

A Quota Approach to Degressively Proportional Allocation

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Abstract: - The study of degressively proportional allocations has attracted growing attention since degressive proportionality became the guiding principle for determining the composition of the European Parliament. One of the key unresolved practical issues is the identification of a single allocation among all admissible ones in a manner supported by a sound theoretical justification. Although this problem has been widely discussed in the literature, no generally accepted method has yet been established that combines objectivity, reproducibility, and theoretical consistency. This paper contributes to the debate by introducing a method founded on an extension of the notion of quota sequences, familiar from the theory of proportional apportionment, to integer-valued degressively proportional allocations satisfying given boundary conditions. The resulting allocation may then be derived through procedures analogous to those used in classical proportional allocation methods. Owing to its general formulation, the proposed approach is not limited to the allocation of seats in the European Parliament, but can also be applied to a broad spectrum of allocation problems involving the distribution of goods, resources, or responsibilities.

Key-Words: - apportionment problem, degressive proportionality, fair division, elections, European Parliament, voting

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1 Introduction

The principle of proportionality is one of the most commonly applied rules in the distribution of goods and obligations. In the case of divisible goods, this principle leads to the allocation of shares proportional to the entitlements of the participants involved in the distribution process. When indivisible goods are considered, such as seats in representative or collegial bodies, the final allocation is determined through specific approximation or rounding procedures applied to the proportional quotas.

The analysis of the properties of proportional divisions is not a new issue. The literature includes numerous works developed over more than two hundred years. In addition to articles, the authors' contributions also include extensive monographs, among which at least two are worth mentioning. One is [1], and the other is [2].

In practice, the proportionality principle functions satisfactorily when differences among the entitlements of participants are relatively small. However, when these disparities become substantial, the application of strict proportionality may lead to undesirable consequences, including the

effective exclusion of certain participants from the allocation process.

Such a situation arose in the distribution of seats in the European Parliament among the Member States of the European Union. The significant demographic disparity between the largest and the smallest Member States — namely Germany and Malta, respectively — combined with a chamber size fixed at 751 members and the application of a strictly proportional allocation system, would result either in Malta being deprived of parliamentary representation or in that country being assigned only a single seat. Given the political significance of the European Parliament and the principle of solidarity among Member States, the Treaty of Lisbon introduced an alternative allocation principle known as degressive proportionality. Article 14(2) of the Treaty of Lisbon [3] states: “The European Parliament shall be composed of representatives of the Union’s citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats.”

In the Treaty of Lisbon, the principle of degressive proportionality was referred to only by

name, without providing details as to what it actually entails. This was later supplemented in Article 1 of European Council Decision 2023/2061, [4], where degressive proportionality was defined as follows: “Degressive proportionality is defined as follows: the ratio between the population and the number of seats of each Member State before rounding up or down to the nearest whole number is to vary in relation to their respective populations in such a way that each Member of the European Parliament from a more populous Member State represents more citizens than each Member of the European Parliament from a less populous Member State and, conversely, that the larger the population of a Member State, the greater its entitlement to a large number of seats in the European Parliament.”

The essence of degressive proportionality lies in the principle that the number of citizens represented by a single Member of Parliament increases with the population of the respective Member State. However, the formulation contained in the Treaty remains too imprecise to determine a unique allocation in the same manner as proportional apportionment based on explicitly calculated quotas. Consequently, despite the Treaty having entered into force in 2009, no universally accepted method or algorithm has yet been established that would enable the regular and objective implementation of degressively proportional seat allocation. As a result, the composition of the European Parliament for the 2024–2029 parliamentary term was once again determined through political negotiations rather than by the application of a formally defined allocation mechanism.

Research on degressive proportionality has focused primarily on identifying one of the many possible allocations that may be applied in practice. Part of this research has been conducted under the auspices of the Committee on Constitutional Affairs and has resulted in two significant proposals, namely the Cambridge Compromise [5] and the Power Compromise [6]. In both cases, the underlying idea is to adapt divisor methods of proportional apportionment to the conditions of degressively proportional allocation. A third noteworthy proposal within this group is that of Muler [7], who modifies the Power Compromise by additionally proposing that part of the seats in the Parliament be allocated according to a transnational party-based criterion.

In the scientific literature, particular attention should be paid to the special issue of the journal *Mathematical Social Sciences*, in which several articles devoted to the problem of degressively proportional allocations were collected. Among

these, papers [8], [9], [10], and [11] deserve special mention. In the first of these works, the authors approach the problem from a general perspective and provide a comprehensive analysis of allocation functions that may serve as a basis for constructing degressively proportional allocations. The subsequent three papers present proposals for specific allocation methods. As works of major importance for the development of this line of research, published outside the aforementioned special issue of *Mathematical Social Sciences*, at least four papers should be indicated: [12], [13], [14], and [15]. These studies represent different approaches to solving the problem and employ a variety of mathematical foundations, ranging from discrete mathematics and relation theory to classical methods of mathematical analysis.

Of particular importance for the considerations presented in this article is paper [16], in which the authors propose the use of the LaRSA algorithm to identify degressively proportional allocations. The algorithm makes it possible to determine all allocations satisfying given boundary conditions. The LaRSA algorithm was also employed in studies [17] and [18]; in the latter, it was applied to implement proportional divisor methods within the framework of degressively proportional allocations. A similar idea is presented in the current study. In the proposed method for determining degressively proportional allocations, the LaRSA algorithm is likewise employed; however, the starting point is the second major group of proportional allocation methods, namely quota methods.

2 Problem Formulation

According to the principle of proportional distribution, the participants involved in the allocation process share the distributed good in proportion to their respective characteristics or entitlements. Although these quantities are typically represented numerically, their interpretation may vary substantially depending on the context. For instance, in the distribution of a company's profits, the relevant criterion is the number of shares held by each shareholder; in electoral representation systems, it is the population size of the respective electoral constituencies; whereas in the allocation of the costs of a joint investment, the relevant quantities may be determined by the opportunity costs borne by the individual investors.

Let a sequence of positive numbers $\mathbf{P} = (p_1, p_2, \dots, p_n)$ represent entitlements assigned to n agents. A corresponding allocation $\mathbf{S} =$

$(s_1, s_2, \dots, s_n)$ is said to be proportional with respect to $\mathbf{P}$ if and only if $\frac{s_i}{p_i} = \frac{H}{V}$ for every i , where $H = s_1 + s_2 + \dots + s_n$ and $V = p_1 + p_2 + \dots + p_n$. In the case of indivisible goods, it is additionally required that all terms of $\mathbf{S}$ be integers. The sequence $\mathbf{Q} = (q_1, q_2, \dots, q_n)$, where $q_i = \frac{p_i H}{V}$ provides a uniquely determined allocation under the proportionality principle. Consequently, the only difficulty in this setting arises from the necessity of approximating the fractional vector $\mathbf{Q}$ by an integer allocation when indivisible goods are distributed.

The situation is fundamentally different under the principle of degressive proportionality. An allocation $\mathbf{S} = (s_1, s_2, \dots, s_n)$ is called degressively proportional with respect to a nondecreasing sequence of positive entitlements $\mathbf{P} = (p_1, p_2, \dots, p_n)$ if and only if the following conditions hold:

- (1) $s_i \leq s_{i+1}$ holds for every $1 \leq i < n$, and
- (2) $s_{i+1}/p_{i+1} \leq s_i/p_i$ holds for every $1 \leq i < n$ [4].

As can be readily observed, condition (2), in contrast to proportional allocation, replaces the equality relation with a weak inequality. This relaxation leads to a loss of uniqueness in the resulting allocation rule and, consequently, generates significant challenges in its practical implementation. In general, it is not possible to identify a single, objectively determined solution without imposing additional criteria or constraints.

In the analysis of apportionment problems, an additional assumption commonly adopted is the integrality of the allocation. In certain situations, such as the allocation of seats in the European Parliament, so-called boundary conditions of the allocation are additionally introduced, specifying a minimum value m and a maximum value M for the good being distributed:

- (3) $s_1 = m$ and $s_n = M$ [3].

Although in the Treaty of Lisbon these conditions are formulated in the form of the inequalities $s_1 \geq m$ and $s_n \leq M$, in accordance with the recommendations of the Committee on Constitutional Affairs, they have been treated in all theoretical considerations and practical applications as equalities (3).

It turns out that even under such additional constraints, there may exist a large number of integer sequences satisfying conditions (1)–(3). For example, in the practical problem of allocating seats in the European Parliament for the 2024–2029 term, where the sequence of entitlements is given by the population sizes of the Member States in 2023, it is possible to identify more than eight hundred million

admissible allocations. Under the currently binding legal provisions, however, there exists no objective and unambiguous method for selecting a unique solution to this problem. The set of all feasible integer allocations $\mathbf{S} = (s_1, s_2, \dots, s_n)$ of H goods satisfying conditions (1)–(3) for a given sequence $\mathbf{P}$ will be denoted by $DP(\mathbf{P}, H, m, M)$.

The aim of the present paper is to propose one of the many possible solutions to the problem of the ambiguity of the degressively proportional allocation rule. A characteristic feature of this approach is its strong connection to the classical quota methods of proportional allocation. By appropriately defining the notion of a quota for degressively proportional division, a reference point is obtained that serves to determine, on the basis of well-known and widely applied rules in apportionment problems, a unique allocation. The proposed modification concerns Hamilton's largest remainder method, which is both well known and applied in practice.

3 Problem Solution

The selection of representation on the basis of quota methods [19], [20] is carried out through a two-stage procedure. In the first stage, a fixed divisor q is chosen. In the classical setting, this divisor is interpreted as the number of voters or votes corresponding to a single seat, that is, $q = q_H = \frac{V}{H}$. The divisor q_H defined in this manner is referred to as the simple quota or Hare quota. Another proposal known from electoral practice is the so-called shifted quota, of which the simple quota constitutes a special case. The shifted quota is defined as the quotient $q_t = \frac{V}{H+t}$, where $t \in [-1, 1)$. It is easy to observe that the simple quota is equal to the shifted quota for $t = 0$. A related concept is the Droop quota q_D , whose origin lies in the search for such a number of votes that, in the election of a single candidate, would guarantee that candidate a majority victory. Clearly, this number is equal to one half of the votes cast. Analogously, in the case of electing two candidates, the number of votes cast should be divided by three. In the general case, that is, in the allocation of H seats, this quantity is given precisely by the Droop quota, whose value is $q_D = \frac{V}{H+1}$.

After selecting and fixing a given quota q , a sequence of quotas $\mathbf{Q} = (q_1, q_2, \dots, q_n) = (qp_1, qp_2, \dots, qp_n)$, is determined, where the individual terms represent the theoretical allocation

of goods to the respective participants of the distribution in accordance with the adopted proportionality rule. The elements of the sequence $\mathbf{Q}$ need not be integers; therefore, in the case of integer allocations — such as the distribution of seats in collective representative bodies — it is necessary to determine an integer-valued representation of this sequence.

An integer representation of a quota sequence can be obtained using a variety of methods, the best known and most widely used in practice being Hamilton's largest-remainder method. Alexander Hamilton, U.S. Secretary of the Treasury, proposed this method as a rule for apportioning seats in the House of Representatives. First, the components of the previously obtained quota sequence $\mathbf{Q}$ are rounded down, yielding the sequence $\lfloor \mathbf{Q} \rfloor = (\lfloor q_1 \rfloor, \lfloor q_2 \rfloor, \dots, \lfloor q_n \rfloor)$, where $\lfloor x \rfloor$ denotes the greatest integer not exceeding x . Thus, each agent i is initially assigned $\lfloor q_i \rfloor$ goods. With this rounding rule, it is evident that each agent receives at least one good, and that $\lfloor q_1 \rfloor + \lfloor q_2 \rfloor + \dots + \lfloor q_n \rfloor \leq H$. If $\lfloor q_1 \rfloor + \lfloor q_2 \rfloor + \dots + \lfloor q_n \rfloor = H$, then the allocation is given by the sequence $\mathbf{S} = \lfloor \mathbf{Q} \rfloor$. If, however, $\lfloor q_1 \rfloor + \lfloor q_2 \rfloor + \dots + \lfloor q_n \rfloor < H$, the remaining $H - \lfloor q_1 \rfloor - \lfloor q_2 \rfloor - \dots - \lfloor q_n \rfloor$ goods must be distributed. This constitutes the second stage of the procedure. According to Hamilton's method, the remaining goods are allocated one at a time in descending order of the remainders $q_i - \lfloor q_i \rfloor$, starting with the largest remainder and continuing until all H goods have been allocated. Ultimately, allocation $\mathbf{S}^{HAM} = \lfloor \mathbf{Q} \rfloor + \mathbf{R} = (\lfloor q_1 \rfloor + r_1, \lfloor q_2 \rfloor + r_2, \dots, \lfloor q_n \rfloor + r_n)$, where $r_i = 0$ or $r_i = 1$, depending on the remainder $q_i - \lfloor q_i \rfloor$ and the number $H - \lfloor q_1 \rfloor - \lfloor q_2 \rfloor - \dots - \lfloor q_n \rfloor$ of goods remaining to be allocated.

An important result for the considerations that follow is a theorem establishing a key optimization property of the allocation $\mathbf{S}^{HAM}$ obtained by applying Hamilton's largest-remainder method. It turns out that, when the Euclidean distance, or equivalently the ℓ_2 -norm, is used as the criterion of fit, Hamilton's largest-remainder method provides the best approximation to the ideal proportional allocation represented by the quota vector $\mathbf{Q}$ [21].

Formally, this means that

$$\mathbf{S}^{HAM} = \arg \min_{\mathbf{A} \in S_H} \sum_{i=1}^n (a_i - q_i)^2,$$

where $S_H = \{\mathbf{A} = (a_1, a_2, \dots, a_n) : a_i \in \mathbb{Z} \wedge a_1 + a_2 + \dots + a_n = H\}$.

The Hamilton rule is the most popular and the most frequently applied method for implementing

the second stage of allocation under quota methods. It is, however, by no means the only possible approach. Apart from the fairly natural solution according to which “the winner takes all,” that is, assigning all remaining units of the good to the agent who obtained the largest number in the first stage, one may also encounter practical implementations in which the distribution of the remaining seats takes into account the “unused” votes or entitlements. Let $\mathbf{S}^1 = (s_1^1, s_2^1, \dots, s_n^1)$ denote the allocation of goods obtained in the first stage. Subsequently, for the quota q applied in the first stage, the sequence of “unused” entitlements is determined as $\mathbf{P}^{UVP} = (p_1 - s_1^1 q, p_2 - s_2^1 q, \dots, p_n - s_n^1 q)$. The quantity $s_i^1 q$ represents the number of entitlements or votes required by agent i to obtain s_i^1 units of the good. Consequently, the difference $p_i - s_i^1 q$ represents the “unused” entitlements, that is, those without which the allocation for agent i would remain unchanged.

The second stage consists in performing an additional allocation in which the sequence of entitlements is replaced by the sequence $\mathbf{P}^{UVP}$. In Greece, where this idea was applied in practice in 2009 to distribute 22 seats in the European Parliament among political parties, the second stage employed the rounded-down Droop quota, and the four seats remaining after the first stage were allocated one by one according to the largest values of the quotients $\frac{p_i^{UVP}}{\lfloor q_D \rfloor}$ [2].

The idea underlying quota methods cannot be directly applied in the case of degressively proportional allocation. The impossibility of meaningfully specifying a single quota q common to all participants in the allocation process necessitates a substantial modification of the first stage. Regardless of its particular form, the quota q , when used in proportional allocation, always embodies the principle of proportionality. The principle of proportional allocation is therefore realized already at the stage of selecting the quota. This is particularly evident in the case of the Hare quota, where $q = \frac{V}{H}$. The remaining elements of the algorithm for determining a specific allocation are merely modifications of the quota sequence $\mathbf{Q} = (q_1, q_2, \dots, q_n) = (qp_1, qp_2, \dots, qp_n)$ necessitated by the requirement of integer-valued allocations. For any given $\mathbf{P}$ and H , the sequence $\mathbf{Q}$ is uniquely determined and represents the theoretical allocation consistent with the principle of proportionality. It therefore constitutes both the benchmark of

proportionality and the basis from which the actual allocation is derived. Consequently, when adapting quota methods to the implementation of the principle of degressive proportionality, attention should primarily be focused on an appropriate modification of the sequence $\mathbf{Q}$.

For certain values of $\mathbf{P}, H, m$ and M , the set $DP(\mathbf{P}, H, m, M)$ may be empty, in which case it is impossible to determine an integer-valued degressively proportional allocation. In practice, however, these parameters are chosen so that such allocations do exist; moreover, the set $DP(\mathbf{P}, H, m, M)$ typically contains many elements. This means that the principle of degressive proportionality is not uniquely determined and, in such cases, permits multiple integer-valued allocations, each of which is equally consistent with the principle. Therefore, in the absence of any additional criteria, the search for a benchmark of degressive proportionality—that is, a sequence $\mathbf{Q}^{DP}$ serving as the counterpart of the sequence $\mathbf{Q}$ in the case of proportional allocation—should take into account all allocations permitted by the principle of degressive proportionality.

Every allocation belonging to the set $DP(\mathbf{P}, H, m, M)$ may constitute a valid solution to a particular allocation problem, which implies that each participant in the allocation process may legitimately expect such an outcome. The expectations concerning both the number of goods received by a given participant and the allocations assigned to others are, for fixed $\mathbf{P}$ and H , fully determined by the boundary conditions m and M , and thus by the set $DP(\mathbf{P}, H, m, M)$. Consequently, if all feasible allocations are known, the average of these allocations may reasonably be regarded as an appropriate reference point for the principle of degressive proportionality. Accordingly, the elements of the sequence $\mathbf{Q}^{DP} = (q_1^{DP}, q_2^{DP}, \dots, q_n^{DP})$ for $i = 1, 2, \dots, n$ may be defined as follows:

$$(4) \quad q_i^{DP} = \frac{1}{\alpha} \sum_{s \in DP(\mathbf{P}, H, m, M)} s_i,$$

where α denotes the cardinality of the set $DP(\mathbf{P}, H, m, M)$, that is, $\alpha = \text{card } DP(\mathbf{P}, H, m, M)$.

The sequence $\mathbf{Q}^{DP}$ of arithmetic means of all possible allocations is uniquely defined and, being an arithmetic mean, clearly satisfies conditions (1)–(3). Moreover, $q_1^{DP} + q_2^{DP} + \dots + q_n^{DP} = H$. As in the case of the sequence $\mathbf{Q}$ of quotas in a proportional allocation, the elements of $\mathbf{Q}^{DP}$ need not be integers. For this reason, in order to obtain an integer allocation, one must specify a method for determining its integer representation. Preserving the underlying idea of Hamilton's largest-remainder method, the desired allocation $\mathbf{s}^{DPHAM}$ can be

obtained by exploiting its characteristic optimization property: among all allocations in $DP(\mathbf{P}, H, m, M)$ we seek one that minimizes the Euclidean distance from the sequence $\mathbf{Q}^{DP}$. Thus, the allocation is taken to be the sequence

$$(5) \quad \mathbf{s}^{DPHAM} = \arg \min_{s \in DP(\mathbf{P}, H, m, M)} \sum_{i=1}^n (s_i - q_i^{DP})^2.$$

It should be emphasized that the use of the optimization property of Hamilton's largest-remainder method to determine the optimal allocation is motivated by the fact that the method cannot always be applied directly. Specifically, if the remaining $H - \lfloor q_1^{DP} \rfloor - \lfloor q_2^{DP} \rfloor - \dots - \lfloor q_n^{DP} \rfloor$ units are added successively to the terms of the sequence $\lfloor \mathbf{Q}^{DP} \rfloor = (\lfloor q_1^{DP} \rfloor, \lfloor q_2^{DP} \rfloor, \dots, \lfloor q_n^{DP} \rfloor)$ starting with the largest remainder, the resulting allocation may fail to satisfy condition (2) and, consequently, may not belong to the set of feasible allocations $DP(\mathbf{P}, H, m, M)$.

It is worth adding that degressively proportional allocations are susceptible to well-known electoral paradoxes. This issue was analyzed in the work of Dniestrzański and the author of *Paradoxes of Degressively Proportional Allocations*, where examples of such situations were identified. The lack of robustness to paradoxes is, of course, a consequence of the strong dependence of the set $DP(\mathbf{P}, H, m, M)$ on the integer parameters $\mathbf{P}, H, m, M$. One can find values of these parameters for which the set $DP(\mathbf{P}, H, m, M)$ is a singleton, in which case it is not difficult to construct examples of paradoxes such as the Alabama paradox, the population paradox, or the new-state paradox.

Consequently, it is difficult to obtain positive results concerning monotonic relationships between the sequence of averages $\mathbf{Q}^{DP}$ and the parameters defining the set $DP(\mathbf{P}, H, m, M)$. The set is finite and therefore the problem is discrete. Given that its cardinality depends on the integer parameters $\mathbf{P}, H, m, M$, and that these parameters consequently affect the calculated averages in a stepwise manner, one should not expect the sequence $\mathbf{Q}^{DP}$ to exhibit particularly interesting analytical properties beyond the obvious optimization-related ones.

3.1 Case study

The above generalization of the Hamilton method was applied to determine a degressively proportional allocation of seats in the European Parliament for the 2024-2029 term. For this

purpose, the same sequence $\mathbf{p}^{EU23}$ of Member States' populations from 2023 was used as that adopted by the Committee on Constitutional Affairs for determining the composition of the European Parliament for the current parliamentary term.

In this term, 720 seats were allocated, and the boundary conditions were the same as those specified in the Lisbon Treaty and treated as binding maxima, namely $s_1 = m = 6$ and $s_n = M = 96$. To determine all possible integer allocations, i.e., the set $DP(\mathbf{p}^{EU23}, 720, 6, 96)$, the LaRSA algorithm [16] was applied. On this basis, it was established that $\alpha = \text{card } DP(\mathbf{p}^{EU23}, 720, 6, 96) = 874025775$ and subsequently, according to formula (4), all terms of the sequence $\mathbf{Q}^{DP}$ were determined.

Table 1. The allocation applied in the 10th parliamentary term and the allocation determined on the basis of the modified Hamilton method

Country	$\mathbf{p}^{EU23}$	$\mathbf{s}^{EU24}$	$\mathbf{Q}^{DP}$	$\mathbf{s}^{DPHAM}$	$\lfloor \mathbf{Q}^{DP} \rfloor + R$
DE	83203320	96	96	96	96+0
FR	67842582	81	81.877	82	81+1
IT	59607184	76	75.396	75	75+0
ES	47432805	61	63.35	63	63+0
PL	37654247	53	53.768	54	53+1
RO	19038098	33	29.63	30	29+1
NL	17734036	31	28.645	29	28+1
BE	11631136	22	20.668	21	20+1
GR	10603810	21	19.931	20	19+1
CZ	10545457	21	19.931	20	19+1
SE	10440000	21	19.931	20	19+1
PT	10352042	21	19.931	20	19+1
HU	9689010	21	19.367	19	19+0
AT	8967500	20	18.82	19	18+1
BG	6838937	17	16.514	17	16+1
DK	5864667	15	15.471	15	15+0
FI	5541241	15	15.461	15	15+0
SK	5434712	15	15.461	15	15+0
IE	5060004	14	15.036	15	15+0
HR	3862305	12	13.443	13	13+0
LT	2805998	11	11.815	12	11+1
SI	2107180	9	10.596	11	10+1
LV	1875757	9	10.156	10	10+0
EE	1331796	7	8.83	9	8+1
CY	904700	6	7.485	7	7+0
LU	643648	6	6.487	7	6+1
MT	520971	6	6	6	6+0
Total	447533143	720	720	720	705+15

Source: Own study based on Eurostat data.

Next, once again using the LaRSA algorithm, the allocation $\mathbf{s}^{DPHAM}$ was determined in accordance with formula (5). The results of these calculations, compared with the currently binding allocation $\mathbf{s}^{EU24}$, are presented in Table 1.

It is worth noting that in this case the final stage did not require the application of the LaRSA algorithm. The obtained result is identical to that

produced by the classical Hamilton method, namely by distributing $720 - \lfloor q_1^{DP} \rfloor - \lfloor q_2^{DP} \rfloor - \dots - \lfloor q_{27}^{DP} \rfloor = 15$ seats successively according to the largest remainders of the sequence $\mathbf{Q}^{DP}$ as shown in column $\lfloor \mathbf{Q}^{DP} \rfloor + R$ of Table 1.

4 Conclusion

The sequence $\mathbf{Q}^{DP}$, consisting of the arithmetic means of all possible allocations, possesses both properties characteristic of the quota sequence $\mathbf{Q}$ of proportional allocation determined on the basis of the Hare quota $q_H = \frac{V}{H}$. For this reason, it may be regarded as a benchmark of degressive proportionality. First, it satisfies conditions (1)–(3), and thus conforms to the principle of degressive proportionality with respect to the sequence $\mathbf{P}$, just as the sequence $\mathbf{Q}$ satisfies the principle of proportionality with respect to the same sequence. Second, the elements of both sequences $\mathbf{Q}$ and $\mathbf{Q}^{DP}$ sum to H , that is, $q_1 + q_2 + \dots + q_n = q_1^{DP} + q_2^{DP} + \dots + q_n^{DP} = H$.

The only, albeit significant, difference is that for fixed $\mathbf{P}$ and H , the sequence $\mathbf{Q}$ is uniquely determined, whereas there are generally many sequences satisfying conditions (1)–(3) for fixed $\mathbf{P}, H, m, M$, whose elements also sum to H . The choice of $\mathbf{Q}^{DP}$ among them is justified by its neutrality with respect to the expectations of the participants in the allocation process. Given knowledge of all feasible allocations, that is, of the set $DP(\mathbf{P}, H, m, M)$, such a choice has the potential to be accepted by all parties involved. The definition of the sequence $\mathbf{Q}^{DP}$ opens the way for the development of new methods of degressively proportional allocation based on the achievements of quota methods in proportional apportionment. The allocation of seats in the 10th term of the European Parliament analyzed in this paper constitutes a particularly clear example of this approach.

Examining this example, one may generally conclude that, for the 10th term of the European Parliament, the allocation obtained using the method proposed in this paper is more advantageous than the currently applied allocation for Member States with medium-sized populations within the European Union. The results presented in Table 1 further demonstrate how knowledge of the sequence $\mathbf{Q}^{DP}$, consisting of the arithmetic means of all feasible allocations, may help avoid certain surprising individual solutions produced through political negotiations. A particularly illustrative example is the assignment of 33 seats to Romania by political

decision. Given the demographic data and the legal provisions determining the allocation parameters P, H, m, M , Romania could have received between 24 and 37 seats in the 10th term of the European Parliament. Consequently, assigning it 33 seats should be regarded as a highly favorable outcome for that country. However, this is not as clearly visible until all feasible allocations are taken into account. It turns out that allocations in which Romania receives 33 or fewer seats constitute less than 6% of all possible allocations; thus, this outcome may be viewed as the acceptance of a marginal solution. A similar situation may be observed in the case of Estonia. In this instance, the assignment of 7 seats in the 10th term may be regarded as unfavorable from the perspective of the average allocation. The feasible range for Estonia extends from 6 to 13 seats in the European Parliament, whereas allocations granting Estonia 7 or fewer seats constitute approximately 10% of all feasible allocations.

In conclusion, it may be stated that the application of the LaRSA algorithm provides knowledge of the entire set of admissible degressively proportional allocations for given values of P, H, m, M . This makes it possible to evaluate individual proposals and, above all, creates the opportunity to formulate objective solutions, for example those based on average allocation values. The method proposed in this paper constitutes a good example of such an approach. Its advantages lie not only in its objectivity, but also in its strong connection with classical quota methods used to solve analogous problems under the principle of proportionality.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work, the author(s) utilized artificial intelligence (AI) technologies solely for the purpose of linguistic refinement and grammatical correction. Following the application of these tools, the author(s) comprehensively reviewed, verified, and edited the resulting text. The author(s) maintain full accountability and responsibility for the accuracy, integrity, and final content of the publication.

References:

- [1] Balinski M. L. & Young H. P., *Fair representation: Meeting the ideal of one man, one vote (2nd ed.)*, Brookings Institution Press, 2001. ISBN: 0-8157-0090-3
- [2] Pukelsheim F. (2017). *Proportional representation: Apportionment methods and their applications (2nd ed.)*, Springer International Publishing AG, 2017. <https://doi.org/10.1007/978-3-319-64707-4>
- [3] Consolidated version of the Treaty on the European Union, *Official Journal of the European Union C 326*, Vol 55, pp. 13-45. <https://doi.org/10.3000/1977091X.C 2012.326.eng>
- [4] European Council Decision (EU) 2023/2061 of 22 September 2023 establishing the composition of the European Parliament, Decision - 2023/2061 - EN - EUR-Lex (europa.eu). <http://data.europa.eu/eli/dec/2023/2061/oj>
- [5] Grimmett, G., Laslier, J.F., Pukelsheim, F., Gonzalez, V., Rose, R., Slomczynski, W., Zachariasen, M., & Życzkowski, K., *The allocation between the EU member states of the seats in the European Parliament, Cambridge Compromise*, 2011. [https://www.europarl.europa.eu/.../IPOL-AFCO_NT\(2011\)432760](https://www.europarl.europa.eu/.../IPOL-AFCO_NT(2011)432760)
- [6] Grimmett, G. & Pukelsheim, F., *Power Compromise. An objective, fair, durable, and transparent fix for the EP composition*. Workshop on Permanent System for the allocation of seats in the European Parliament, 2024. https://www.europarl.europa.eu/.../EP_composition_Pukelsheim.pdf
- [7] Muller M. *Reconciling degressive proportionality and electoral equality through proportional completion*. Workshop on Permanent System for the allocation of seats in the European Parliament, 2024. [https://www.europarl.europa.eu/thinktank/en/document/IPOL_IDA\(2024\)759467](https://www.europarl.europa.eu/thinktank/en/document/IPOL_IDA(2024)759467)
- [8] Słomczyński, W. & Życzkowski, K., Mathematical aspects of degressive proportionality, *Mathematical Social Sciences* 63(2), 2012, pp. 94–101. <https://doi.org/10.1016/j.mathsocsci.2011.12.002>
- [9] Florek, J., A numerical method to determine a degressive proportional distribution of seats in the European Parliament, *Mathematical Social Sciences* 63(2), 2012, pp. 121-129.

- <https://doi.org/10.1016/j.mathsocsci.2011.07.003>
- [10] Kellermann T., The minimum-based procedure: A principled way to allocate seats in the European Parliament, *Mathematical Social Sciences* 63(2), 2012, pp. 102-106. <https://doi.org/10.1016/j.mathsocsci.2011.10.004>
- [11] Serafini, P., Allocation of the EU Parliament seats via integer linear programming and revised quotas, *Mathematical Social Sciences* 63(2), 2012, pp. 107-1113. <https://doi.org/10.1016/j.mathsocsci.2011.08.006>
- [12] Charvat, J., Member States' Representation in the European Parliament: history and present, *Civilia*, 9, 2018, pp. 4-22. <https://doi.org/10.5507/civ.2018.005>
- [13] Dniestrzański, P., The proposal of allocation of seats in the European Parliament - The shifted root, *Procedia - Social and Behavioral Sciences*, 124, 2014, pp. 536-543. <https://doi.org/10.1016/j.sbspro.2014.02.517>
- [14] Haman, J., The concept of degressive and progressive proportionality and its normative and descriptive applications, *Studies in Logic, Grammar and Rhetoric* 50 (63), 2017, pp. 67-91. <https://doi.org/10.1515/SLGR-2017-0019>
- [15] Martinez-Aroza, J. & Ramirez-Gonzalez, V., Several methods for degressively proportional allotments. A case study, *Mathematical and Computer Modelling*, 48, 2008, pp. 1439 - 1445. <https://doi.org/10.1016/j.mcm.2008.05.026>
- [16] Łyko J. & Rudek R. (2013). A fast exact algorithm for the allocation of seats for the EU Parliament, *Expert Systems with Applications*, 40, 2013, pp. 5284-5291. <https://doi.org/10.1016/j.eswa.2013.03.035>
- [17] Lyu, W. & Zhao, L., Axioms and Divisor Methods for a Generalized Apportionment Problem with Relative Equality, *Mathematics*, 11(15), 2023, 3270. <https://doi.org/10.3390/math11153270>
- [18] Cegiełka, K., Łyko, J. & Rudek, R., Beyond the Cambridge Compromise algorithm towards degressively proportional allocation, *Operational Research* 19 (2), 2019, pp. 317-332. <https://doi.org/10.1007/s12351-017-0292-y>
- [19] Balinski, M.L. & Young, H.P., The quota method of apportionment, *Amer. Math. Monthly* 82, 1975, pp. 701-730. <https://doi.org/10.1080/00029890.1975.11993911>
- [20] Gallagher, M., Comparing Proportional Representation Electoral Systems: Quotas, Thresholds, *Paradoxes and Majorities*, *British Journal of Political Science*, 22(4), 1992, pp. 469-496. <https://doi.org/10.1017/S0007123400006499>
- [21] Birkhoff, G., House Monotone Apportionment Schemes, *Proceedings of the National Academy of Sciences of the United States of America*, 73(3), 1976, pp. 684-686. <https://doi.org/10.1073/pnas.73.3.684>

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