Earthing Resistance Tester developed using Resonant Circuit Technology with No Auxiliary Electrodes

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Abstract: - This paper presents an earthing resistance tester that uses resonant circuit technology to operate with a simple earthing electrode and without auxiliary electrodes. The measurement mechanism is analyzed using an earth return circuit model. The earthing electrode being tested, a lead wire, a return wire and the soil itself comprise the earth return circuit (LRC serial circuit). The requirements for this measurement method are obtained from numerical simulations, taking the lengths of the lead wire and the return wire to be 1 m and 20 m, and soil resistivity less than 10-2 s/m. The developed tester can measure the earthing resistance by evaluating the loop impedance of the earth return circuit at its resonant frequency, using a resonant frequency in the range of 100 kHz to 1.5 MHz. The return wire need only be placed on the soil surface without any termination. Earthing resistance measurements made by the developed tester and a conventional tester (3-pole method) differed by less than ±20% for earthing resistance values between 50 Ω and 400 Ω. The developed tester is a good way to test a simple earth electrode with an earthing resistance value of around 100 Ω.

Key-Words: - Earthing resistance, an earth return circuit model, tester, without auxiliary electrodes, resonant frequency, experiments and examinations

1 Introduction

Earthing is very important for both human and equipment safety in the case of power faults and lightning events, and is also is required for a signal ground, a voltage reference and so on. Examples of required resistance are 10 Ω and 100 Ω, values set by national regulations or international requirements. An earthing resistance value of 100 Ω is normally required for both human and equipment safety for low-voltage equipment in Japan.

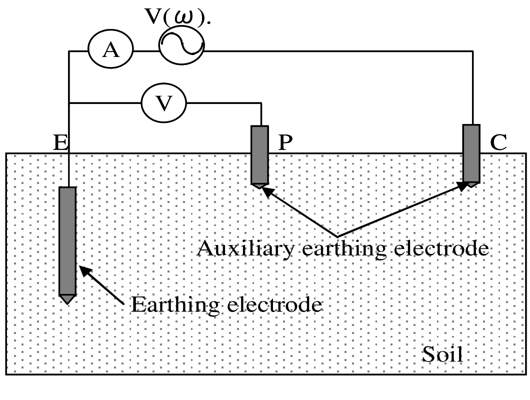
Many methods have been developed to measure the resistance of earth electrodes. The best known is the 3-pole method [1]. One pole is the earthing electrode to be tested, and the other two are auxiliary electrodes. The auxiliary electrodes must be directly connected to the soil itself for current injection and voltage reference. However, it may be difficult to measure the earthing resistance value this way because the soil surface is usually covered by concrete or stones in developed areas. Therefore we have proposed an earthing resistance measurement method that requires no auxiliary electrodes [2]. The proposed method measures the loop impedance [3] [4] at the resonant frequency of a circuit composed of the electrode itself, a return wire and the ground itself. An insulated copper wire about 20 m long used as the return wire is simply laid out straight on the soil surface, with no termination. A sophisticated network analyzer with a battery power supply was required to test the method because of the electrical noise in the field, including radio waves and stray currents.

In this paper, we would first like to explain our proposed earthing resistance measurement method, and also to show the voltage ratio technique. We also outline our development and testing of an earthing resistance tester based on our proposal.

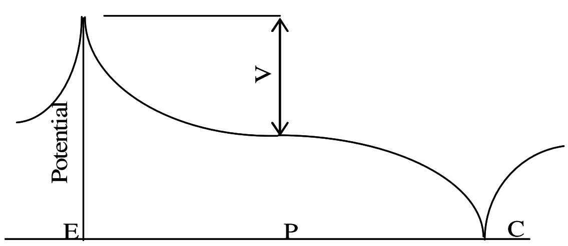
2 Conventional measurement methods

2.1 3-pole method

An existing earthing resistance measurement method is shown in Fig.1. Fig.1 (a) shows the measurement configuration and Fig.1(b) the potential distribution. In Fig.1 (a), E is the earthing electrode being tested, P is the auxiliary electrode for potential measurement, C is the auxiliary electrode for current injection.



(a) Measurement system



(b) Potential distribution

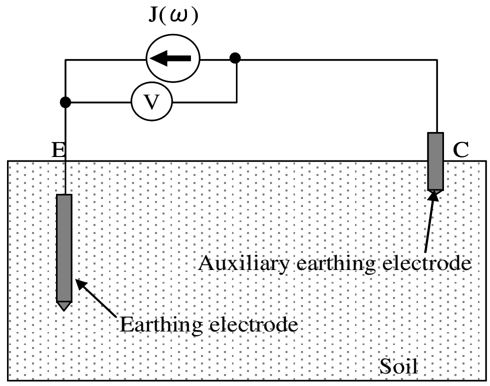
Fig.1. 3-pole method

In Fig.1(a), $V(\omega)$ is a signal generator whose frequency is usually about 500 Hz, A is a current meter, V is a voltage meter.

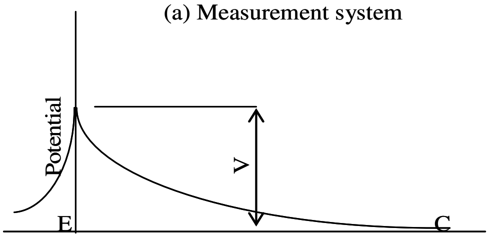
In Fig.1 (b), the horizontal axis shows distance, the vertical axis shows potential (voltage). V is the potential difference between electrodes E and P. The earthing resistance value at E is defined as the ratio of V and A. This is the most widely used method in the world. However, when the soil is stony or paved, it is a difficult method to use.

2.2 Loop impedance method

The loop impedance method is shown in Fig.2. Fig.2 (a) shows the measurement configuration and Fig.2 (b) the potential distribution, as in Fig.1 (b). In Fig.2 (a), E is the earthing electrode being tested, and C is the auxiliary electrode for current injection. J is the current source, its frequency usually about 1 kHz, V is a voltage meter. E, C and the soil form the earth return loop, and the measurement is made by evaluating the loop impedance. If the earthing resistance at electrode C is smaller than that at E, the measured loop impedance is almost the same as the earthing resistance of electrode E, and the earthing resistance value of E is given by the ratio of J and V. This method only uses electrode C for current injection, and while it is often easy to measure earthing resistance, it is difficult to determine a low earthing resistance value for C in the field. Commonly, the building earth is used as a C electrode, but this is dangerous when there are power faults in buildings.



(a) Measurement system



(b) Potential distribution

Fig. 2 A loop impedance method

3 A measurement method without using auxiliary electrodes

As the loop impedance method explained in section 2.2 is quite a simple way to measure earthing resistance, we would like to consider a loop impedance method without a C electrode, as shown in Fig.3. The difference between Fig.2 and Fig.3 is only the presence or absence of a C electrode. In this new measurement system, a lead wire, a return wire, and an impedance meter are used, and the return wire is just placed on the ground surface without any termination.

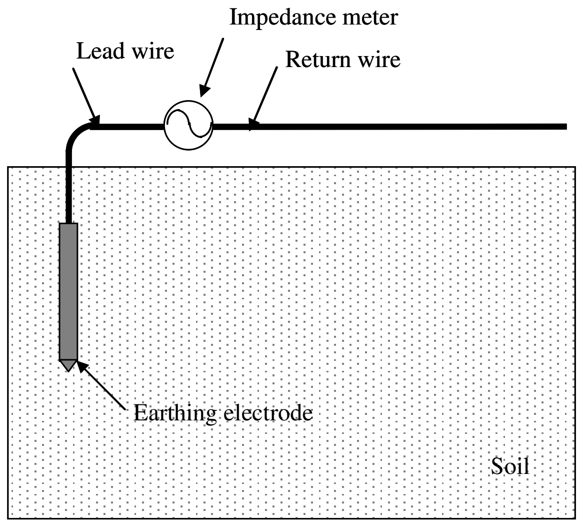


Fig.3. Measurement system without using auxiliary electrodes

3. 1 Earth return model

To explain the measurement mechanism of the new method, an earth return model [(6) (7)] is shown in Fig.4. A return wire and a lead wire are placed on an infinite ground soil; the electrode being tested is buried in the soil. In Fig.4, both horizontal axis x and vertical axis z indicate distance. In Fig.4, a lead wire is connected to the electrode being tested and to a voltage oscillator (Vs), and the return wire is connected to Vs with the other end of the return wire open. ε_w (about 3 - 5) is the permeability of these wires' insulators, and σ_w is the conductivity of the wires. ε_e (about 10 - 80) and σ_e are the permeability and conductivity values for the soil. In Fig.4, 'a' is the diameter of the wire conductors, 'h' is the distance from the soil surface to the wires' centers; these two wires are covered with PE insulation, so they do not contact to the soil directly. $V(x)$ and $I(x)$ are the potential and current of the wire conductor at x, where the potential is the potential from infinity (z = -∞). S is the rectangular surface below the wire as shown in Fig.4, and C is the contour of the surface S. $P(x,z)$ is a point on the contour C.

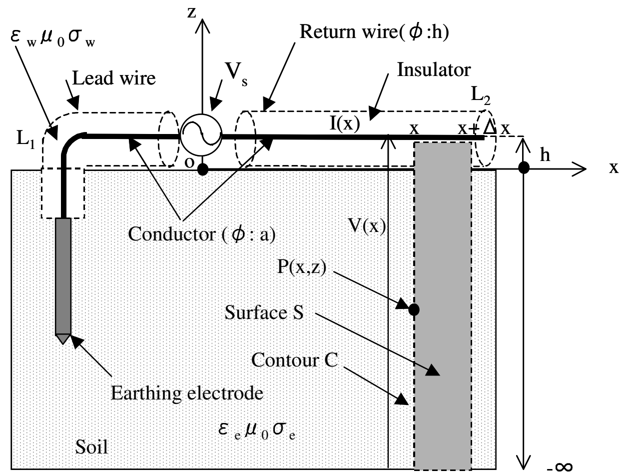


Fig.4 Earth return model of new system

3.2 Earth return circuit analysis

Where $\vec{E}$ and $\vec{H}$ are electric and magnetic field vectors induced by the voltage oscillator Vs, the following equation applies:

$$\int_{-\infty}^{h-a}\{E_z(x+\Delta x,z)-E_z(x,z)\}dz-\int_{x}^{x+\Delta x}\{E_z(x,h)-E_z(x,-\infty)\}dx$$
$$=-j\omega\mu_0\int_{-\infty}^{h-a}\int_{x}^{x+\Delta x}H_y(x,h)dxdz \quad \text{......(1)}$$

where E_z and H_y are the z component of electric field $\vec{E}$ and the y component of $\vec{H}$. "j" is a complex number, ω is an angular frequency. In Eq. (1), the term $E_z(x,-\infty)$ on the right-hand side of Eq. (1) can be neglected because the electric field can be assumed to vanish at $z=-\infty$.

Here, the potential of the wire conductor is given by:

$$V(x)=-\int_{-\infty}^{h-a}E_z(x,z)dz \quad \text{......(2)}$$

When $\Delta x\to 0$, Eq.(1) can be written:

$$\frac{dV(x)}{dx}+Z_wI(x)=-j\omega\mu_0\int_{-\infty}^{h-a}H_y(x,z)dz \quad \text{......(3)}$$

where $Z_w(\approx\frac{1}{\pi a^2\sigma_w}+\frac{j\omega\mu_0}{8\pi})$ is the impedance of the wire conductor[(4)]. The right hand side of Eq. (3) is a physical value related to magnetic field contribution of the wire under the soil. Therefore, Eq. (3) can be expressed as follows.

$$\frac{dV(x)}{dx} = -Z(\omega)I(x) \quad (4)$$

$$Z(\omega) = Z_w + Z_e + j\omega L \quad (5)$$

$$\frac{dI(x)}{dx} = -Y(\omega)V(x) \quad (6)$$

$$Y(\omega) = j\omega C + Y_e \quad (7)$$

$$Z_e Y_e \approx \gamma_e^2 = j\omega\mu_0(\sigma_e + j\omega\varepsilon_e\varepsilon_0) \quad (8)$$

where L is an inductance of the wire when the soil is assumed to be a perfect conductor, Z_e is the impedance contribution of the wire conductor under the soil. C is the capacitance of the wire when the soil is assumed to be a perfect conductor, Y_e is the admittance contribution of the wire conductor under the soil. L and C are given by:

$$L = \frac{\mu_0}{2\pi}\ln\frac{2h}{a} \quad (9)$$

$$C = \frac{2\pi\varepsilon_w\varepsilon_0}{\ln\frac{2h}{a}} \quad (10)$$

as the frequency is less than 3 MHz and σ_e is more than 10^{-2} (s/m) or 10^{-3}, Z_e can be written as follows.(4)(6)

$$Z_e \approx \frac{j\omega\mu_0}{2\pi}\ln\frac{1+\gamma_e h}{\gamma_e h} \quad (11)$$

where the equivalent earth return circuit of the wire conductor over the soil can be expressed as shown in Fig.5.

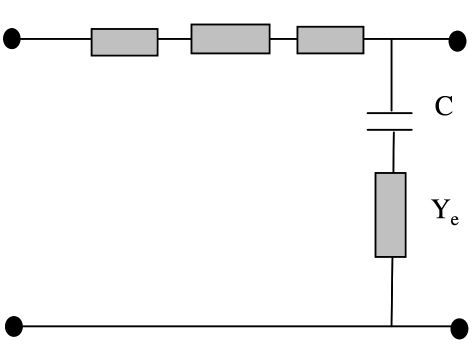


Fig.5 An equivalent earth return circuit of the wire over the soil

In this case, the transmission coefficient and the characteristic impedance shown in Fig.5 are as follows.

$$\gamma_0^2 = (Z_w + Z_e + j\omega L)(j\omega C + Y_e) \quad (12)$$

$$z_0 = \sqrt{\frac{Z_w + Z_e + j\omega L}{j\omega C + Y_e}} \quad (13)$$

According to the equivalent circuit shown in Fig.5 and Eqs.(12) and (13), Fig.4 can be represented schematically as shown in Fig.6. In Fig.6, Z_g is the earth impedance of the earthing electrode being tested. The signal oscillator, the earth electrode being tested and the end of the return wire are at x = $-L_1$, x = 0 and x=L_2.

In Fig.6, L_a is the additive inductance, and Cs is the stray capacitance of Vs. Fig.6 is assumed to be a transmission line model in which the left-hand side is terminated by R_g and the right-hand side is open. Numerical results based on the transmission line model show that the earthing resistance value can be obtained as the impedance at the resonant frequency of the transmission line (See Appendix). The resonant frequency depends on the length of the return wire, so in Fig.6, L_a must be added so that the resonant frequency is less than 3 MHz.

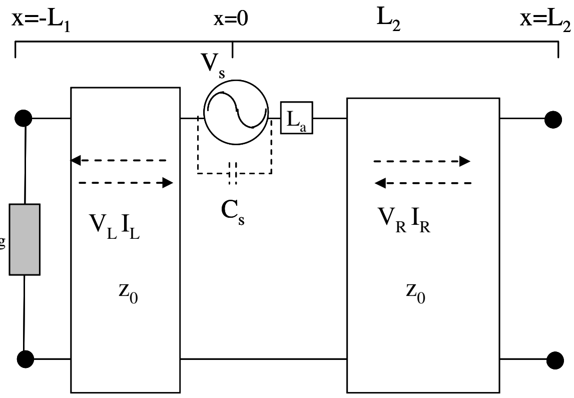


Fig.6 Earth return circuit model for Fig.4

3.3 Numerical results

The additive inductance L_a, the lengths of the lead wire and the return wire, and the soil conductivity are very important parameters for the proposed new measurement method. In this section, the requirements for these parameters are examined.

(1) Additive inductance L_a impedance

Calculated earthing resistances with/without additive inductance are shown in Fig.7 (8) where L_1 = 1 m and L_2 = 20 m, h/a = 2, ε_w =4, ε_e = 10 and σ_e =

10^{-2} s/m. Typical soil resistivity[1] is shown in Table.1.

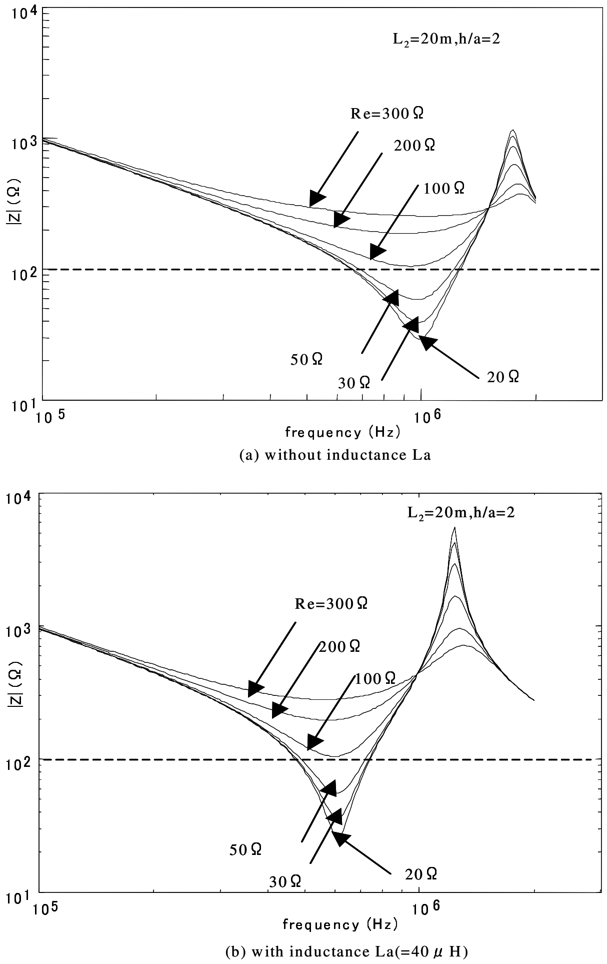


Fig.7 L_a dependency of calculated earthing resistance

Fig.7 (a) is the calculated result without the additive inductance L_a for 20, 30, 50, 100, 200 and 300 Ω. Fig.7 (b) is the calculated result when the additive inductance L_a is 40 μH. In Fig.7 (a) and Fig.7 (b), the horizontal axis shows the frequency in Hz and the vertical axis the impedance in Ω. In Fig.7 (a), the resonant frequency is about 900 – 1 MHz, in Fig.7, frequencies where impedances are the lowest are the resonant frequencies. The range is 550 – 600 KHz for Fig.7 (b). Calculated earthing resistances are almost the same as the values given. In earthing resistance measurements, it is important that the return wire should not function as an antenna. Therefore, the length of the return wire must be less than 1/10 or 1/20 of a wavelength. If the length of the return wire is set to 20 m, the measurement frequency must be less than 0.75 - 1.5 MHz.

Table.1 Typical soil resistivity (1/conductivity)

Soil	Soil resistivity (ohm-m)		
	Mini.	Mid.	Max.
Surface soils, loam, etc.	1		50
Clay	2		10^2
Sand and gravel	50		10^3
Surface limestone	10^2		10^4
Limestone	5		$4*10^3$
Shale	5		10^2
Sandstones	20		$2*10^3$
Granites, basalts		10^4	
Decomposed geneisses	50		$5*10^2$
Slates, etc.	10		10^2
Pastoral, low hills, rich soil		30	
Flat country, marshy, densely wooded		10^2	
Pastoral, medium hills and forestation		$2*10^2$	
Rocky soil, steep hills	10	$5*10^2$	10^3
Sandy, dry, flat, typical of coastal country	$3*10^2$	$5*10^2$	10^4

(2) Return wire length L_2 dependency

The calculated earthing resistance values when L_2 is set to 1, 2, 3, 5, 10, and 20 m are shown in Fig.8, where L_1 = 1 m, h/a = 2 , ε_w = 4, ε_e = 10, $\sigma_e = 10^{-2}$ s/m and Rg = 100 Ω.

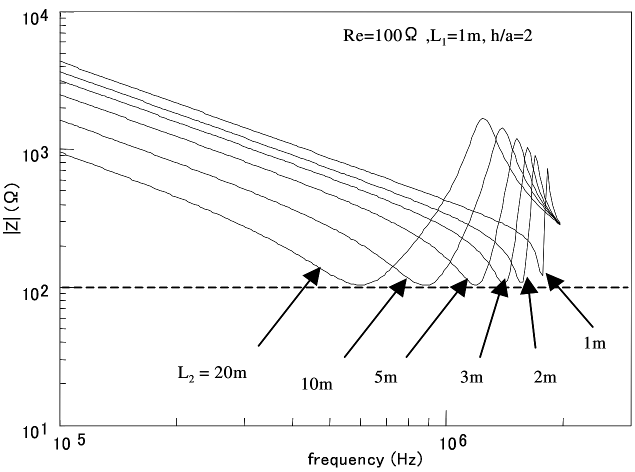


Fig.8 L_2 dependency of calculated earthing resistance

In Fig.8[8], the resonant frequency is proportional to the return wire length, when L2 is 20 m, the resonant frequency is about 550 - 600 kHz and the calculated earthing resistance value is 105 Ω. However, when L2 is 1 m, the resonant frequency is 1.8 MHz and the calculated earthing resistance value is 120 Ω.

(3) Soil conductivity σ_e dependency

Calculated earthing resistance values when σ_e is set to 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, and 10^{0} s/m are shown in Fig.9, where L_1 = 1 m, L_2 = 20 m, h/a = 2 , ε_w = 4, ε_e = 10 and R_g = 100 Ω. When soil conductivity σ_e is less than 10^{-3} s/m, the calculated earthing resistance is more than 140 Ω, and the error is more than 40%. However, when soil conductivity σ_e is higher than 10^{-2}, the calculated earthing resistance is about 103 - 105 Ω. In general, in an urban area, soil conductivity is usually higher than 10^{-2} s/m, which means that the new method can be used in a wide variety of environments.

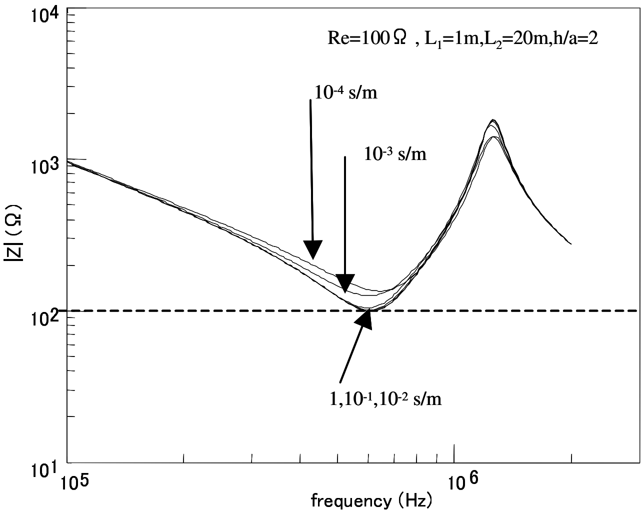


Fig.9 σ_e dependency of calculated earthing resistance

4. Development of earthing resistance tester

4.1 S parameter method (S_{11})

In general, earthing resistance measurement is not easy because there exists electrical noises in the field and ground soil. For examples, under 3MHz, radio waves, amateur radio wave, electric power noises, electric fault currents. The network analyzer can evaluate impedance under the test by using S_{11} measurement mode. The network analyzer has a bridge for impedance measurements as shown in Fig.10, and its characteristic impedance is set to 110Ω in this case. The network analyzer has two terminals for a lead wire and a return wire as shown in Fig.10. After doing open, short and standard (110Ω) corrections at the two terminals, we can obtain impedances between two terminals of the lead wire and the return wire by using S_{11} at each frequency.

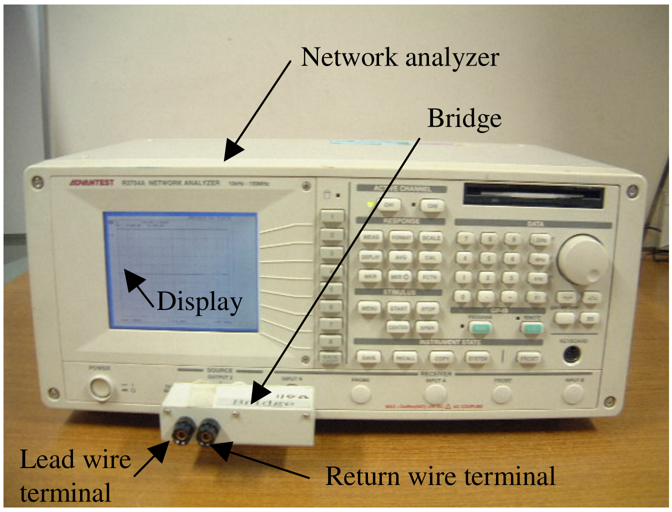
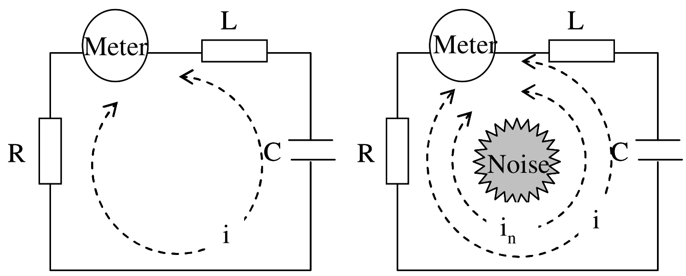


Fig.10 A developed network analyzer for measurement powered by batteries

4.2 New Tester

A network analyzer is a powerful piece of equipment for evaluating impedance in noisy environments. However, for our purposes it must be modified to be simple, portable and not heavy in real on-site measurement. There are several methods to evaluate impedances:

(1) Resonance method (RLC)
(2) Bridge method
(3) I-V method
(4) RF I-V method
(5) Other (S parameter method)

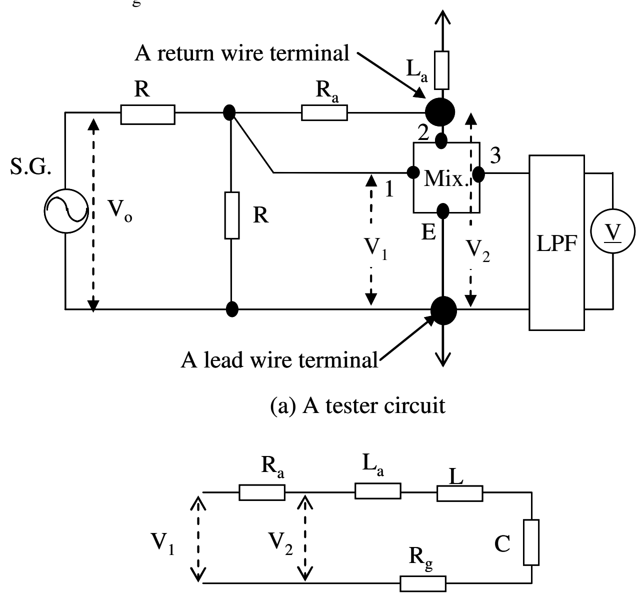


(a) without noise (a) with noise

Fig.11 Measurement systems for resonant method

For the loop impedance measurement of an earth return circuit, the resonance method [1] is suitable in this case. A measurement system using the resonance method is shown in Fig.11. Fig.11(a) is the measurement system without noise, where “i” indicates the circuit current. Fig.11 (b) is a measurement system with noise, where “i” is the circuit current and “i_n” is the current induced by noise. In Fig.11, R is a resistance that represents the earthing resistance, L is the inductance and C is the capacitance of the earth return circuit. ‘Meter’ is an impedance meter that can vary in frequency. At the resonant frequency, R is given where Meter shows the minimum resistance at the resonant frequency in Fig.11 (a). However, in real on-site measurement, there is noise, as shown in Fig.11 (b), such as radio waves and electrical fault currents, so it is not easy to evaluate R.

To overcome these problems, we propose the tester circuit shown in Fig.12. Fig.12(a) is the tester circuit, Fig.12(b) is an equivalent circuit. In Fig.12, S.G. is a signal generator, R is 50 Ω, R_a is the additive resistance 500 Ω, La is the additive inductance 40 μH, Mix. is an analog mixer with two input ports, one dc-out port and one earth port. LPF is a low pass filter; V is a dc voltmeter as shown in Fig.12 (a). In Fig.12 (b), R_g, L and C comprise the loop impedance of the earth return circuit being tested. R_g is the earthing resistance of the electrode being tested. R_a is the additive resistance, where R_a >> R and R_g.



(a) A tester circuit

(b) Equivalent circuit

Fig.12 A tester circuit

In Fig.12(b), V_1 and V_2 can be written as follows.

$$V_1 = V_1(\omega,t) = \frac{V_o(\omega,t)}{2} \quad \text{......................................} (14)$$

$$V_2 = V_2(\omega,t) = \frac{R_g V_0(\omega,t)/2}{R_a + R_g + j\{\omega(L+L_a) - \frac{1}{\omega C}\}} + V_n(t) \quad (15)$$

Where $V_0(\omega,t)$ is a signal generator producing a sinusoidal wave with an angular frequency ω. In Eq.(15), $V_n(t)$ represents the noise as a function of time. We define the following time integral:

$$\alpha = \frac{1}{T}\int V_1(\omega,t) * V_2(\omega,t)dt \quad \text{.................................} (16)$$

Eq.(16) shows the correlation between $V_1(\omega,t)$ and $V_2(\omega,t)$ where T is a finite duration. When the lead wire and the return wire are not connected, then are connected, to the earth return circuit, the following equations hold at the resonant frequency.

$$\alpha_{open} = \frac{1}{T}\int \frac{V_0(\omega,t)^2}{4}dt \quad \text{..} (17)$$

$$\alpha_{R_g} = \frac{R_g}{R_a + R_g}\frac{1}{T}\int \frac{V_0(\omega,t)^2}{4}dt \quad \text{...........................} (18)$$

where

$$\int V_0(\omega,t)V_n(t)dt = 0 \quad \text{...} (19)$$

Then we can write the following equation,

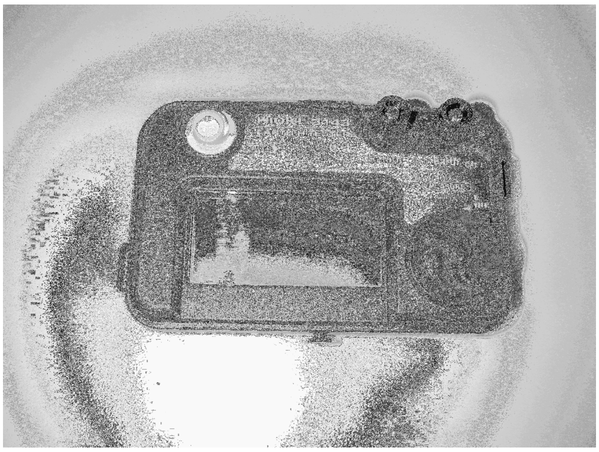
$$\beta = \frac{\alpha_{R_g}}{\alpha_{open}} = \frac{R_g}{R_a + R_g} \quad \text{..} (20)$$

and we can finally obtain the resistance of the earth electrode being tested as follows.

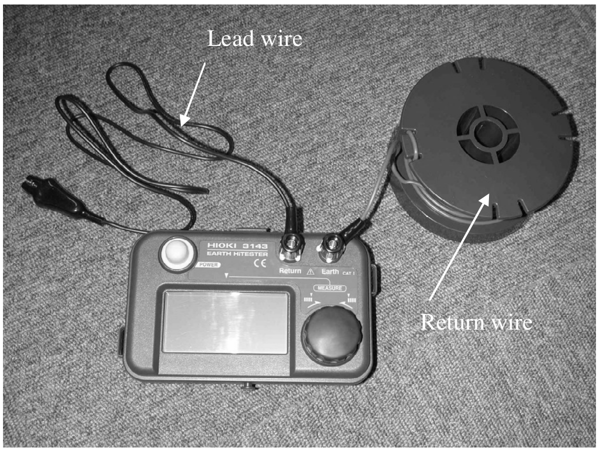
$$R_g = \frac{\beta R_a}{1-\beta} \quad \text{...} (21)$$

4.3 Experimental results

We have developed an earthing resistance meter provided by HIOKI.E.E CORP. as shown in Fig.13 with the test circuit shown in Fig.12. Fig.13 (a) shows a view of the new tester powered by 4 x 1.5 V dry cells. The tester has a power switch, a dial that can vary the oscillation frequency between 500 kHz and 1.5 MHz, and also has a lead wire terminal and a return wire terminal. Fig.13 (b) shows the new tester, and also shows the lead and return wires (1 m and 20 m respectively) connected to the terminals.



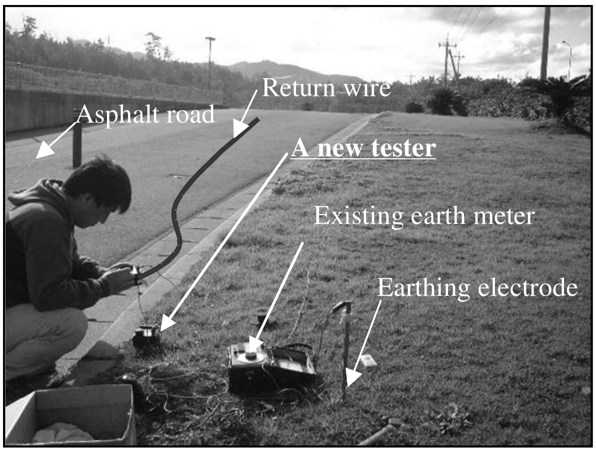
(a) Outlook of a new tester



(b) A new tester and a return wire and a lead wire (rolled)

Fig.13 A developed new tester

We have examined the performance of the new tester at about 20 sites in Japan. Several of the experimental setups are shown in Fig.14. Figs.14 (a) and (b) show the return wire placed on asphalt road, Fig.14(c) shows the return wire on tiles. We use an earth electrode 0.8 m long and 0.014 m in diameter, which is driven to the soil.



(a)A return wire is put on an asphalt road 1

(b) A return wire is put on an asphalt road 2

(c) A return wire is put on tiles

Fig.14 Experiment situations

By changing the depth of the electrode, the earthing resistance can be controlled. Measurement results are shown in Fig.15. The horizontal axis is the R_g value determined by the existing method (3-pole method) and the vertical axis is the R_g value determined by the new tester. Solid squares represent measured data, and the solid line is an approximated curve. The broken line is the ideal curve, which would be obtained if the R_g measured by the existing method were equal to that from the new tester. The actual measurements show some deviation between the methods, which is small around 100 - 150 Ω, but increases for earthing resistances above 200 Ω because the condition that $R_a >> R_g$ is not satisfied. Therefore, the new tester requires a correction table (values shown in Fig.15). The corrected measurements are shown in Fig.16. With corrections, the deviation between the methods is less than ±20% (Fig.16). The new tester can be used for earthing resistance measurements between 100 - 300 Ω, as shown in Fig.16.

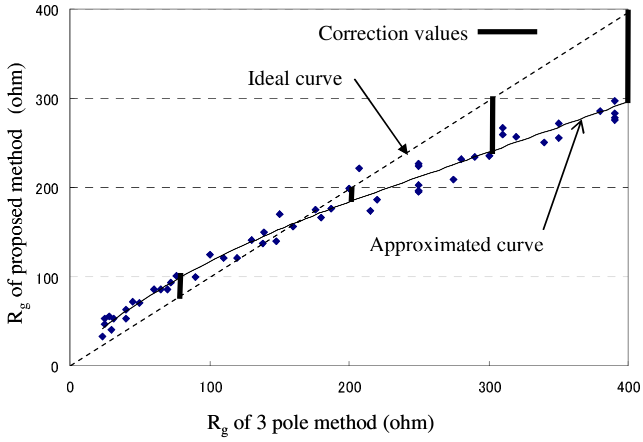


Fig.15 Experiment results without corrections

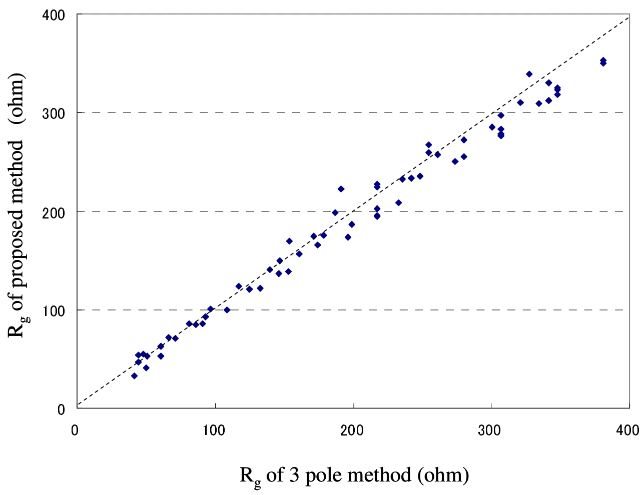


Fig.16 Experiment results with corrections

5. Conclusions

This paper proposes an earthing resistance measurement method without auxiliary electrodes. A theoretical analysis of the proposal was conducted using earth return circuit models. In this method, insulated lead and return wires are simply placed on soil surfaces without any termination. Numerical simulations of measurements were used to explore the expected outcomes, using a 1 m lead wire and a 20 m return wire. The soil resistivity must be less than 10^{-2} s/m, so the new method is applicable in urban or rural areas except where sand or stone creates high soil resistivity. Therefore, a new earthing resistance tester was developed based on voltage ratio measurements. The new tester was tried at more than 20 sites across Japan, including places where the soil surface was covered by asphalt, tiles, stones and sand. It was found that measurement deviations between the new tester and existing methods were not small, but that by using correction tables, the new tester can be used to measure earthing resistance where the value lies between 100 – 300 Ω. The new tester is a very simple measurement tool that can be very useful in the field, especially when auxiliary electrodes cannot be used. In the future, we would like to improve the measurement accuracy for low earthing resistances, perhaps down to the 10 - 30 Ω range.

References

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APPENDIX Calculation of earthing resistance

The transmission equations in Fig.6 can be represented as follows.

$$V_L(x) = V_L e^{-\gamma_0 x} + r_L e^{2\gamma_0 L_1} V_L e^{+\gamma_0 x} \quad \text{..................(A1)}$$

$$I_L(x) = I_L e^{-\gamma_0 x} - r_L e^{2\gamma_0 L_1} I_L e^{+\gamma_0 x} \quad \text{..................(A2)}$$

$$V_R(x) = V_R e^{-\gamma_0 x} + r_R e^{-2\gamma_0 L_2} V_R e^{+\gamma_0 x} \quad \text{.....................(A3)}$$

$$I_R(x) = I_R e^{-\gamma_0 x} - r_R e^{-2\gamma_0 L_2} I_R e^{+\gamma_0 x} \quad \text{.................... (A4)}$$

where the time function is $\exp(j\omega t)$ and is omitted in Eqs.(A1-A4), V_L and I_L are the voltage and current in the lead wire at x = 0, and V_R and I_R are the voltage and current in the return wire at x = 0, and r_L and r_R are the reflection coefficients of the lead wire and the return wire at each termination. Termination conditions of each wire are given as follows.

$$\frac{V_L(-L_1)}{I_L(-L_1)} = Z_0 \frac{1+r_L}{1-r_L} = Z_g \quad \text{................................ (A5)}$$

$$\frac{V_R(L_2)}{I_R(L_2)} = Z_0 \frac{1+r_R}{1-r_R} = \infty \quad \text{..................................... (A6)}$$

where

$$Z_0 = \frac{V_L}{I_L} = \frac{V_R}{I_R} \quad \text{... (A7)}$$

$$Z_g = \frac{R_g}{1 + j\omega C_g R_g} \quad \text{.. (A8)}$$

$$C_g = \frac{\varepsilon_e \varepsilon_0}{2\pi} \frac{1}{F(\frac{2\ell}{a})} \quad \text{... (A9)}$$

$$F(x) = \ln(x + \sqrt{1+x^2}) + \frac{1}{x} - \sqrt{1+\frac{1}{x^2}} \quad \text{................ (A10)}$$

where Z_g , R_g , C_g , ℓ and a are the earth electrode impedance, and its real part Z_g , its capacitance Z_g , the earth electrode length and the diameter of the earth electrode (7) , respectively.

In this case, $Z(\omega)$ at the signal generator Vs is given as the sum of the impedances of the lead wire and the return wire as follows.

$$Z(\omega) = \left.\frac{V_L(x)}{I_L(x)}\right|_{x=0} + \left.\frac{V_R(x)}{I_R(x)}\right|_{x=0} \quad \text{........................ (A11)}$$

Eq.(A.11) can be rewritten using Eqs.(A1)-(A4).

$$Z(\omega) = Z_0 \frac{1+r_L e^{+2\gamma_0 L_1}}{1-r_L e^{+2\gamma_0 L_1}} + Z_0 \frac{1+r_R e^{-2\gamma_0 L_2}}{1-r_R e^{-2\gamma_0 L_2}} \quad \text{................... (A12)}$$

When $|\gamma_0 L_1| << 1$, Eq.(A12) can be represented as follows.

$$Z(\omega) \cong Z_g + Z_0 \frac{1+r_R e^{-2\gamma_0 L_2}}{1-r_R e^{-2\gamma_0 L_2}} \quad \text{......................... (A13)}$$

When ε_e is 10, σ_e is 0.01 s/m , the length of the earth electrode is 0.8 m , and its diameter is 0.14 m , then C_g is about 0.32 nF. When R_g is 100 Ω and the frequency is 1 MHz, according to Eq.(A8), the earth impedance is estimated to be $96.3 - j18.9$ Ω. Therefore Eq.(13) can be rewritten as follows if the frequency is less than 1 MHz.

$$Z(\omega) \approx R_g + jXg + Z_0 \frac{1+r_R e^{-2\gamma_0 L_2}}{1-r_R e^{-2\gamma_0 L_2}} \quad \text{............. (A14)}$$

where X_g is the imaginary part of the earth electrode's impedance.

If the following resonant frequency condition is satisfied by changing the frequency,

$$jXg + Z_0 \frac{1+r_R e^{-2\gamma_0 L_2}}{1-r_R e^{-2\gamma_0 L_2}} = 0 \;\; or \;\; << R_g \quad \text{................ (A15)}$$

the earthing resistance of the earth electrode can be given as follows.

$$Z(\omega) \approx R_g \quad \text{..(A16)}$$