New Results and Applications about the Level Crossing Rate of SC Receiver output Signal in the Presence of Gamma Shadowing and k-μ or Rician Multipath Fading

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Abstract: - In this paper, the wireless communication system with dual SC receiver operating over shadowed multipath fading channel is considered. The multipath fading is k-μ or Rician. The received signal experiences the short term fading which is resulting in SC receiver envelope variation and Gamma long term fading resulting in SC receiver envelope power variation. The closed form expressions for joint probability density functions of SC receiver output signal envelope and their first derivative of SC receiver output signal envelope are calculated for both, k- μ and Rician fading. These expressions are used for evaluation of average level crossing rate of SC receiver output signal envelopes. The numerical expressions are plotted to show the effect of Rician fading severity parameter and Gamma shadowing severity parameter on the average level crossing rate of SC receiver output signal envelope.

Key-Words: Systems Theory, Signal Processing, Gamma shadowing, k-μ fading, level crossing rate (LCR), Rician fading, SC receiver, wireless communication systems

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1 Introduction

The short term fading and long term fading degrade average level crossing rate (LCR) and average fade duration of wireless communication system [1]. The received signal of wireless communication system is subjected to multipath fading and shadowing. The short term fading causes signal envelope variation. In shadowed multipath fading environment, received signal envelope has small scale fading distribution and received signal envelope power is long scale fading distributed. There are more distributions which can be used to described signal envelope variation and signal envelope power variation. The most frequent statistical models that can be used to describe small scale signal envelope variation are: Rayleigh, Rician, Nakagami-*m*, Weibull and α-μ distributions. The long scale signal envelope variation can be described by using Gamma and log-normal distributions [2].

Rayleigh and Nakagami-*m* distributions can be used to describe small scale signal envelope variation in linear, non line-of-sight multipath fading environment. In line-of-sight multipath fading environments, small scale signal envelope variation can be described by using Rician distribution. Rician distribution has Rician factor. Rician factor is defined as ratio of dominant component power and scattering component power.

Putting Rician factor to be zero, Rician distribution reduces to Rayleigh distribution. The α-μ distribution is general distribution. Rayleigh, Nakagami-*m* and Weibull distributions can be derived from α-μ distribution. The Weibull distribution can be obtained from α-μ distribution by setting μ=1; and by setting α=2, α-μ distribution reduces to Nakagami-*m* distribution. The α-μ distribution reduces to Rayleigh distribution by setting α=2 and μ=1 [3].

The k-μ distribution is also general distribution. Rayleigh, Nakagami-m and Rican distribution can be derived from k-μ distribution. By setting k = 0, the k-μ distribution reduces to Nakagami-m distribution and for μ = 1, Rician distribution can be derived from k-μ distribution. By setting k = 0 and μ = 1 the k-μ distribution approximates Rayleigh distribution [G].

There are combining techniques used to combat the influence of short term fading effects and long term fading effects on level crossing rate and average fade duration of wireless communication systems. The most frequently used combining techniques are maximal ratio combining (MRC) [D], [H], equal gain combining (EGC) and selection combining (SC). MRC combining enables the best performance and the SC combining provides the least implementation complexity. The SC combiner output signal is equal to the maximum of input signals.

The second order performance measures of wireless communication system are average level crossing rate of output signal envelope and average fade duration of wireless system. The average level crossing rate is defined as average value of the first derivative of output signal envelope [2]. The average fade duration is defined as the ratio of outage probability and average level crossing rate.

2 Related Works

There are more papers in open technical literature considering second order statistics of wireless communication systems operating over composite shadowed multipath fading environment. The multipath fading has different distributions (Rayleigh, Rician, Weibull,or Nakagami-*m*) [4]-[7] and shadowing is described by log-normal [5] or Gamma distribution.

The second order statistic analysis of selection macro-diversity combining over Gama shadowed Rayleigh fading environments is given in [4] and the second order statistics of the signal in Ricean-lognormal fading channel with selection combining in [5].

Average LCR and AFD for SC diversity over correlated Weibull fading channels are investigate in [6]. The two formulae for the average LCR and AFD at the output of dual-branch selection diversity receivers are performed and some earlier published results given in a more general and compared.

Some expressions for average LCR and AFD for dual-branch maximum ratio combining (MRC) and selection combining (SC) schemes which exist in the correlated fading channels, are derived in [7]. It is supposed that channel model of the diversity branches is correlated small scale with Nakagami-*m* statistics. The numerical results point out that the average LCR and AFD of MRC and SC schemes are significantly affected by the correlation between each branch when they are performing in the correlated environments.

In [8], macrodiversity SC receiver with two microdiversity MRC receivers operating over composite Gamma shadowed Nakagami-*m* multipath fading channel is considered. Microdiversity MRC receivers are used to reduce short term fading effects on system performance and macrodiversity SC receiver is used to reduce long term fading effects on system performance. The closed form expressions for average level crossing rate and average fade duration are calculated. Performance of selected diversity techniques over α-μ fading channels are analized in [j].

The wireless communication system operating over interference limited, α-k-μ multipath fading channel is considered in [C]. The closed form expressions for cumulative distribution function and LCR of the ratio of two k-μ random variables and the ratio of the two α-k-μ random variables are calculated. They are used then for evaluation the LCR of wireless communication system with L-branches, SIR based SC receiver operating over. α-k-μ multipath fading environment in the presence of co-channel interference affected to α-k-μ multipath fading.

In paper [9], the average level crossing rate and average fade duration of wireless communication system operating over composite Gamma shadowed Rician multipath fading channel are evaluated.

Infinite-series expressions for the second-order statistical measures of a macro-diversity structure operating over the Gamma shadowed

Ricean fading channels are provided in [10]. MRC combining at each base station (micro-diversity), and selection combining (SC), based on output signal power values, between base stations (macro-diversity) are considered.

Selection combining (SC) based on signal-to-interference ratio (SIR) over κ-μ fading channels is performed in [A], [B]. Probability density function (PDF) and cumulative distribution function (CDF) of the received SIR are determined in [A]. Based on the results obtained for PDF and CDF, infinite-series expressions are derived for the output level crossing rate (LCR) and average fade duration (AFD).

The wireless communication system with dual branch selection combining (SC) diversity receiver operating over k-μ multipath fading environment is considered in [B]. The closed form expressions for average level crossing rate of SC receiver output signal envelope and average fade duration of proposed system are evaluated. Numerical results are presented graphically to show the influence of Rican factor k and fading severity μ on average level crossing rate and average fade duration.

In our paper, the wireless communication system with SC receiver operating over composite shadowed multipath fading environment is analyzed. The received signal is subjected simultaneously to Rician multipath fading and Gamma shadowing.

The short term fading causes signal envelope variation and long term fading causes signal envelope power variation. The closed form expression for joint probability density function of SC receiver output signal envelope and the first derivative of SC receiver output signal envelope is calculated. This expression is used for calculation of average level crossing rate of SC receiver output signal envelope. It can be used for evaluation of the average fade duration of wireless communication system with SC receiver operating over composite Gamma shadowed Rician multipath fading channel.

To the best authors' knowledge the average level crossing rate of wireless system with SC receiver operating over composite Gamma shadowed Rician multipath fading channels is not reported in open technical literature. The obtained result can be used in performance analysis and designing of wireless communication system with SC receiver in the presence of Gamma large scale fading and Rician small scale fading.

3 Level Crossing Rate of Rician random variable with Gamma distributed power

Squared Rician random variable can be written as sum of two independent Gaussian random variables:

$$x^2 = x_1^2 + x_2^2 \tag{1}$$

where x_1 and x_2 are independent Gaussian random variables with the same variances σ^2. The first derivative of x is:

$$x = \frac{1}{x}\left(x_1 x_1 + x_2 x_2\right) \tag{2}$$

The first derivative of Gaussian random variable is Gaussian random variable. Thus, $\dot{x}_1$ and $\dot{x}_2$ are also Gaussian random variables. The linear transformation of Gaussian random variable is Gaussian random variable. Therefore, $\dot{x}$ follow conditional Gaussian distribution. The average value of $\dot{x}$ is:

$$\overline{x} = \frac{1}{x}\left(x_1 \overline{x}_1 + x_2 \overline{x}_2\right) = 0 \tag{3}$$

since $\overline{x}_1 = \overline{x}_2 = 0$.

The variance of the first derivative of Rician random variable with Gamma distributed power is:

$$\sigma_x^2 = \frac{1}{x^2}\left(x_1^2 \sigma_{x_1}^{\ 2} + x_2^2 \sigma_{x_2}^{\ 2}\right) \tag{4}$$

where

$$\sigma_{x_1}^{\ 2} = \sigma_{x_2}^{\ 2} = 2\sigma^2 \pi^2 f_m^2 = \Omega \pi^2 f_m^2 . \tag{5}$$

After substituting (5) in (4), the expression for variance of $\dot{x}$ becomes:

$$\sigma_x^2 = \frac{\Omega \pi^2 f_m^2}{x^2}\left(x_1^2 + x_2^2\right) = \Omega \pi^2 f_m^2 \tag{6}$$

The joint probability density function of Rician random variable with Gamma distributed power and the first derivative of Rician random variable with Gamma distributed power is:

(7)

where $p_x(x)$ is Rician probability density function:

$$p_x(x) = \frac{2(k+1)}{e^k\Omega} e^{-\frac{(k+1)x^2}{\Omega}} \cdot I_0\left(2\sqrt{\frac{(k+1)k}{\Omega}}x\right). \tag{8}$$

where $I_0(z)$ is the modified Bessel function of the first kind with order zero. A Rician fading channel is described by two parameters, k and Ω. k is the ratio between the power in the direct path and the power in the other, scattered, paths. Ω is the total power from both paths ($\Omega = \nu^2 + \sigma^2$), and acts as a scaling factor to the distribution. Therefore, Rician factor k increases as dominant component power increases or scattering components power decreases.

After substituting (8) in (7), the expression for the joint probability density function becomes:

$$p_{x\dot{x}}(x\dot{x}) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{\dot{x}^2}{2\sigma_x^2}} \frac{2(k+1)}{e^k\Omega} \cdot e^{-\frac{(k+1)x^2}{\Omega}} \cdot I_0\left(2\sqrt{\frac{(k+1)k}{\Omega}}x\right) =$$

$$= \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{\dot{x}^2}{2\sigma_x^2}} \frac{2(k+1)}{e^k\Omega} \cdot e^{-\frac{(k+1)x^2}{\Omega}} \sum_{i=0}^{\infty}\left(\frac{k(k+1)}{\Omega}\right)^i x^{2i} \tag{9}$$

The average level crossing rate is calculated as average value of the first derivative of Rician random variable with Gamma distributed power:

$$N_x = \int_0^\infty \dot{x} p_{x\dot{x}}(x\dot{x})\,d\dot{x} =$$

$$= \frac{1}{\sqrt{2\pi}\sigma_x}\int_0^\infty d\dot{x}\,\dot{x} e^{-\frac{\dot{x}^2}{2\sigma_x^2}} \frac{2(k+1)}{e^k\Omega} \cdot e^{-\frac{(k+1)x^2}{\Omega}} \sum_{i=0}^{\infty}\left(k(k+1)\right)^i x^{2i}\Omega^i =$$

$$= \frac{1}{\sqrt{2\pi}} \pi f_m \Omega^{1/2} \cdot \frac{2(k+1)}{e^k\Omega} \cdot e^{-\frac{(k+1)x^2}{\Omega}} \sum_{i=0}^{\infty}\left(k(k+1)\right)^i x^{2i}\Omega^i \tag{10}$$

By averaging conditional average level crossing rate, average level crossing rate becomes:

$$N_x = \int_0^\infty d\Omega\, N_{x/\Omega}\, p_\Omega(\Omega) =$$

$$= f_m\sqrt{2\pi}\frac{1}{\beta^c\Gamma(c)} \cdot \frac{k+1}{e^k} \cdot \sum_{i=0}^{\infty}\left(k(k+1)\right)^i \frac{1}{(i!)^2} x^{2i} \cdot$$

$$\cdot \int_0^\infty d\Omega\,\Omega^{c-1-1/2} e^{-\frac{(k+1)x^2}{\Omega} - \frac{\Omega}{\beta}} =$$

$$= f_m\sqrt{2\pi}\frac{1}{\beta^c\Gamma(c)} \cdot \frac{k+1}{e^k} \cdot \sum_{i=0}^{\infty}\left(k(k+1)\right)^i \frac{1}{(i!)^2} x^{2i} \cdot$$

$$\cdot \left(\beta(k+1)x^2\right)^{\frac{6}{2}-1/4} K_{c-1/2}\left(2\sqrt{\frac{(k+1)x^2}{\beta}}\right) \tag{12}$$

Here, $\Omega > 0$, and c, $\beta > 0$. $\Gamma(c)$ is the gamma function evaluated at c.

The cumulative distribution function of Rician random variable is:

$$F_x(x) = \int_0^x p_x(t)dt = \int_0^x dt \cdot \frac{2(k+1)}{e^k\Omega} t \cdot e^{-\frac{(k+1)t^2}{\Omega}} \cdot I_0\left(2\sqrt{\frac{(k+1)k}{\Omega}}t\right) =$$

$$= \frac{2(k+1)}{e^k\Omega}\int_0^x dt\, t \cdot e^{-\frac{(k+1)t^2}{\Omega}} \sum_{i=0}^{\infty}\left(k(k+1)\right)^i \Omega^{-i} t^{2i} =$$

$$= \frac{2(k+1)}{e^k\Omega}\sum_{i=0}^{\infty}\left(k(k+1)\right)^i \Omega^{-i}\int_0^x dt\, t^{2i+1} \cdot e^{-\frac{(k+1)t^2}{\Omega}} =$$

$$= \frac{2(k+1)}{e^k\Omega}\sum_{i=0}^{\infty}\left(k(k+1)\right)^i \Omega^{-i}\frac{1}{2}\Omega^{i+1}\gamma\left(i, \frac{k+1}{\Omega}x^2\right) =$$

$$= \frac{k+1}{e^k}\sum_{i=0}^{\infty}\frac{\left(k(k+1)\right)^i}{(i!)^2}\gamma\left(i, \frac{k+1}{\Omega}x^2\right) \tag{13}$$

4 Level Crossing Rate of SC Receiver output signal

The wireless communication system with dual SC receivers operating over composite Gamma shadowed Rician multipath fading channel is considered. Signal envelopes at inputs of SC receiver are denoted with x_1 and x_2, and signal envelope at the output of SC receiver is denoted with x. The joint probability density function of SC receiver output signal x and its first derivative is:

$$p_{x\dot{x}}(x\dot{x}) = p_{x_1\dot{x}_1}(x\dot{x})F_{x_2}(x) + p_{x_2\dot{x}_2}(x\dot{x})F_{x_1}(x) =$$

$$= 2p_{x_1\dot{x}_1}(x\dot{x})F_{x_2}(x) \tag{14}$$

where $p_{x_1 x_1}(x\dot{x})$ is given with (9) and $F_x(x)$ is given with (13).

After substituting, $p_{x\dot{x}}(x\dot{x})$ becomes:

$$p_{x\dot{x}}(x\dot{x}) = 2\frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{\dot{x}^2}{2\sigma_x^2}} \frac{2(k+1)x}{e^k\Omega} \cdot e^{-\frac{(k+1)x^2}{\Omega}} \sum_{i=0}^{\infty}\left(\frac{k(k+1)}{\Omega}\right)^i \Omega^{-i} x^{2i} \cdot$$

$$\cdot \frac{k+1}{e^k}\sum_{i_1=0}^{\infty}\left(k(k+1)\right)^{i_1} \gamma\left(i_1, \frac{k+1}{\Omega}x^2\right) \tag{15}$$

The random variable Ω follows Gamma distribution. The joint probability density function of SC output random variable and the first derivative of SC receiver output random variable can be calculated by averaging (15):

$$p_{x\dot{x}}(x\dot{x}) = \int_0^\infty d\Omega\, p_\Omega(\Omega)\, p_{x\dot{x}}\left(x\dot{x}/\Omega\right) \tag{16}$$

where $p_{x\dot{x}}(x\dot{x}/\Omega)$ is given with (15).

Average level crossing rate is:

$$N_x = \int_0^\infty d\dot{x}\,\dot{x}\, p_{x\dot{x}}(x\dot{x}) =$$

$$= \int_0^\infty \frac{2}{\sqrt{2\pi}} \pi f_m \Omega^{1/2} \cdot \frac{2(k+1)x}{e^k\Omega} \cdot e^{-\frac{(k+1)x^2}{\Omega}} \sum_{i=0}^{\infty} \frac{\left(k(k+1)\right)^i}{(i!)^2}\Omega^{-i}x^{2i} \cdot$$

$$\cdot \frac{(k+1)}{e^k} \cdot \sum_{i_1=0}^{\infty} \frac{\left(k(k+1)\right)^{i_1}}{(i_1!)^2} \gamma\left(i_1, \frac{k+1}{\Omega}x^2\right) \cdot \frac{1}{\beta^c\Gamma(c)} \Omega^{c-1} e^{-\frac{1}{\beta}\Omega} d\Omega \tag{17}$$

where

$$\gamma\left(i_1, \frac{k+1}{\Omega}x^2\right) = \Gamma(i_1) - \frac{1}{i_1+1}\frac{(k+1)^{i_1}}{\Omega^{i_1}} \cdot$$

$$\cdot x^{2i_1} e^{-\frac{k+1}{\Omega}x^2}\, {}_1F_1\left(i_1+1, 1, \frac{k+1}{\Omega}x^2\right) \tag{18}$$

and

$${}_1F_1\left(i_1+1, 1, \frac{k+1}{\Omega}x^2\right) = \sum_{i_2=0}^{\infty} \frac{(i_1+i_2)!}{(i_2!)^3} \frac{(k+1)^{i_2}}{\Omega^{i_2}} x^{2i_2} \tag{19}$$

After substituting, the expression for level crossing rate becomes:

$$N_x = \frac{f_m\sqrt{2\pi}}{\Gamma(c)\beta^c} \cdot \frac{2(k+1)^2}{e^{2k}}\, x \sum_{i=0}^{\infty} \frac{\left(k(k+1)\right)^i}{(i!)^2} x^{2i} \cdot$$

$$\cdot \sum_{i_1=0}^{\infty} \frac{\left(k(k+1)\right)^{i_1}}{(i_1!)^2} \Gamma(i_1) \int_0^\infty d\Omega\, \Omega^{c-1-1/2-i} e^{-\frac{(k+1)x^2}{\Omega} - \frac{1}{\beta}\Omega} -$$

$$- \frac{f_m\sqrt{2\pi}}{\Gamma(c)\beta^c} \cdot \frac{2(k+1)^2}{e^{2k}}\, x \sum_{i=0}^{\infty} \frac{\left(k(k+1)\right)^i}{(i!)^2} x^{2i} \cdot \sum_{i_1=0}^{\infty} \frac{\left(k(k+1)\right)^{i_1}}{(i_1!)^2} \cdot$$

$$\cdot \frac{1}{i_1+1}(k+1)^{i_1} x^{2i_1} \sum_{i_2=0}^{\infty} \frac{(i_1+i_2)!}{(i_2!)^3} (k+1)^{i_2} x^{2i_2} \cdot$$

$$\cdot \int_0^\infty d\Omega\, \Omega^{c-1-1/2-i-i_1-i_2} e^{-\frac{2(k+1)x^2}{\Omega} - \frac{1}{\beta}\Omega} =$$

$$= \frac{f_m\sqrt{2\pi}}{\Gamma(c)\beta^c} \cdot \frac{2(k+1)^2}{e^{2k}}\, x \sum_{i=0}^{\infty} \frac{\left(k(k+1)\right)^i}{(i!)^2} x^{2i} \cdot$$

$$\cdot \sum_{i_1=0}^{\infty} \frac{\left(k(k+1)\right)^{i_1}}{(i_1!)^2} \Gamma(i_1) \cdot \left(\beta(k+1)x^2\right)^{\frac{c}{2}-\frac{1}{4}-\frac{i}{2}} \cdot K_{c-\frac{1}{2}-i}\left(2\sqrt{\frac{(k+1)x^2}{\beta}}\right) -$$

$$- \frac{f_m\sqrt{2\pi}}{\Gamma(c)\beta^c} \cdot \frac{2(k+1)^2}{e^{2k}}\, x \sum_{i=0}^{\infty} \frac{\left(k(k+1)\right)^i}{(i!)^2} x^{2i} \cdot \sum_{i_1=0}^{\infty} \frac{\left(k(k+1)\right)^{i_1}}{(i_1!)^2} \cdot$$

$$\cdot \frac{1}{i_1+1}(k+1)^{i_1} x^{2i_1} \sum_{i_2=0}^{\infty} \frac{(i_1+i_2)!}{(i_2!)^3} (k+1)^{i_2} x^{2i_2} \cdot$$

$$\cdot \left(2\beta(k+1)x^2\right)^{\frac{c}{2}-\frac{1}{4}-\frac{i}{2}-\frac{i_1}{2}-\frac{i_2}{2}} \cdot K_{c-\frac{1}{2}-i-i_1-i_2}\left(2\sqrt{\frac{2(k+1)x^2}{\beta}}\right) \tag{20}$$

5 Level Crossing Rate of SC Receiver output signal in the Presence of Gamma Shadowing and k-μ Multipath Fading

The selection receiver with two inputs operating over independent identically distributed k-μ short term fading and Gamma long term fading is studied in this section. The signal envelopes at the input of SC receiver are denoted with x_1 and x_2, and the output signal with x. Probability density function of SC receiver output signal envelopes:

$$p_{x\dot{x}}(x\dot{x}) = p_{x_1 \dot{x}_1}(x\dot{x}) F_{x_2}(x) + p_{x_2 \dot{x}_2}(x\dot{x}) F_{x_1}(x) =$$

$$= 2 p_{x_1 x_1}(xx) F_{x_2}(x) \tag{21}$$

The joint probability density function of k-μ rtandom variable and its first time derivative is:

$$p_{x_1 \dot{x}_1}(x \dot{x}_1) = \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega^{\frac{\mu+1}{2}}} \cdot x_1^{\mu} e^{-\frac{\mu(k+1)}{\Omega}} \cdot x_1^2 \cdot$$

$$\cdot I_{\mu-1}\left(2\sqrt{\frac{(k+1)k}{\Omega}}\, x_1\right) \cdot \frac{1}{\sqrt{2\pi}\,\beta_1} e^{-\frac{\dot{x}_1^2}{2\beta_1^2}} \tag{22}$$

where k and μ are fading parameters, **Ω** is average power and

$$\beta_1^2 = \pi^2 f_m^2 \frac{\Omega}{\mu(k+1)} \tag{23}$$

Gamma long term fading causes signal envelope power variation resulting that **Ω** is random variable with:

$$p_{\Omega}(\Omega) = \frac{1}{\Gamma(0)\beta^c} \Omega^{c-1} e^{-\frac{1}{\beta}\Omega},\ \Omega \geq 0. \tag{24}$$

The conditional average level crossing rate of SC receiver output signal envelope is

$$N_{x/\Omega} = \int_0^{\infty} d\dot{x}\, \dot{x}\, p_{x\dot{x}}(x\dot{x}) = \int_0^{\infty} d\dot{x}\, \dot{x}\, 2\, p_{x_1 \dot{x}_1}(x\dot{x}) F_{x_2}(x) =$$

$$= 2F_{x_2}(x) \int_0^{\infty} d\dot{x}\, \dot{x}\ p_{x_1 \dot{x}_1}(x\dot{x}) = 2F_{x_2}(x) N_{x_1}(x) \tag{25}$$

Average level crossing rate of k-μ random variable is:

$$N_{x_1} = \int_0^{\infty} d\dot{x}_1\, \dot{x}_1\ p_{x_1 \dot{x}_1}(x_1 \dot{x}_1) =$$

$$= \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega^{\frac{\mu+1}{2}}} \cdot x_1^{\mu} e^{-\frac{\mu(k+1)}{\Omega}} \cdot x_1^2 \cdot$$

$$\cdot I_{\mu-1}\left(2\mu\sqrt{\frac{(k+1)k}{\Omega}}\, x_1\right) \cdot \frac{\beta_1}{\sqrt{2\pi}} \tag{26}$$

The cumulative distribution function of k-μ random variable is:

$$F_{x_1}(x_1) = \int_0^{x_1} dt\, p_x(t) =$$

$$= \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega} \cdot \sum_{i_1=0}^{\infty} \left(\mu\sqrt{\frac{(k+1)k}{\Omega}}\right)^{2i_1+\mu-1} \frac{1}{i_1!\,\Gamma(i_1+c)} \cdot$$

$$\cdot \int_0^{x_1} dt\ t^{2i_1+\mu-1} e^{-\frac{\mu(k+1)}{\Omega} t^2} =$$

$$= \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega} \cdot \sum_{i_1=0}^{\infty} \left(\mu\sqrt{\frac{(k+1)k}{\Omega}}\right)^{2i_1+\mu-1} \frac{1}{i_1!\,\Gamma(i_1+c)} \cdot$$

$$\cdot \left(\frac{\Omega}{\mu(k+1)}\right)^{i_1+\mu} \gamma\left(i_1+\mu, \frac{\mu(k+1)}{\Omega} x_1^2\right) \tag{27}$$

After substituting (27) and (26) in (25), the expression for conditional average level crossing rate becomes:

$$N_{x/\Omega} = \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\, \Omega^{\frac{\mu+1/2}{2}}} \cdot$$

$$\cdot \sum_{i_1=0}^{\infty} \left(\mu\sqrt{\frac{(k+1)k}{\Omega}}\right)^{2i_1+\mu-1} \frac{1}{i_1!\,\Gamma(i_1+c)} \cdot$$

$$\cdot \left(\frac{\Omega}{\mu(k+1)}\right)^{i_1+\mu} \gamma\left(i_1+\mu, \frac{\mu(k+1)}{\Omega} x_1^2\right) =$$

$$= \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\, \Omega^{\frac{\mu+1/2}{2}} \sqrt{2\pi}} \cdot$$

$$\cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{(k+1)k}{\Omega}}\right)^{2i_2+\mu-1} \frac{1}{i_2!\,\Gamma(i_2+c)} \cdot$$

$$x^{2i_2+2\mu-1} \cdot e^{-\frac{\mu(k+1)}{\Omega} x^2} \cdot \pi f_m \frac{\Omega^{1/2}}{\mu^{1/2}(k+1)^{1/2}} =$$

$$= \frac{4\mu^{3/2}(k+1)^{\mu+1/2} \pi f_m}{k^{\mu-1} e^{2k\mu}\, \Omega^{\mu+1/2}} \cdot$$

$$\cdot \sum_{i_1=0}^{\infty} \left(\mu\sqrt{\frac{(k+1)k}{\Omega}}\right)^{2i_1+\mu-1} \frac{1}{i_1!\,\Gamma(i_1+c)} \cdot$$

$$\cdot \left(\frac{\Omega}{\mu(k+1)}\right)^{i_1+\mu} \gamma\left(i_1+\mu, \frac{\mu(k+1)}{\Omega} x^2\right) \cdot$$

$$\cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{(k+1)k}{\Omega}}\right)^{2i_2+\mu-1} \frac{1}{i_2!\,\Gamma(i_2+c)} \cdot$$

$$x^{2i_2+2\mu-1} \cdot e^{-\frac{\mu(k+1)}{\Omega}x^2} \tag{28}$$

The average level crossing rate of SC receiver output signal envelope is

$$N_x = \int_0^\infty dx\, N_{x/\Omega}\, p_\Omega(\Omega) =$$

$$= \frac{2\mu^{3/2}(k+1)^{\mu+1/2}\pi f_m}{k^{\mu-1}e^{2k\mu}} \cdot$$

$$\cdot \sum_{i_1=0}^{\infty} \left(\mu\sqrt{(k+1)k}\right)^{2i_1+\mu-1} \frac{1}{i_1!\,\Gamma(i_1+c)} \cdot$$

$$\cdot \left(\frac{1}{\mu(k+1)}\right)^{i_1+\mu} \cdot \frac{1}{i_1+\mu}\left(\mu(k+1)x^2\right)^{i_1+\mu} \cdot$$

$$\cdot \sum_{j_1=0}^{\infty} \frac{1}{(i_1+\mu+1)j_1}\left(\mu(k+1)x^2\right)^{j_1} \cdot$$

$$\cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{(k+1)k}\right)^{2i_2+\mu-1} \cdot \frac{1}{i_2!\,\Gamma(i_2+c)} \cdot x^{2i_2+2\mu-1} \cdot \frac{1}{\Gamma(c)\beta^c} \cdot$$

$$\cdot \int_0^\infty d\Omega\, \Omega^{-\mu-1/2-i_1-\mu/2+1/2+i_1+\mu-i_1-\mu-j_1-i_2-\mu/2+1/2+c-1} \cdot$$

$$\cdot e^{-\frac{2\mu(k+1)}{\Omega}-\frac{1}{\beta}\Omega} =$$

$$= \frac{2\mu^{3/2}(k+1)^{\mu+1/2}\pi f_m}{k^{\mu-1}e^{2k\mu}} \cdot$$

$$\sum_{i_1=0}^{\infty} \left(\mu\sqrt{(k+1)k}\right)^{2i_1+\mu-1} \frac{1}{i_1!\,\Gamma(i_1+c)} \cdot$$

$$\cdot \left(\frac{1}{\mu(k+1)}\right)^{i_1+\mu} \cdot \frac{1}{i_1+\mu}\left(\mu(k+1)x^2\right)^{i_1+\mu}$$

$$\cdot \sum_{j_1=0}^{\infty} \frac{1}{(i_1+\mu+1)j_1}\left(\mu(k+1)x^2\right)^{j_1} \cdot$$

$$\cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{(k+1)k}\right)^{2i_2+\mu-1} \cdot \frac{1}{i_2!\,\Gamma(i_2+c)} \cdot x^{2i_2+2\mu-1} \frac{1}{\Gamma(c)\beta^c} \cdot$$

$$\cdot 2\left(\mu(k+1)x^2\beta\right)^{-\mu-i_1/2-i_2/2-i_1/2+1/4+c/2} \cdot$$

$$K_{-2\mu-i_1-i_2-j_1+1/2+c}\left(2\sqrt{\frac{\mu(k+1)x^2}{\beta}}\right) \tag{29}$$

6 Numerical Results

In next two figures, the level crossing rate of SC receiver output signal envelope is presented for different values of Rice factor k, Gamma distribution parameters c and β (b in the figures) and signal envelope.

In Fig. 1, the level crossing rate of SC receiver output signal envelope versus input signal envelope is presented for different values of Rice factor k, and Gamma distribution parameters c and b.

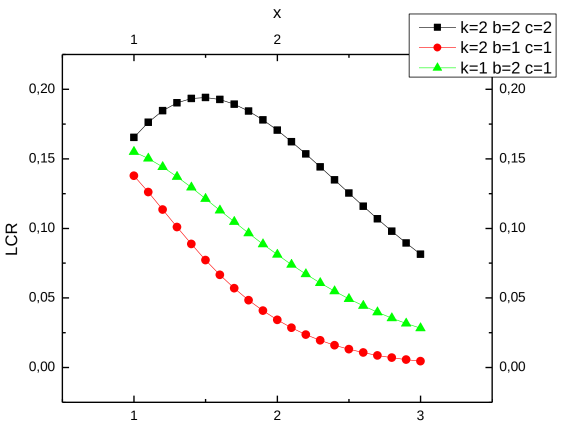

Fig. 1. The level crossing rate versus signal envelope

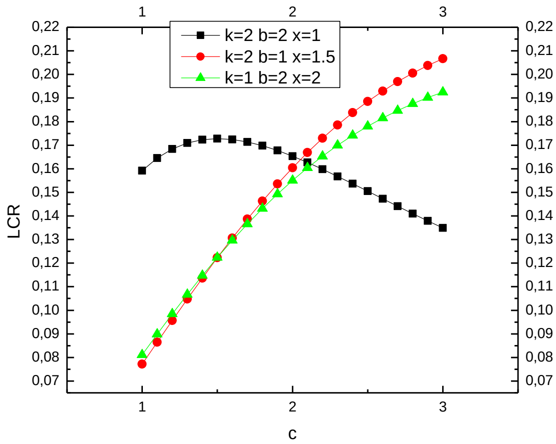

Fig. 2. The level crossing rate of SC receiver output signal versus parameter c

The level crossing rate has lower values for lower values of parameter b. Also, the LCR has smaller values for smaller values of parameter c and higher values of signal envelope.

The system performance is better for smaller values of the average level crossing rate.

The level crossing rate of SC receiver output signal versus Gamma distribution parameter c is shown in Fig. 2. The parameters of curves are

Rice factor *k*, Gamma distribution parameters *b*, and signal envelope.

From this figure one can see the influence of distribution's parameters on the LCR of SC receiver output signal and choose the most appropriate values for designing of wireless systems.

It is visible that the LCR is lesser for smaller values of parameter c for some selected values of the parameters *k* and *b*.

7 Conclusion

In this paper, the wireless communication system with dual SC receiver operating over shadowed multipath fading channel is considered. The received signal is subjected simultaneously to Gamma long term fading and k- μ or Rician short term fading. The short term fading causes signal envelope variation and Gamma long term fading causes signal envelope power variation.

SC receiver is used to reduce short term fading effects and long term fading effects resulting in system performance improvement. The second order statistics of proposed systems are analyzed. The joint probability density function of SC receiver output signal envelopes and their first derivatives of SC receiver output signal envelopes are calculated. The closed form expressions for average level crossing rate of SC receiver output signal envelopes are calculated by using these results. The level crossing rate is calculated as average value of the first derivative of SC receiver output signal envelope.

The k-μ and Rician distributions are general distributions. By setting $k = 0$, and $\mu = 1$, Rician distribution can be derived from k-μ distribution. By setting Rician factor to be zero, in obtained expression for average level crossing rate can be derived the expression for average level crossing rate of wireless communication system with SC receiver operating over composite Gamma shadowed Rayleigh multipath fading environment.

The numerical results are presented graphically to show the influence of Rician factor and Gamma shadowing severity parameter on average level crossing rate of wireless communication system. The system performance is better for lower values of average level crossing rate. The level crossing rate increases as Gamma shadowing severity parameter decreases.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.