A Comparative Analysis of Minimum Initial Capital Requirements in Discrete-Time Insurance Surplus Models

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Abstract: - This study determines the requisite minimum initial capital for an insurance company to prevent bankruptcy, ensuring that the ruin probability does not exceed 0.01. We analyze discrete-time insurance surplus models across three categories: non-investment, bond investment, and stock investment. We examine the minimum initial capital necessary for non-investment, bond investment, and stock investment across various safety loadings. The ruin probabilities of three models are obtained through simulation, whereas the minimum initial capital is calculated using the linear least squares method. The results demonstrate that for safety loading levels ranging from 0.1 to 0.4, increasing by 0.1, the model's stock investment necessitates a reduced minimum initial capital in comparison to both non-investment and bond investment. For safety loading values ranging from 0.5 to 1.0, with increments of 0.1, the minimum initial capital for the three types is nearly identical.

Key-Words: - ruin probability, discrete-time insurance surplus model, minimum initial capital requirements.

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1 Introduction

The assessment of discrete-time insurance surplus models encompasses many different aspects. In [1], [2], the authors examine the standard discrete-time surplus process, establishing the fundamental theory, determining an adjustment coefficient known as the Lundberg exponent, and establishing a bound on the ruin probability. The procedure typically develops as follows:

The surplus
= the initial capita + the premium − the claims.

The assumptions suggest that the claims take place during the times when $T_i, T_1 \le T_2 \le, \ldots,$ it is satisfied. We refer to these as claim times. The i-th claim occurs at time T_i and currently has a size of Y_i.. The claim time process $\{T_i\}$ and the claim size process $\{Y_i\}$ are independent of each other.

An insurance company has to manage the minimum initial capital (MIC) needed for preventing bankruptcy, generally through a study of ruin probability.

The study [3] presents the compound binomial model as follows:

$$U(n) = u + n - \sum_{k=1}^{N(n)} Y_k,\ \ U(0) = u;\ \ n \in \mathbb{N},$$

where u represents the initial capital, the premiums for each period are one, $N(n)$ denotes the number of claims in the initial n period, which is assumed to follow a binomial process, and $\{Y_i\}$ consists of a collection of non-negative random variables that are independent and identically distributed (i.i.d.).

The function

$$\Phi(u) = P\big(\{U(n) < 0 \text{ for some } n \in \mathbb{N}\} \mid U(0) = u\big)$$

It is referred to as the ruin probability, indicating the likelihood of the event U descending below zero for the very first time. He formulated recursive equations for the ruin probabilities.

The study [4] examines a discrete-time insurance risk model defined by claims that follow a heavy-tailed distribution, appropriate for situations with significant inequality or rare but catastrophic events. They propose asymptotic outcomes for ruin probability within this model. The study [5] proposes a discrete-time surplus model in which claims occur daily. The model can be written as:

$$U(n) = u + cn - \sum_{k=1}^{n} Y_k, \ \ U(0) = u; \ \ n \in \mathbb{N}, \qquad (1)$$

where c is the premium rate, and $\{Y_i\}$ is a set of non-negative random variables that are independent and have the same distribution. 2 Examine the following instances of $\{Y_i\}$: In the first instance $\{Y_i\}$, it can be considered an exponential random variable, with the density function expressed as

$$f(y) = \alpha e^{-\alpha y}.$$

They derive the recursive and explicit formulas for the ruin probability as follows. For each, $n \in \mathbb{N}$, the formula is:

$$\Phi_n(u) = \Phi_{n-1}(u) + \frac{\left(\alpha c_n(u)\right)^{n-1}}{(n-1)!} e^{-\alpha c_n(u)} \frac{c_1(u)}{c_n(u)}$$
$$= \sum_{k=1}^{n} \frac{\left(\alpha c_k(u)\right)^{k-1}}{(k-1)!} e^{-\alpha c_k(u)} \frac{c_1(u)}{c_k(u)}$$

where

$$c_n(u) = u + nc.$$

In the second instance, $\{Y_i\}$ it is identified as the geometric random variable, with the mass function expressed as:

$$p_k = P(Y = k) = pq^k, \ k \geq 0,$$

$p + q = 1, 0 < p < 1.$ To simplify, they establish the premium rate c as 1 in model (1) and formulate the recursive formula for ruin probabilities accordingly.

$$\Phi_1(u) = q^{u+2}$$
$$\Phi_{n+1}(u) = \Phi_n(u) + h_{n+1}(u),$$

for $n \geq 2,$ where $h_{n+1}(u)$ is defined by

$$h_{n+1}(u)$$
$$= p^n q^{d(u)+n}$$
$$\times \frac{d(u)[d(u)+(n+1)][d(u)+(n+2)]\ldots[d(u)+(2n-1)]}{n!},$$

where $d(u) = u + 2.$

Substituting $T_n = n$ and $Z_n = T_n - T_{n-1}, \ n \in \mathbb{N}, T_0 = 0$ into equation (1) yields

$$U_n = U_{n-1} + cZ_n - Y_n, U_0 = u. \qquad (2)$$

The study [6] builds on the outcomes of [5]. The collection $\{Y_i\}$ is considered an arbitrary random variable, consequently avoiding the need to classify it as either an exponential or geometric random variable. They derive the recursive formula for ruin as follows:

$$\Phi_n(u) = \Phi_1(u) + \int_{-\infty}^{u+c} \Phi_n(u + c - y) dF_{Y_1}(y), n \in \mathbb{N}.$$

In a discrete-time surplus model integrating investment, [7], [8], include the constant interest rate r in their model, which is represented as

$$U_n = U_{n-1}(1+r) - Y_n, \ \ U_0 = u, \ n \in \mathbb{N}.$$

This model presents the recursive form and Lundberg's bounds for the ruin probabilities. Using stochastic control and convex optimization, [9] demonstrated how reinsurance improves an insurer's solvency, a significant component of ruin theory.

It is noted that in the study conducted in [3], [4], [5], [6], [7], [8], [9], the exact calculation of the ruin probability cannot be derived. Furthermore, [8] exclusively employs a constant interest rate. In a discrete-time surplus process, [10] examines the minimum initial capital of investment when r is a constant interest rate, similar to what [7], [8] studied. The study [10] separated claims using the 50th, 60th, 70th, and 80th percentiles criteria, which represents one of the differences.

This research examines model (2) under the condition of daily claims occurrence ($Z_n = 1$) and the discrete-time surplus model incorporating investment, where r is both a constant and an arbitrary variable. Owing to the intricacy of computing ruin probability, we employ the simulation method for its estimation. Our models are delineated in equations (3), (4), and (5), respectively.

$$U_n = U_{n-1} + c - Y_n, \ \ U_0 = u, \ n \in \mathbb{N}, \qquad (3)$$

$$U_n = U_{n-1}(1+r) + c - Y_n, \ \ U_0 = u, \ n \in \mathbb{N}, \qquad (4)$$

$$U_n = U_{n-1}(1+r_n) + c - Y_n, \ \ U_0 = u, \ n \in \mathbb{N}. \qquad (5)$$

where c is the premium rate, $\{Y_n, n \in \mathbb{N}\}$ denotes the set of i.i.d., $r = (1+r_0)^{1/365} - 1$ with r_0 being the annual compound interest rate of 0.02, and r_n represents the rate of return. Equation (3) describes a discrete-time insurance surplus model void of investment. Whereas equation (4) delineates the discrete-time insurance surplus model with investment in a risk-free asset (bond or fixed account), equation (5) represents the discrete-time insurance surplus model with investment in a risky asset (stock).

2 Methodology

2.1 Estimation of Parameters

2.2.1 Estimation of Claims Parameters

This research utilizes data on daily automobile insurance claims from an insurance company. The aggregate claims for 365 days are illustrated in Figure 1.

The 356 claims Y_n are accepted in a log-normal distribution $\mu = 11.257, \sigma = 0.95923$ at a 95% confidence level.

2.2.2 Estimation of Rate of Return Parameters

The rate of return r_n, , as stated in equation (5), is defined by:

$$r_n = \frac{S_n - S_{n-1}}{S_{n-1}},$$

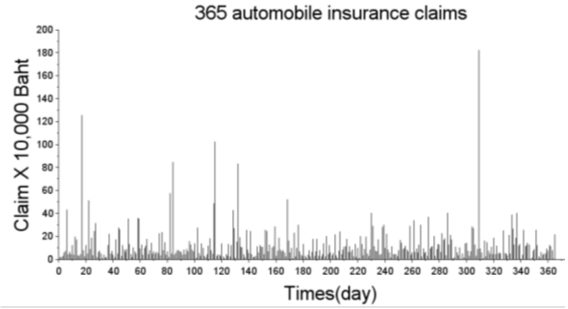

Fig. 1: The 365 Claims of automobile insurance
Source: created by the authors

where S_n represents the stock price on the nth day. Our research involves making investments in SCB stock, which pertains to the financial services and banking sector. SCB is a prominent commercial bank in Thailand, and according to Morgan Stanley's 2024 report, it has received an overweight rating (bullish), its value is deemed attractive relative to regional competitors, and digital banking is projected to account for approximately 30% of total income by 2026. The rate of returns from July to December 2004 is accepted as a logistic distribution $\mu = 9.9187 \times 10^{-4}$ $\sigma = 0.00537$ at a 95% confidence level.

2.2 Simulation

The relationship between the initial capital and the ruin probability of equations (3), (4), and (5) will be determined in this section. We calculate the

$$E(Y_n) = e^{11.257 + \frac{1}{2}(0.95923)^2} = 122,646.4489,$$

Keeping in mind that Y_n it is the log-normal distribution. According to the expected premium principle,

$$c = (1+\theta)\frac{EY_n}{EZ_n} = (1+\theta)EY_n, = (1+\theta)122,646.4489,$$

where θ is a safety loading, the premium rate c can be calculated using equations (3), (4), and (5).

We have set the initial capital at u = 0, 20000, 1980000 Baht, and the number of simulations at 40000. In addition, we modify the safety loading θ to values ranging from 0.1 to 1 in increments of 0.1. Figure 2 illustrates the flowchart for simulating equation (3). The flowchart for the simulation of equation (4) is similar to that of equation (3); however, we modify the step $U_i = U_{i-1} + c - Y_i$ by including the interest rate r, , resulting in

$$U_i = U_{i-1}(1+r) + c - Y_i,$$

where r is as previously defined in equation (4). Figure 3 illustrates the flowchart for the simulation of equation (5).

Figures 4, 5, and 6, respectively, display the outcomes of the simulation for equations (3) to (5).

Figures 4, 5, and 6 illustrate that the relationship between initial capital and ruin probabilities is described by the exponential function.

$$y = ae^{-bu} \tag{6}$$

where y represents the ruin probability, u denotes the initial capital, and a and $b > 0$ are constants.

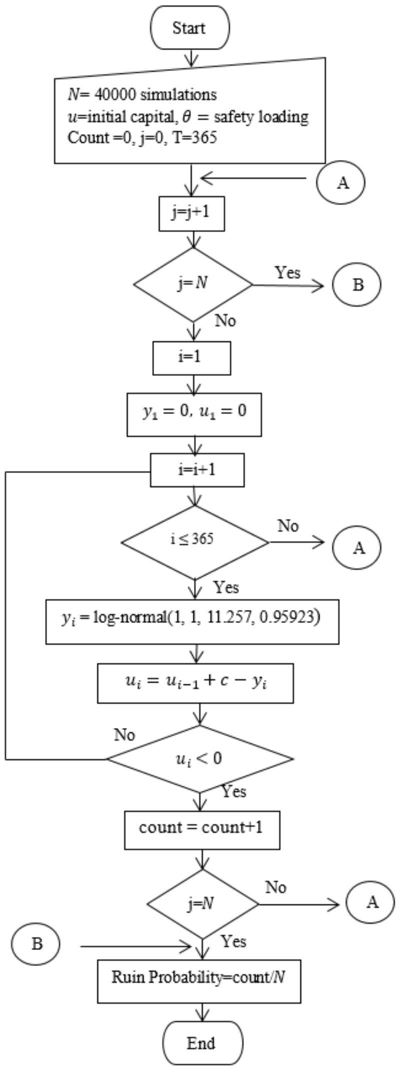

Fig. 2: The flowchart for simulating equation (3)
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We use the linear least squares regression method to calculate a and b. For a data set $(x_1, y_1), (x_2, y_2), \ (x_n, y_n)$, if we aim to forecast y_i using the linear equation $s = mt + c$. We compute

$$\begin{pmatrix} m \\ c \end{pmatrix} = \begin{pmatrix} \dfrac{n\sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \\ \dfrac{\left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} x_i y_i\right)}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \end{pmatrix}.$$

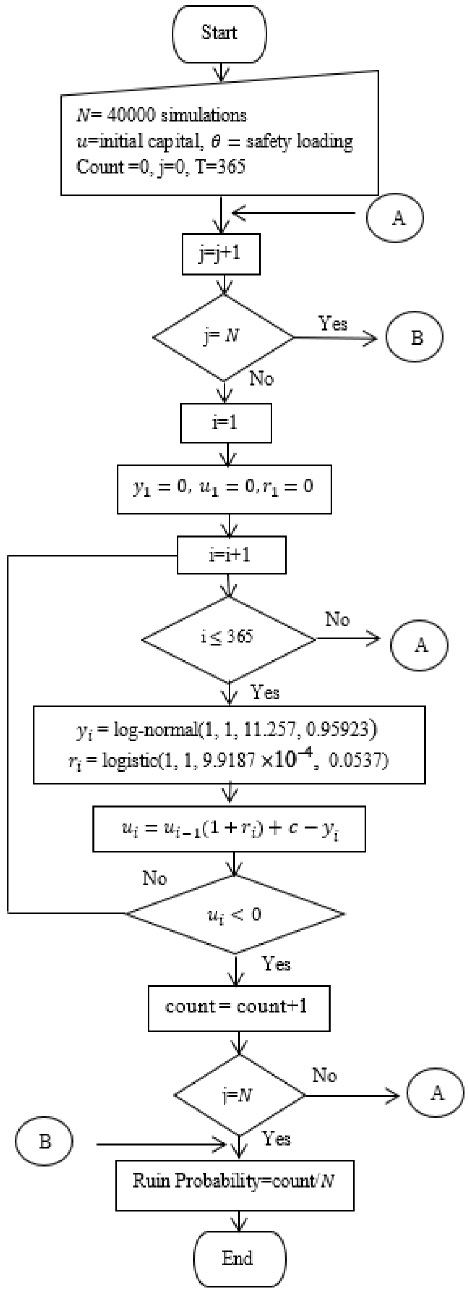

Fig. 3: The flowchart for simulating equation (5)
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We apply the natural logarithm function to both sides of (6), and the result is:

$$\ln y = -bu + \ln a.$$

By defining $s = \ln y,\ m = -b,$ and $c = \ln a$, we obtain:

$$m = \frac{n\sum_{i=1}^{n} u_i \ln y_i - \left(\sum_{i=1}^{n} u_i\right)\left(\sum_{i=1}^{n} \ln y_i\right)}{n\sum_{i=1}^{n} u_i^2 - \left(\sum_{i=1}^{n} u_i\right)^2}$$

and

$$c = \frac{\left(\sum_{i=1}^{n} u_i^2\right)\left(\sum_{i=1}^{n} \ln y_i\right) - \left(\sum_{i=1}^{n} u_i\right)\left(\sum_{i=1}^{n} u_i \ln y_i\right)}{n\sum_{i=1}^{n} u_i^2 - \left(\sum_{i=1}^{n} u_i\right)^2}$$

Recall that $m = -b$, and $c = \ln a$; so we derive.

$$b = -\frac{n\sum_{i=1}^{n} u_i \ln y_i - \left(\sum_{i=1}^{n} u_i\right)\left(\sum_{i=1}^{n} \ln y_i\right)}{n\sum_{i=1}^{n} u_i^2 - \left(\sum_{i=1}^{n} u_i\right)^2},$$

$$a = \exp\left(\frac{\left(\sum_{i=1}^{n} u_i^2\right)\left(\sum_{i=1}^{n} \ln y_i\right) - \left(\sum_{i=1}^{n} u_i\right)\left(\sum_{i=1}^{n} u_i \ln y_i\right)}{n\sum_{i=1}^{n} u_i^2 - \left(\sum_{i=1}^{n} u_i\right)^2}\right).$$

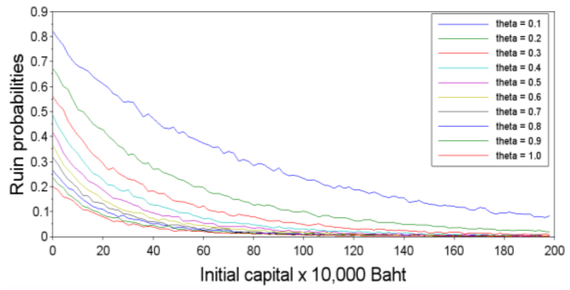

Fig. 4: The correlation between initial capital and the ruin probability in equation (3)
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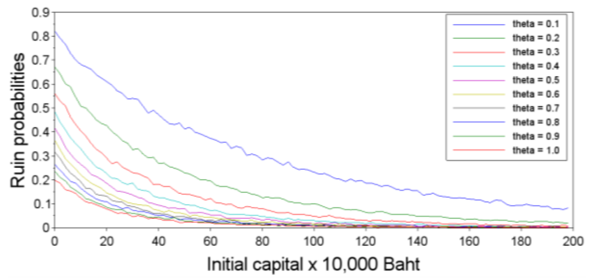

Fig. 5: The correlation between initial capital and the ruin probability in equation (4)
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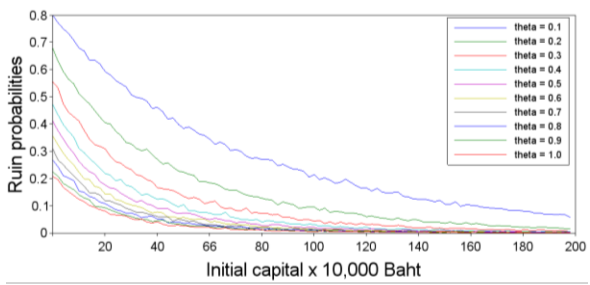

Fig. 6: The correlation between initial capital and the ruin probability in equation (4)
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2.3 Minimum Initial Capital

This section will determine the minimum initial capital (MIC) via equation (6), provided that the insurance company requires the ruin probability y, to be less than or equal to α, , implying that

$$ae^{-bu} \le \alpha. \qquad (7)$$

By applying the natural logarithm to both sides of (7) and resolving the inequality, we derive that

$$u \ge \frac{1}{b}(\ln a - \ln \alpha).$$

Consequently, the MIC is equal to

$$\frac{1}{b}(\ln a - \ln \alpha). \qquad (8)$$

3 Results

Utilizing equation (8), we ascertain the MIC by modifying the safety loading θ from 0.1 to 1 in increments of 0.1. This study references equation (3), which represents the non-investment model; equation (4), which illustrates the investment model in a risk-free asset (bond); and equation (5), which pertains to the investment model in a risky asset (SCB stock). The insurance company requires the ruin probability to remain below $\alpha = 0.01$; the results are presented effectively in Table 1, Table 2, and Table 3, which also indicate that R^2 falls between [0.9680, 0.9980], [0.9738, 0.9967], and [0.9703, 0.9963], respectively. Figure 7 illustrates the MIC as a function of varying θ for equations (3) to (5).

Table 1. MIC(Baht) of equation (3), $\alpha = 0.01$.

safety loading θ	a	b	MIC
0.1	0.77714150	0.00000120	3,627,531.13
0.2	0.60675412	0.00000186	2,207,278.79
0.3	0.47947494	0.00000230	1,682,655.02
0.4	0.38262212	0.00000258	1,412,582.47
0.5	0.31474641	0.00000277	1,245,192.12
0.6	0.26880982	0.00000296	1,111,965.89
0.7	0.22259423	0.00000301	1,030,819.08
0.8	0.19055878	0.00000303	972,731.22
0.9	0.16192232	0.00000307	907,013.56
1.0	0.14483456	0.00000319	837,933.24

Source: created by the authors

Table 2. MIC(Baht) of equation (4), $\alpha = 0.01$.

safety loading θ	a	b	MIC
0.1	0.77351587	0.00000120	3,623,634.24
0.2	0.60481967	0.00000187	2,193,767.52
0.3	0.47910269	0.00000230	1,682,317.33
0.4	0.39232337	0.00000260	1,411,346.67
0.5	0.32199833	0.00000282	1,231,191.94
0.6	0.26337233	0.00000289	1,131,828.25
0.7	0.22656736	0.00000305	1,023,302.03
0.8	0.19747327	0.00000313	953,040.94
0.9	0.17083458	0.00000322	881,400.82
1.0	0.14378240	0.00000318	838,275.46

Source: created by the authors

Table 3. MIC(Baht) of equation (5), $\alpha = 0.01$.

safety loading θ	a	b	MIC
0.1	0.767910640	0.00000130	3,339,298.68
0.2	0.600264860	0.00000192	2,132,700.99
0.3	0.480427330	0.00000238	1,626,928.94
0.4	0.381096920	0.00000263	1,384,208.61
0.5	0.318281400	0.00000284	1,218,433.38
0.6	0.271995910	0.00000300	1,101,067.31
0.7	0.225414430	0.00000304	1,024,788.00
0.8	0.196617460	0.00000315	945,611.09
0.9	0.164556180	0.00000313	894,781.77
1.0	0.146912850	0.00000322	834,551.07

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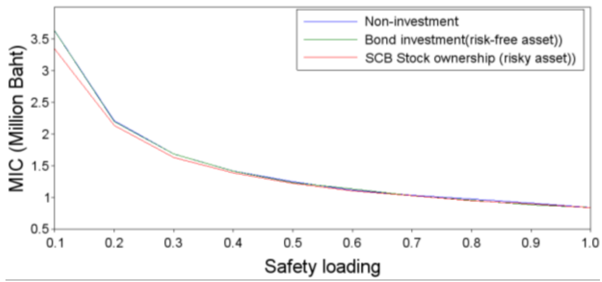

Fig. 7: The MIC of equations (3), (4), and (5).
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Figure 8 is obtained by enlarging the graph presented in Figure 7. The objective is to present MICs comprising three cases.

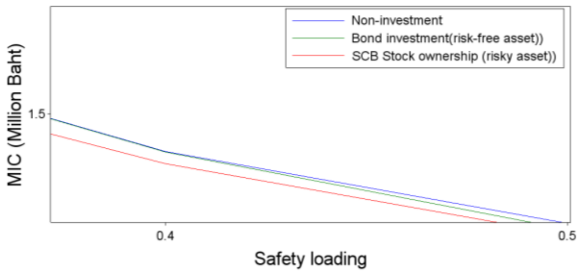

Fig. 8: The MIC of equations (3), (4), and (5) for θ 0.4 and 0.5
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4 Conclusion

The aim is to present MICs that contain three cases. From Table 1, Table 2, Table 3, and Figure 7, we may ascertain whether the insurance company necessitates reserving the MIC, given that the ruin probability does not exceed 0.01. With a $\theta = 0.01$, the insurance company ought to invest in SCB stock, since the marginal investment cost (MIC) is lower compared to refraining from investment and investing in bonds. The return on investment in SCB, as opposed to non-investment: $\dfrac{3,627,531.13 - 3,339,298.68}{3,339,298.68} \times 100 = 7.9\%$. For θ values of 0.2 and 0.3, the insurance company should likewise invest in SCB; however, the returns are diminishing. In fifty percent of the instances $\theta = 0.1$, the returns are 3.38% and 3.31%, respectively. For θ a value of 0.4, the insurance company should further invest in SCB shares, yielding a return of 2.01%. Ultimately, when the θ range is from 0.5 to 1.0, the insurance company should refrain from investment, as the MIC for the three scenarios—non-investment, risk-free asset investment, and risky asset investment—are closely aligned.

We will compute MIC if the claims are not independent, which is an intriguing topic for future research.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed to the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflict of Interest

The authors have no conflict of interest to declare.