) for the four types of IER.

The second elementary step is to assume that one or both parties of the ER may also be the IER. More precisely, economic relations are defined as relations between IERs, with the clarification that the operands in them can be both subjects and IERs.

That is, in the concept of economic relations we also include relations between economic relations.

The most significant features of the real evolutionary process of economic relations are presented in Table 2, which contains the exhaustive list of all possible evolutionary stages of economic relations.

The most complete list of stages of ER evolution includes 16 types of ER and, accordingly, management systems of economic relations.

Moreover, each subject is described by the entire four IERs, and each type of IER has a specific character (Public or private).

By combining character values for each element of the fours, we get a complete variety of sets of private-authoritarian or public-democratic characters of their relationships.

The theory of evolution of ER created in this way defines a complete system of stages of economic relations development both in the past and present and future and, accordingly, defines all possible types of management systems of economic relations.

It can be assumed that the Economic Relations of types 9, 11, 13 and 15 are expected to be stable, but types 10, 12 and 14 are expected to be transient. It is interesting to note that the System of absolutely public economic relations and, therefore, the corresponding Universally-Democratic Management System are the economic interpretation of idea of a purposeful-oriented system, [10].

Table 2. Evolutionary Stages of Economic Relations (PR -private nature, SOC – public (social) nature)

Types of ER \ Types of IER		Impact/ Appropriation	Exchange	Distribution/ Concentration	Production
ERs of a fully private nature (Universally-Authoritarian Management System)	Type 1.	PR	PR	PR	PR
ERs with partially private nature to which Partially authoritarian MS correspond	Type 2	SOC	PR	PR	PR
	Type 3	PR	SOC	PR	PR
	Type 4	SOC	SOC	PR	PR
	Type 5	PR	PR	SOC	PR
	Type 6	SOC	PR	SOC	PR
	Type 7	PR	SOC	SOC	PR
	Type 8	SOC	SOC	SOC	PR
ERs of a partially Public (social) nature to which Partially Democratic MSs (DMSs) correspond	Type 9	PR	PR	PR	SOC
	Type10	SOC	PR	PR	SOC
	Type 11	PR	SOC	PR	SOC
	Type12	SOC	SOC	PR	SOC
	Type13	PR	PR	SOC	SOC
	Type14	SOC	PR	SOC	SOC
	Type15	PR	SOC	SOC	SOC
Absolutely public (social) ER (Universally-democratic Management System)	Type16	SOC	SOC	SOC	SOC

The important difference of the final stage of economic relations evolution from all previous ones is that the process of achieving the final characteristics of all the elements of the quad (the

fully social nature of Economic Relations and, accordingly, the fully democratic nature of the Management System) is endless – it is the eternal pursuit of the ideal.

It should be noted that for systems of Economic Relations that are "private in nature" (PR), von Neumann's Theory of Game is applicable since the private nature of ER means competition between the parties.

For systems of Economic Relations that have a "public (social) nature," The Theory of Games is inapplicable and must be improved since the social nature means the cooperation of the parties (common goals), and not the competition.

3.4 Stages of Evolution

It is generally recognized that the universal private nature of economic relations and the authoritarian form of their management were the most ancient, [11].

However, at each stage of evolution, the private nature of one of the aspects of IER is replaced by the public one. The corresponding authoritarian forms of economic relations management systems are being replaced by more and more democratic forms.

Note that the evolution of management systems of ERs corresponds to the forms of ER evolution.

All four types of IER are contained in each type of economic relationship. Thus, the type of ER is described by four components, each of which can be of a "private" or "public" nature.

The subsequent types of ER are contained in the previous types in the form of undisclosed contradiction, namely the contradiction associated with the independence of subjects of economic relations.

At each stage, one aspect of independence is eliminated, acquiring the public character instead of the private one.

The elimination of aspects of independence continues until all the aspects of the independence of the subjects of economic relations are eliminated. Thus, the last stage of ER development is characterized by the complete dependence of the subjects.

In any society, it is easy to detect the phenomena of almost any form of ER, from form 1 (family) to form 16 (commune).

The terms "stages" or "formations" are usually used in the field of political economy, to denote the forms given in Table 2.

3.5 An Example of the Development of Rational Behavior of Subjects within a Community

By the thesis that the utility of commodity A is not its monetary value, but some measure depending on its quantity, let us denote the utility of commodity A as q(A) (the quantity of the commodity).

Each product A has a conversion vector into any other product (in the multidimensional space of goods), that is:

$$\overrightarrow{K_A}(t)=(K_A^1(t),K_A^2(t),...) \tag{1}$$

Then the utility of product A in terms of product B will be:

$$U_{AB}(t) = q(A)\bullet K_A^B(t) \tag{2}$$

Suppose that the community must decide which of the subjects S_1, S_2, ... is more rational to transfer product A from subject S_0. This decision is made based on a comparison of the utility of product B produced by the subject S_i, provided that he receives product A. It should be taken into account that in addition to product A, many other products were used in the production of product B.

There are many levels of solving this issue, depending on the depth of "prognostic" capabilities:
Level 1

Having received product A (the product being sold), subjects can produce the following products:
Subject S_1 - product B1, having spent the set of products

$$I_{B_1} = \{P_i\}_{B_1} \tag{3}$$

...
Subject S_n- product B_n, having spent the set of products

$$I_{B_n} = \{P_i\}_{B_n} \tag{4}$$

Then the comparable level 1 utility for the community will be:
For product B_1 – the utility is equal to:

$$U_1^1 = B_1 - \sum_{i\in I_{B1}} q(P_i)\bullet K_{P_i}^{B_1} \tag{5}$$

...
For product B_n – the utility is equal to:

$$U_n^1 = B_n - \sum_{i\in I_{B_n}} q(P_i)\bullet K_{P_i}^{B_n} \tag{6}$$

It would be rational to transfer product A to subject S_l :

$$S_l: U_l = \max (U_i, i \{I_{B_1}, I_{B_2}, \dots, I_{B_n}\} \quad (7)$$

(maximization is carried out on products produced with the use of product A. Product B_k is produced using product A).

Carrying out similar reasoning for the purchased product C in an exchange transaction and taking into account the rationality of purchasing the product that, during its production process "takes away" the minimal utility from society, we find that rational behavior would mean the receiving product C from the subject:

$$S_m: U_m = \min (U_j, j\{J_{D_1}, J_{D_2}, \dots, J_{D_p}\}) \quad (8)$$

Where $J_{D_1}, J_{D_2}, \dots, J_{D_p}$- sets of products necessary for the production of products D_l, D_2, …, D_p.

(minimization is carried out for products that are used in the production of product C. product D_k are used in the production of product C). Then the utility of the operation of exchanging product A for product C will be:

$$U_{AC} = U_l - U_m \quad (9)$$

Or in general form:

$$U_{AC} = \max(U_i - U_j), i \{I_{B_1}, I_{B_2}, \dots, I_{B_n}\}, j \{J_{D_1}, J_{D_2}, \dots, J_{D_p}\} \quad (10)$$

Level 2 (without a background, the option of changing the history of the exchange of goods in the past is not considered, i.e. the history of commodity exchanges in the past cannot be changed).

Let us assume that from product B_l it will be possible to obtain products:

$$\left\{B^2_{l_j}\right\}, j \in J^2_{B_l} \quad (11)$$

and the utility of each of them is equal to:

$$U^2_{l_j}, j \in J^2_{B_l} \quad (12)$$

which is calculated in the way as described above.

Accordingly, it would be rational to transfer product B_l to subject p:

$$U^2_p = \max(U_j) \quad (13)$$

Then the second-order utility of transferring product A to subject S_l will be:

$$U^2_{B_l} = U^2_p \quad (14)$$

Using the reasoning for the purchased product C from level 1, the utility of the operation of exchanging product A for product C can be represented as:

$$U_{AC}^{1,5} = U_p^2 - U_m \quad (15)$$

where U_m is from (8).

But such utility is not yet level 2, but level "one and a half", as it counts products produced two levels down (produced with the use of product A), but only 1 level up (used to produce product C).

Having carried out similar to (8) reasoning for purchased products that are used in the process of production of products used to produce product C in the exchange transaction, and understanding the rationality for the society of acquisition products that "remove" minimal utility from the society, we can designate the utility of the second level of exchange product A for product C, as:

$$U_{AC}^2 = U_p^2 - U_k^2 \quad (16)$$

The above reasoning can be expressed in the language of the theory of sets, more precisely in the language of the Theory of Structures (N. Bourbaki):

Let us introduce sets of subjects of economic relations:

$$S = \{S_i\} \quad (17)$$

The set of objects (products) of economic relations:

$$Ob = \{Ob_i\} \quad (18)$$

The set of objects that are used in the production of product i:

$$ObUp_i = \{ObUp_i^j\} \ j \ JU \quad (19)$$

The set of objects for the production of which product i is used:

$$ObDown_i = \{ObDown_i^j\}, j \ JD \quad (20)$$

Then the "downward utility" of object (product) i can be defined as:

$$UDown_i = P(Ob_i \text{ X } ObDown_i) \quad (21)$$

where P is the double Descartian or direct multiplication of sets.
It's obvious that:
$ObUp_i = B(\{Ob_i\})$, and $ObDown_i$ $B(\{Ob_i\})$,

where B(A) – Boolean of the set A (22)

The "upward utility" of object (product) i can be defined as:

$$UUp_i = P(Ob_i \text{ X } ObUp_i) \quad (23)$$

"The utility of object (product)" i can be defined as: $U_i = P(UDown_i \text{ X } UUp_i)$ (24)

The utility of the ER "Exchange of object A for C" of the first level can be defined as:

$$U_{AC} = P(U_A \text{ X } U_C) \quad (25)$$

The second level of utility is expressed as:

The "second level downward utility" of the object (product) i can be defined as follows:

$$UDown_i^2 = P(Ob_i X \{ObDown_j^k\}), jJD \quad (26)$$

The "upward utility" of object (product) i can be defined as:

$$UUp_i^2 = P(Ob_i \text{ X } \{ObUp_j^k\}) , \ j \ JU \quad (27)$$

The "second level utility" of object (product) i can be defined as:

$$U_i^2 = P(UDown_i^2 \text{ X } UUp_i^2) \quad (28)$$

Similarly, we can construct the definition of rational behavior of the 3rd level, in which we formulate not only the utility of products produced by using products produced by using product A, but also of products produced by using products produced by using products produced by using product A ... The above arguments are carried out in a similar way for all basic forms of economic relations - not only for Exchange, but also for Concentration/ Distribution and Production. If in Exchange the relationship between two subjects is considered, then in Concentration the relationship "many subjects - one subject" is considered, and in Distribution "one subject - many subjects". Accordingly, the next step of complexity leads to the consideration of the relationship "many subjects - many subjects," which is the basic form of Production.

4 Conclusions

Thus, the Theory of "rational" behavior of the subjects (TRES) should include, in addition to the Theory of Games, the theory designed to develop decisions in conditions of the social nature of economic relations. That is, in contrast to the conditions when "each participant tries to maximize a certain function (his profit), not all elements of which are under his control", [1], TRES should describe the rational behavior of the subjects seeking to maximize the "utility" for the entire community.

In the transition from a private to a public nature, the concept of "rational" behavior changes, and, accordingly, the corresponding theory of "rational behavior" changes. This transition adds features of planning rather than a game to the theory of rational behavior.

TRES for the public ER must solve two main problems:

1. Determining the "usefulness" of product A for the community, knowing its "usefulness" for each subject of the community. By this utility, the rational behavior of the subject of the community is determined. The complexity of this task is determined by two factors:
   a. Utility is not the immanent property of product A but depends on many factors – time, geography, etc. A bottle of water on the shores of Lake Ontario is not as useful as in the Sahara Desert (which is not entirely true during the rainy season).
   b. The utility of product A for society must take into account not only the utility of product A for each of the subjects (UA(S)) but also the utility for society of product B produced by a given subject with the use of product A (UB(S)) (a kind of weighting coefficient of the subject, which also depends on time, place, etc.).
2. A method for finding the optimum in the problem of enormous dimensions. When the community consists of millions of subjects producing hundreds of millions of goods, the purely computational task of finding the optimum is not a simple matter.
3. In case when economic relations are at the public-private stage, there are interacting communities within which the public nature of economic activity predominates, and the communities themselves act on the external market with a private nature, TRES retains all the main features of von Neumann's Theory of Games. With the clarification that the

usefulness of product A is determined not for an individual entity, but for the entire community interacting with another community. Similarly, for product B received from another community, its utility for the entire community receiving that product must be determined.

4. In addition, the theory of evolution of economic relations defines smaller stages of ER-associated with the attribution of economic relations to four basic forms: 1. Impact-Appropriation, 2. Exchange, 3. Distribution/Concentration, 4. Production.

It can be argued that the assignment of a given economic relationship, for which rational behavior is determined, to a specific element of these basic forms will influence the determination of the rational behavior of the subject participating in these ERs. The specific form of ER (one of 16) plays a decisive role in the distribution of the decision-making process and in the choice of rational actions of the actors involved.

5. Another aspect of the generalization of the Theory of Games is the removal of the limitations of the theory associated with the hypothesis introduced by von Neumann about the role of money as a universal "transferable numerical utility".

When studying the evolution of economic relations, we consider money as performing only the exchange function and, as noted in several works, for example, [12], this function of money as a means of exchanging goods and a means of comparing the "utility" of goods loses its monopoly and uniqueness as the property of global availability of information and the property of global availability of computing power for all participants in economic relations are achieved.

Such global accessibility conditions mean the availability of a complete report on current commodity exchanges, taking into account all regional and other circumstances.

Thus, the utility of the product in TRES must be expressed not by means of money, but means of pure numerical exchange rate of product A for any other product B at any given region at any given point of time.

6. The article presents the following new results:

- an approach to generalizing game theory is proposed to expand the scope of its applicability not only to economic relations based on competition but also to economic relations based on the cooperation of economic entities,
- an exhaustive classification of the forms of economic relations covering the entire history from the moment of the origin of economic relations to the forms in the future, is proposed,
- the starting point of the development of Economic Relations (human-human relations) has been determined. Pre-economic Relations (human-nature relations) present this starting point,
- the principle of transformation of the initial pre-economic relations into economic relations of increasingly developed forms is indicated,
- the article indicates for what systems of economic relations the classical game theory is applicable, and for what ones it should be improved,
- the "utility" indicator of goods for the community of economic entities has been introduced. This measure is not based on the concept of money, but is based on the coefficients of exchange of some goods for others ones and on the interdependence of the utility (for society) of goods in the technological chain of production,
- the mathematical analysis of the introduced utility measure has been carried out.

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