

Small Scale Drainage Network Problem Detection Moving beyond the Concept of ARF. A Preliminary Study.

CAMYLYN LEWIS¹, STEFANO MAMBRETTI²

¹Aqua-Assets LLC,
401 Barford Court, Chantilly,
VA, 20151,
USA

²Department of Civil and Environmental Engineering (DICA),
Politecnico di Milano,
Piazza Leonardo da Vinci 32, Milano,
ITALY

Abstract: - Historically, assessments of drainage networks rely on applying design rainfall to existing catchments over a set return period. Nevertheless, for large catchments, it is unrealistic to believe that long-return-period rainfalls would impact the entire region. As a result, in the past, design rainfall was decreased based on the total area of the catchment using specific Areal Reduction Factors (ARF). This methodology presents difficulties because rainfall is localised; it may rain on one side of the street but not on the other. Achieving a statistically significant length of rainfall records and a sufficient density of rainfall gauges is challenging, which complicates the calculation of design rainfall depths. Further complications arise with uncertainties in projecting a design rainfall depth in a changing climate. Modern calculation methods are making obsolete the traditional approach of simply applying a design rainfall on a whole catchment. This paper presents an alternative approach to designing and then testing the performance of a system. This is done by applying the rainfall centre to various sub-catchments of the network, identifying the worst-case scenario, which, by definition, corresponds to the critical situation for the given return period. The proposed procedure is applied to an ideal catchment, specifically created for the purpose of evaluation (an ideal catchment, being theoretically “perfect”), which facilitates a better understanding of the problems, because a real catchment with its own critical issues can mask the underlying problems.

Key-Words: - Drainage network assessment; areal reduction factor; .dynamic simulation.

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1 Introduction

The estimation of design precipitation at the catchment scale is a fundamental requirement in hydrology, underpinning flood risk assessment, hydraulic infrastructure design, and water resources management. While precipitation is inherently a spatial process, it is most commonly measured at discrete locations through rain gauges, resulting in point-based estimates of rainfall extremes. To reconcile this mismatch between point observations and areal hydrological applications, the concept of the Areal Reduction Factor (ARF) has been widely adopted. ARF is conventionally defined as the ratio between the areal average precipitation over a given domain and the corresponding point precipitation for the same duration and return period [1]. This factor provides a practical means to transform

Intensity-Duration-Frequency (IDF) relationships derived from point data into areal estimates suitable for hydrological modeling and engineering design.

The ARF concept has been embedded in hydrological practice for decades and remains a standard component in many national design guidelines. Its importance stems from the recognition that extreme rainfall observed at a point is unlikely to occur uniformly over larger areas due to the spatial variability of precipitation fields. Empirical studies consistently show that ARF values decrease with increasing catchment size and increase with rainfall duration, reflecting the scale-dependent structure of storm systems [2]. Consequently, ARFs serve as a key scaling mechanism linking local rainfall extremes to basin-scale hydrological responses.

Despite its widespread use, the ARF framework is subject to several conceptual and practical limitations. First, ARFs are typically derived empirically from historical datasets, making them highly sensitive to data quality, record length, and spatial coverage. In regions with sparse rain gauge networks, the estimation of areal precipitation – and thus ARFs – remains highly uncertain [1]. Second, different methodological approaches often yield significantly different ARF estimates, with substantial variability [3]. This methodological inconsistency raises concerns regarding the transferability and robustness of ARF relationships across regions and climatic conditions.

Moreover, ARFs implicitly assume a simplified representation of rainfall spatial structure, often neglecting storm dynamics, orientation, and intra-event variability. Recent studies have demonstrated that factors such as storm type, spatial variability, and rainfall organization can induce considerable deviations in ARF values, sometimes exceeding 20% for the same nominal conditions [2]. These findings highlight the limitations of representing complex precipitation fields through a single scalar reduction factor. Additionally, traditional ARF approaches may lead to systematic biases in design applications, including the potential overestimation or underestimation of design rainfall, with direct implications for infrastructure cost and safety [4].

In recent years, advances in remote sensing technologies, particularly weather radar and satellite-based precipitation products, have significantly improved the spatial and temporal resolution of rainfall observations. These developments challenge the traditional reliance on ARFs by enabling the direct estimation of spatially distributed precipitation fields. Radar-based analyses, for instance, reveal systematic differences between point-derived and spatially aggregated rainfall extremes, suggesting that conventional ARF relationships may not adequately capture the true scaling behavior of precipitation [5]. Furthermore, high-resolution gridded datasets allow for the explicit representation of rainfall heterogeneity, reducing the need for empirical scaling factors.

Given these advancements, there is a growing recognition within the hydrological community that the ARF concept may be increasingly inadequate for modern applications. Rather than relying on static reduction factors, contemporary approaches advocate for the use of spatially distributed rainfall inputs and physically based modeling frameworks that explicitly account for precipitation variability. Moving beyond ARFs has the potential to reduce

epistemic uncertainty, improve the reliability of extreme rainfall estimation, and enhance the design of resilient hydraulic systems under non-stationary climate conditions. Accordingly, re-evaluating and ultimately overcoming the ARF paradigm represents a critical step toward next-generation hydrological modeling and risk assessment.

The aim of this paper is to verify the differences that can be found in identifying the critical points of a drainage network, when using a hyetograph for the entire catchment and when using concentrated hyetographs, applied to the different sub-catchments.

2 Literature Review

2.1 General Considerations

The complexity of hydrological phenomena occurring in a catchment during an intense event is still far from being fully understood and, therefore, simulated through calculations. Several factors increase the uncertainty, including the spatial and temporal variability of the forcing (rainfall) and the terrain response, which in turn depends in part on the conditions preceding the event being considered [6].

In this paper, however, the focus is on the rainfall simulation only. In this case also, the hydrological response in urban areas is highly sensitive to rainfall properties and the same structure of the city alters the rainfall characteristics. For instance, in [7] a resolution of 4- or 5-min and 1 km to assess the mentioned characteristics has been used.

It cannot be denied that on a large scale, for large catchments, the use of areal reduction coefficients (ARF) to assess the global catchment behavior is entirely adequate. In [5] it is shown that the differences obtained, over a 20,000 km² catchment, between radar images and ground-based devices lead to estimates of different ARF values, but within the range of 11-32%. Spatial (2 km²) and temporal (15–30 min) resolutions are very high for a catchment of such large dimensions; but, as will be shown in the next paragraph, for small urban catchments this resolution does not allow the identification of local problems. Similarly, in [8] it has been found that rainfall resolution of 1,000 m-15 min is clearly not sufficient for small catchments.

A wide range of methods have been developed to estimate ARFs, broadly categorized into gauge-based, radar-based, and stochastic approaches.

Gauge-based methods rely on networks of rain gauges to compute areal averages. These methods are limited by spatial sampling density and often require interpolation techniques such as Thiessen polygons or kriging. While widely used, gauge-based ARFs are particularly sensitive to network sparsity and may fail to capture localized rainfall extremes.

Storm-centered approaches represent a significant methodological advancement, focusing on the spatial structure of individual storm events rather than fixed geographic areas. These methods often yield higher ARF values compared to fixed-area approaches, as they better capture the peak intensities within moving storm systems [4].

Radar-based methods have gained prominence with the increasing availability of high-resolution precipitation data. Radar observations provide continuous spatial coverage, allowing for more accurate estimation of rainfall fields and their spatial variability. Studies have shown that radar-derived ARFs can differ substantially from gauge-based estimates, particularly for short-duration, high-intensity events [9].

Stochastic and scaling-based approaches attempt to generalize ARFs using statistical or fractal representations of rainfall fields. These methods often rely on assumptions of spatial self-similarity and have been used to derive theoretical scaling relationships between point and areal rainfall extremes. However, their applicability is often constrained by the complexity of real-world precipitation processes.

2.2 Sources of Uncertainty and Variability

Despite their widespread use, ARFs are subject to significant uncertainty arising from multiple sources.

One major source of uncertainty is data limitation, particularly in regions with sparse rain gauge networks. In such cases, areal rainfall estimates are highly uncertain, leading to unreliable ARF values. Even in well-instrumented regions, measurement errors and inconsistencies in data processing can introduce biases.

Methodological inconsistency is another crucial problem. Different ARF estimation techniques – such as fixed-area Vs. storm-centered approaches – can yield markedly different results. Intercomparison studies have demonstrated that these differences can exceed 20%, even when applied to the same dataset. This lack of methodological standardization limits the transferability of ARFs across regions [3].

ARFs are also highly sensitive to storm characteristics, including storm type (convective Vs. stratiform), spatial organization, and temporal evolution. Convective storms, characterized by high spatial variability, typically result in lower ARFs compared to stratiform systems, which exhibit more uniform rainfall distributions.

Recent studies [10] show that ARFs decrease with increasing area, increase with increasing duration, and decrease with increasing return period. In particular, there is a strong geographical variability across different hydrologic regions, suggesting that ARFs are specific to regional climate patterns and geographical characteristics and should not be applied arbitrarily to other locations.

This raises concerns regarding the use of generalized ARF relationships in design guidelines.

Finally, the assumption of stationarity inherent in traditional ARF estimation is increasingly being challenged. Climate change is altering precipitation patterns, including the intensity and spatial structure of extreme events. As a result, ARFs derived from historical data may no longer be representative of future conditions.

2.3 ARFs Estimation

In [11], an extensive review of the different estimation methods for the ARF value is presented, also considering radar images. The Authors found that the relationship between point and areal extreme rainfall, and consequently the ARF values, depends on several factors: characteristics of the rainfall (as duration, intensity, area extent, seasonality, return period), of the area (catchment features, shape and topography), of the data (as length of time series, density of the rain gauge network, time resolution, data type: gauge- or radar-based). They have to conclude that, considering the wide variety of different empirical algorithms proposed by the literature, a further effort from the scientific community would be required in order to select the most appropriate approach.

A rule-of-thumb can be considered the classification reported in [12] and shown in Table 1 (Appendix), from the papers which study the decorrelation distance (and decorrelation time), defined as the distance (and time) from which two points show independent statistical behavior, and it is obtained as the range of the climatological variogram [13]. The first group, characterized by light rainfall events, presented very high decorrelation distance and time (17 km and 15 min) compared to the second group, with a decorrelation

distance of 5 km and a decorrelation time of 5 min. The last two groups presented a double structure, where small and intense clusters, with low decorrelation distance and time (less than 5 km and 5 min), are located in a random or organized way inside areas with a lower variability (decorrelation of 15 km and 15 min).

Similarly, [14] presented a study about variability in accumulated rainfall within a single radar pixel of 500m x 500m, comparing it with 9 rain gauges located in the same area.

In [15] it is emphasized the need to consider two types of ARF. The former, to simulate real events; the latter, to verify the network with design events. The first case finds ARF values that can be higher than 1, especially in the case of recorded low-intensity events; this appears entirely reasonable because in the case of real events that give rise to low recorded rainfall intensity values, it is reasonable to expect that the rainfall center is not located on the vertical line of the rain gauge. In the second case, the ARF value is always less than one, because the design rainfall is assumed with a given return period and it is not reasonable to expect this to have the same intensity over a very large area.

After a literature review about the discussion related to the importance of the return period for ARF coefficient determination, [4] developed a three-parameter relationship describing the areal reduction factors as a function of rainfall duration and area, calibrated against 6408 individual combinations of storm, area, and duration. In this case, it is to be believed that the return period is implicitly considered, being tied to the duration. However, the confidence limits still appear to be quite large.

2.4 Toward Overcoming the ARF Paradigm

Given the limitations of traditional ARFs, there is a growing body of research advocating for alternative approaches that move beyond empirical reduction factors.

One promising direction is the use of spatially distributed rainfall inputs in hydrological models. By directly incorporating gridded precipitation data, these models can explicitly account for spatial variability without the need for ARFs. This approach is particularly advantageous for small to medium-sized catchments and urban hydrology applications, where spatial heterogeneity plays a critical role.

Another approach involves the development of stochastic rainfall generators that simulate spatially consistent precipitation fields. These models can reproduce the statistical properties of rainfall,

including extremes, and provide a more robust basis for design applications.

Additionally, scaling frameworks based on multifractal theory have been proposed to describe the relationship between point and areal rainfall extremes. While still an active area of research, these approaches offer a more theoretically grounded alternative to empirical ARFs.

Ultimately, the shift toward physically based and data-driven methodologies reflects a broader transition in hydrology, determined by advances in observation technologies and computational capabilities [16], [17]. Overcoming the ARF concept does not necessarily imply its complete abandonment but rather its integration into more comprehensive frameworks that better capture the complexity of precipitation processes.

3 Local Problem Identifications

Comparing data from different raingauges in the city of Milano, in the North of Italy, it is clear that in case of intense events differences are recorded also for short distances – all below three kilometers. In the city 17 raingauges are installed, but here only the results of three of them are reported, for the sake of brevity. The “Monviso” raingauge has been selected as pivotal, because it is the oldest and most controlled in the town. In Table 2 (Appendix), the positions and the GPS coordinates are reported for the three raingauges.

Distances are as follows:

- Marino – Monviso: 2.58 km
- Marino – Sondrio: 2.82 km
- Monviso – Sondrio: 2.57 km

As can be seen in Figure 1 (Appendix), on August 25th, 2005, two events have been recorded (time step equal to 1 min). In the former event, the following data have been recorded (between 11:00 and 13:30).

- Marino: maximum intensity 12 mm/h at 12:48 and 12:49 (one tip of the bucket is 0.2 mm, so two tips have been recorded in two minutes). Total depth: 0.4 mm.
- Sondrio: maximum intensity 48 mm/h at 12:01, 12:02, 12:13, 12:15. Total depth: 20.2 mm.
- Monviso: maximum intensity 120 mm/h at 12:03. Total depth: 50.6 mm.

For the latter event, the following data have been recorded (between 16:00 and 20:30).

- Marino: maximum intensity 120 mm/h at 17:20. Total depth: 44.4 mm.
- Sondrio: maximum intensity 60 mm/h at 17:30. Total depth: 32.8 mm.
- Monviso: maximum intensity 48 mm/h at 17:11, 17:17, 17:24, 17:27, 17:28, 17:30. Total depth: 32 mm.

As can be seen, the peaks are quite intense, but still the return period is below two years. As the return period increases, the differences coherently increase. The rainfalls in the city of Milano have been studied, among others, in [7], with a high-resolution radar, believing that short-duration rainfall events caused by convection are the most relevant for urban flooding. It is possible that these effects are associated or amplified by the presence of heat islands

As a preliminary observation, it can therefore be said that the application of a single hyetograph to an area, even when not excessively large, allows the identification of average behavior and deficiencies, but does not identify local critical issues.

4 The Theoretical Catchment

4.1 Catchment Geometry

In order to evaluate the procedure in the most objective possible way, an ideal catchment was used.

It is fundamental to emphasize that the Authors intentionally designed a theoretically “perfect” ideal catchment, where every single pipe, node, and sub-catchment parameter strictly complies with standard hydraulic design criteria. In a real-world case study, existing structural anomalies - such as historical construction bottlenecks, localized counter-slopes, localized siltation, or sub-optimal pipe dimensioning errors - would inevitably introduce site-specific physical noise. Such localized vulnerabilities would mask the primary objective of this research. By simulating a perfectly dimensioned asset under uniform baseline conditions, any hydraulic failure, backwater effect, or surface flooding that emerges during spatial testing can be exclusively and rigorously attributed to the intrinsic mathematical limitations of the traditional Areal Reduction Factor (ARF) paradigm, rather than to local engineering flaws.

The constructed catchment is of 10 km² (using the well-known SWMM computer program) composed of 100 sub-catchments of ten hectares

each, all equal and with the following characteristics:

- Area: 10 ha = 100,000.00 m² = 0.1 km²
- Imperviousness: 25%
- Length = 237.17 m = $0.75 \cdot \sqrt{A}$ (see below)
- Width = 421.64 m
- Slope 0.5%
- Manning impervious areas: 0.025 m^{-1/3} s
- Manning pervious areas: 0.1 m^{-1/3} s
- Depression storage impervious area: 0.05 mm
- Depression storage pervious area: 0.05 mm

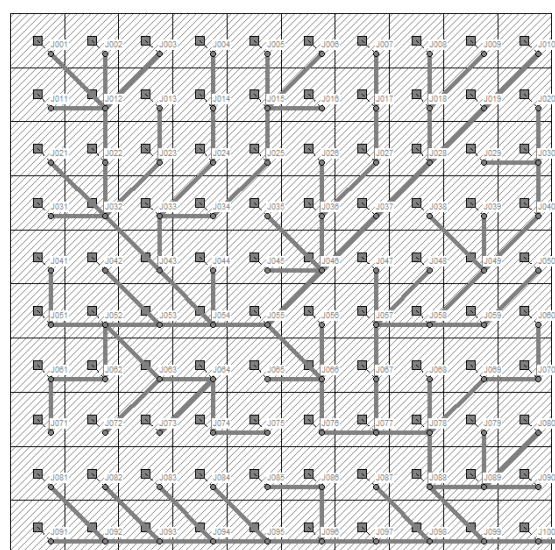


Fig. 2: Topology of the “ideal” experimental catchment

Source: created by the authors

The used infiltration model is Horton’s and the parameters are:

- Max infiltration rate: 3.0 mm/h
- Min infiltration rate: 0.5 mm/h
- Decay constant: 4 h⁻¹
- Drying time: 7 days

The network topology was constructed as shown in Figure 2 to avoid potential biases due to symmetry. All pipes are circular and have the same length (475 m) and slope (0.5%), while the diameter varies as a function of the drained area. The design flow rate was computed using the expression that links the critical duration to the size of the drainage area using the kinematic method on a flat surface, and the diameter was sized such that the water level was approximately 70% of the depth. Return period for dimensioning has been set to 5 years.

4.2 Rainfalls

The Depth-Duration-Frequency (DDF) curves used in this study are derived from the historical climatological dataset of the city of Milano, Italy. The baseline analytical formulation allows the determination of cumulative rainfall depth $h[mm]$ for any assigned return period $T[years]$ and storm duration $\theta[hours]$:

$$h = \mu(\theta) \cdot \xi(T) = \frac{36.9}{(0.104 + \theta)^{0.754}} \cdot \left(0.841 - 4.316 \cdot \left(1 - \left(-\ln \left(1 - \frac{1}{T} \right) \right)^{-0.06} \right) \right)$$

For a specific return period T , the general DDF relationship simplifies into a standard power-law formulation:

$$h = \frac{A(T)}{(0.104 + \theta)^{0.754}}$$

where $A(T)$ is the coefficient depending on the return period T , whose values are reported in Table 3.

Table 3. IDF coefficients for the different return periods.

Return period [years]	$A(T)$
2	34.57
5	46.03
10	54.06
20	62.11

Source: created by the authors on the base of the historical climatological dataset of the city of Milano, Italy

4.3 Pipe Dimensioning

Pipes within the synthetic network were dimensioned by translating hydrological rainfall forcing into hydraulic design flows via the classic rational kinematic method. For any given node draining an aggregated catchment area $A_{Tot} = \sum_i A_i$, the peak design discharge $Q_d[m^3/s]$ is modeled as:

$$Q_d = C \cdot i(t_c) \cdot A_{Tot}$$

where C is the global dimensionless runoff coefficient (weighted based on the 25% impervious layout), and $i(t_c)$ is the critical rainfall intensity evaluated at the basin's aggregate time of concentration t_c . The time of concentration t_c for each individual sub-catchment grid is computed

analytically using the kinematic wave translation formulation [12]:

$$t_c = \frac{(n \cdot L)^{3/5}}{i^{2/5} \cdot s^{3/10}}$$

where:

- n is the Manning roughness coefficient [$m^{-1/3} s$];
- L is the length of the sub-catchment [m]
- i is the rainfall intensity [$m s^{-1}$]
- s is the sub-catchment slope [-]

To capture structural safety margins, the rainfall intensity applied to pervious areas is reduced by the minimum infiltration capacity governed by Horton's infiltration model (*i.e.* equal to 0.5 mm/h). The same applies for the Manning coefficient, which is therefore set equal to 0.01 $m^{-1/3} s$.

Substituting the IDF curve in the concentration time equation gives:

$$t_c = \frac{(n \cdot L)^{3/5}}{\left(\frac{46.03}{(0.104 + t_c)^{0.754}} \cdot \frac{1}{t_c} \right)^{2/5} \cdot s^{3/10}}$$

As mentioned above, to define the catchment length, the following formula has been used:

$$L = 0.75 \cdot \sqrt{A}$$

Pipes also have a slope of 0.5% and a Manning coefficient equal to 0.014 $m^{-1/3} s$. To verify and size the conduit diameters under uniform open-channel flow regimes, the Manning-Strickler equation is deployed.

To prevent premature pressurization and allow safe freeboard dynamics during the baseline 5-year return period design forcing, the maximum filling depth criterion y/D approx 70% $\pm$ 15% was strictly enforced across all commercial diameter selections.

Due to the large aggregated upstream contributing areas processing cumulative volumes without intermediate overflows, the resulting nominal diameters scale logically from an upstream baseline of 0.9 meters up to major downstream collector units of 3.2 meters. While pipes of such dimensions in practice are frequently shaped as horizontal elliptical profiles, a uniform circular cross-section was maintained here to isolate hydraulic boundary responses and validate the spatial permutation methodology without introducing complex cross-sectional geometry artefacts.

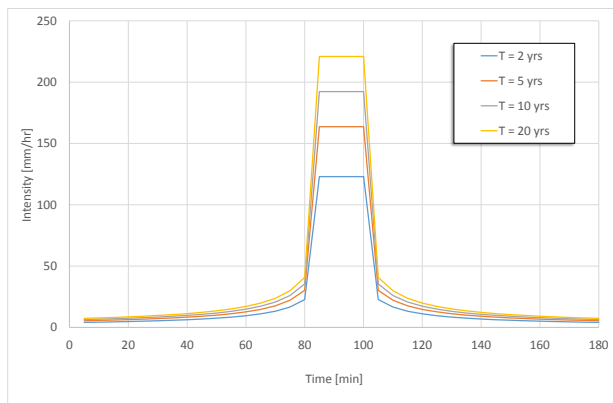


Fig. 3: Chicago hyetographs for the different return periods

Source: created by the authors

5 Simulations

5.1 Hyetographs

The Chicago-type hyetographs were defined for return periods of 2, 5, 10 and 20 years (Figure 3). The base of the hyetograph is equal to 3 hours, *i.e.*, much longer than the maximum time of concentration of the whole catchment, estimated around 1.7 hours. The hyetographs are shown in Figure 3. Moreover, uniform hyetographs have also been used for preliminary tests.

To quantify the calculated criticality of the network, a height value was set exactly equal to the maximum height available for the junction (*i.e.*, up to the manhole cover) and a pond area equal to ten square meters. This area is very small and is intended solely to highlight the amount of water outflowed from the system, as a function of the simulated rainfall.

Parameters to be evaluated are described in the following.

5.2 A Single Hyetograph on the Entire Catchment

The first test performed has been carried out using a constant hyetograph, applied across the entire catchment with an intensity slightly above that related to the concentration period of 1.7 hours for a five-year return period (we used 30 mm/h).

Because the catchment leads to a hydrograph with two peaks in the final portion (Figure 4, Appendix), the pipes are confirmed to be significantly larger than necessary, also because the generated discharge appears to be easily contained within the proposed network. Therefore, no floods are recorded.

Things become significantly more complicated, and the network becomes inadequate, if a Chicago

hyetograph is applied, even with a two-year return period, which is lower than the design return period (five years); in this case, crises occur, as shown in Figure 5 (Appendix). It is observed that even the initial catchments are subject to crisis; this is counterintuitive, because the initial pipe is designed with a correct critical duration for the sub-catchment and a five-year rainfall return period. The Chicago hyetograph, which is critical for any duration, should not create a crisis.

But Figure 5 (Appendix) itself suggests the reason for this by looking at the head grade line, which is less steep than the conduit: this means that the system crisis is triggered from downstream. Ultimately, as mentioned, a crisis condition exists for all sub-catchments.

However, in the Authors' opinion, these crises are not real; it is untrue that a network crisis such as the one in question has such a short return period. Indeed, it should be noted that the use of a Chicago hyetograph implies the assumption that in each sub-catchment there is a critical rainfall event for that sub-catchment. Therefore, critical events occur almost simultaneously across the one hundred sub-catchments that comprise the network under consideration. Since this hypothesis is merely untrustworthy, the application of an ARF should not be taken into consideration in this situation. As mentioned in the previous sections, this approach may have made sense in past years; nowadays, with the computational developments that have taken place, a different solution has to be devised.

5.3 Hyetographs on Different Sub-Catchments

To systematically explore network vulnerability without relying on rigid empirical scalar values like the ARF, this work introduces an automated spatial permutation framework. The catchment C is discretized into a set of N sub-catchments (in this case $N=100$), denoted as $C = \{c_i | i = 1, 2, \dots, N\}$.

For each individual simulation scenario s_i (where $i \in [1, N]$), a localized intense storm event is centered on cell c_i . The spatial forcing function maps a non-uniform precipitation distribution across the domain, applying a high-intensity hyetograph $H(T_{20})$ of 20-year return period to the target core element c_i , while the immediate topological neighborhood sub-catchments receive a lower-intensity hyetograph $H(T_2)$ of 2-year return period.

For instance, when the convective core is assigned to sub-catchment c_{55} , the meteorological forcing dictates that c_{55} is forced by $H(T_{20})$,

whereas its first-order spatial neighbors $\{c_{44}, c_{45}, c_{46}, c_{54}, c_{56}, c_{64}, c_{65}, c_{66}\}$ are subjected to $H(T_2)$. All remaining boundary sub-catchments within the domain are set to a zero-rainfall baseline state.

By iterating this spatial allocation across all N sub-catchments, a comprehensive matrix of 100 deterministic simulations is generated. The resulting systemic response cannot be aggregated into a trivial mono-objective scalar, as the networks' critical limits depend on independent spatial configurations. To quantify this behavior, a multi-objective performance vector is defined:

$$Y(s_i) = [N_f(s_i), V_{max}(s_i), V_{tot}(s_i)]^T$$

where:

- $N_f(s_i)$ is the cardinality of the subset of flooded cells under scenario s_i .
- $V_{max}(s_i) = \max_j (V_j(s_i))$ represents the maximum local flood volume recorded within a single critical node j .
- $V_{tot}(s_i) = \sum_{j=1}^N V_j(s_i)$ represents the total flood volume over the entire network for the event.

The empirical execution of the 100 simulations reveals high objective function divergence, with $N_f(s_i)$ scaling non-linearly from 1 up to 10 flooded cells. This can be obtained by seven distinct spatial centers.

From an engineering perspective, this spatial permutation strategy successfully isolates localized bottlenecks. When the network is subjected to localized rainfall with a 20-year return period, upstream pipes may experience crises due to insufficient storage volume relative to the localized input. Conversely, downstream trunk lines - dimensioned for higher aggregated historical cumulative areas - absorb the localized volumetric shock without immediate surcharging, though local upstream hydraulic failure propagation to contiguous links is observable.

Mathematically, the critical scenarios do not converge toward a unique spatial center, demonstrating a Pareto-like distribution where different locations optimize different vulnerability metrics:

- the maximum local flood volume $V_{max}(s_i)$ is recorded when $i = 89$ and it results $N_f = 4$
- the total flood volume over the entire network is recorded when $i = 79$ and it results $N_f = 6$

Figure 6 (Appendix) shows the results obtained by positioning the rain center over each of the one hundred sub-catchments.

To evaluate the statistical interdependence of these vulnerability objectives, Pearson correlation coefficients ρ were computed across the entire simulation matrix series:

- $\rho(N_f(s_i), V_{tot}(s_i)) = 0.65$
- $\rho(N_f(s_i), V_{max}(s_i)) = 0.36$
- $\rho(V_{tot}(s_i), V_{max}(s_i)) = 0.86$

For the 100 simulations, the correlations between the number of flooded cells and the maximum flooded volume, between the number of flooded cells and the total flooded volume, and between the maximum and total flooded volumes were all statistically significant ($p < 0.001$ in all cases). The different correlation strengths nevertheless indicate that the three indicators capture different aspects of the flood response.

The weak-to-moderate coupling between $N_f(s_i)$ and $V_{max}(s_i)$ (being $\rho = 0.36$) highlights the insufficiency of standard empirical design approaches: a network's "critical status" is not an intrinsic scalar constant, but a function of the selected mathematical target parameter. In real-world urban infrastructure applications, this mathematical optimization space should ideally be coupled with a discrete land-use weight vector to transform hydraulic criticalities into direct economic or social expected risk metrics (see *e.g.* [18]).

In other words, this result mathematically demonstrates that a single, objective "critical" event does not exist; instead, the criticality depends entirely on which metric of the performance vector $Y(s_i)$ is prioritized. To resolve this ambiguity and determine which scenario constitutes the actual "worst-case", an external operational objective function must be introduced, through an expected damage function $D(Y(s_i))$:

$$D(Y(s_i)) = \sum_{j=1}^N w(c_j) \cdot f(V(s_j))$$

where $f(V(s_j))$ represents the local depth-damage curve for node j , and $w(c_j)$ is a land-use socio-economic weight vector reflecting exposed assets (*e.g.*, residential areas, hospitals, or critical transportation nodes versus green spaces). Therefore, the definition of the critical storm center

is not a purely mathematical or physical property of the network, but a multi-criteria decision-making problem where social utility and economic risk tolerance dictate the optimization objective. This framework effectively bridges the gap between raw technical simulations and practical, risk-aware urban planning.

Because this matrix simulation routine is easily automated through scripting wrappers around the SWMM computational engine, it provides a scalable, physically consistent diagnostic framework that outperforms traditional lumped ARF intensity reductions by mapping the real spatial vulnerability topography of the asset.

6 Concluding Remarks and Further Research

This preliminary study formalizes a spatial permutation paradigm that maps localized network vulnerabilities, moving past the structural limitations of lumped empirical scaling. The empirical findings support the theoretical premise that a single, spatially uniform hyetograph cannot adequately capture the hydraulic response of distributed catchment systems.

From a probabilistic perspective, applying a critical Chicago hyetograph simultaneously across an entire basin implies a joint event probability that violates independent spatial correlation structures. If each sub-catchment c_i undergoes a local extreme event E_i with a nominal return period T , the joint probability of simultaneous occurrence across the domain satisfies:

$$P\left(\bigcap_{i=1}^N E_i\right) \ll P(E_i) \Rightarrow T_{joint} \gg T$$

Consequently, traditional lumped approaches drastically overestimate downstream system stress while masking localized upstream bottlenecks.

Future research trajectories will expand this computational framework across three primary dimensions.

First, the scalability of the proposed method must be validated against large-scale, heterogeneous real-world catchments. This validation is critical given the observed breakdown of historical hydrological stationarity due to climate change. Future formulations will model the IDF coefficients as time-dependent stochastic variables to evaluate network resilience under evolving non-stationary forcing fields.

Second, while the current framework assumes a static spatial allocation of the convective core, real-world precipitation fields exhibit dynamic translation. The next iteration of this model will formalize the storm center as a continuous kinematic trajectory vector $\vec{x}_0$:

$$\vec{x}_0 = \vec{x}_{start} + \vec{v} \cdot t$$

where $\vec{x}_{start}$ represents the spatial boundary entry coordinate, and $\vec{v}$ is the translation velocity vector.

The precipitation field will be generalized as a space-time dependent intensity function $I(\vec{x}(t, \tau))$, incorporating a temporal peak decay operator τ .

The third line of research depends closely on the first two above highlighted. While the stationary study as presented in this paper can be performed even if the number of sub-catchments increases considerably, when moving storms with varying peak intensities are considered, the brute-force simulation approach will face the curse of dimensionality, rendering full enumeration computationally intractable.

To isolate the global critical scenario, future work will deploy a hybrid metaheuristic framework combining Genetic Algorithms (GA) and Ant Colony Optimization (ACO).

Genetic Algorithms (GA) will be used as a global search operator to optimize the continuous-discrete vector

$$\vec{u} = [\vec{x}_{start}, \vec{v}, \tau]^T$$

maximizing the system multi-objective vulnerability vector $Y(s_i)$.

To ground this kinematic formulation in empirical reality and validate the boundaries of the optimization space, future benchmarks will integrate spatially distributed rainfall fields derived from weather radar data. Instead of sampling the velocity vector $\vec{v}$ and the translation trajectory $\vec{x}_0$ from an unconstrained uniform distribution, historical radar climatology will be processed to extract empirical probability density functions of storm cell dynamics, such as tracking vectors, spatial morphology, and localized peak decay rates τ . These radar-driven patterns will serve a dual purpose: first, as a real-world benchmark to validate the structural damage configurations identified by the ideal synthetic storms; second, as informative priors within the metaheuristic search space, restricting the Genetic Algorithm and ACO paths to meteorologically feasible trajectories and

significantly reducing computational convergence time.

Ant Colony Optimization (ACO) will be used to: deploy to map the discrete graph-theoretic topology of the drainage network. The pheromone matrix will encode the probability of structural failure propagation and cascade bottlenecks across interconnected pipes, solving the localized multi-path hydraulic routing optimization problem.

By synthesizing kinematic storm modeling with hybrid GA-ACO optimization, this framework will transition from a predictive case study into a mathematical stress-testing environment for resilient urban infrastructure design.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

The authors wrote, reviewed and edited the content as needed and they have not utilised artificial intelligence (AI) tools. The authors take full responsibility for the content of the publication.

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Contribution of individual authors to the creation of a scientific article (ghostwriting policy)

The authors contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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APPENDIX

Table 1. Characterization of rainfall events, spatial and temporal scales, and rainfall estimation uncertainty

	Characterization and Intensity	Spatial range	Temporal range
Light rainfall	1 mm/h	17 km	15 min
Convective cells	Short and intense from 25 mm	5 km	5 min
Organized stratiform	Up to 17 mm/h	< 5 km	< 5 min
Unorganized stratiform	High-intensity core, combined with low-intensity areas	15 km	15 min

Source: [12]

Table 2. Position of three rain gauges in the city of Milano (North of Italy)

Elevation [m asl]	Position (street)	Name	GPS coordinates
120.60	Piazza Scala 2	Marino	45.466890, 9.190612
125.20	Via Muzio 5	Sondrio	45.490795, 9.200154
125.40	Via Monviso 6	Monviso	45.483959, 9.168915

Source: created by the authors

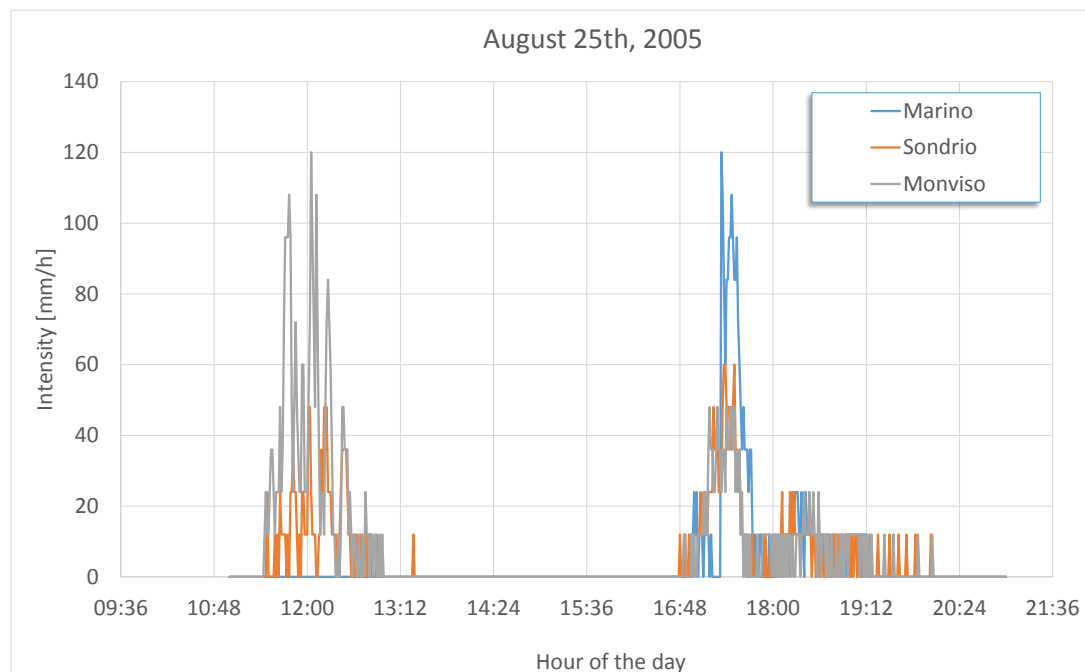


Fig. 1: Hyetographs recorded in three raingauges in Milano

Source: created by the authors

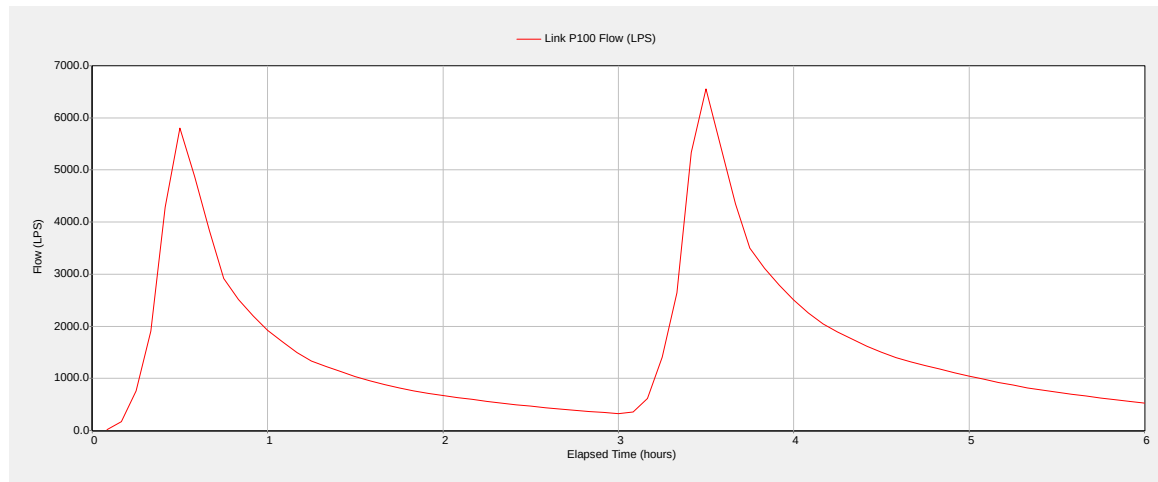


Fig. 4: Hydrograph outflowing the catchment following the application of a uniform hyetograph, with a 5-year return period

Source: created by the authors using SWMM.

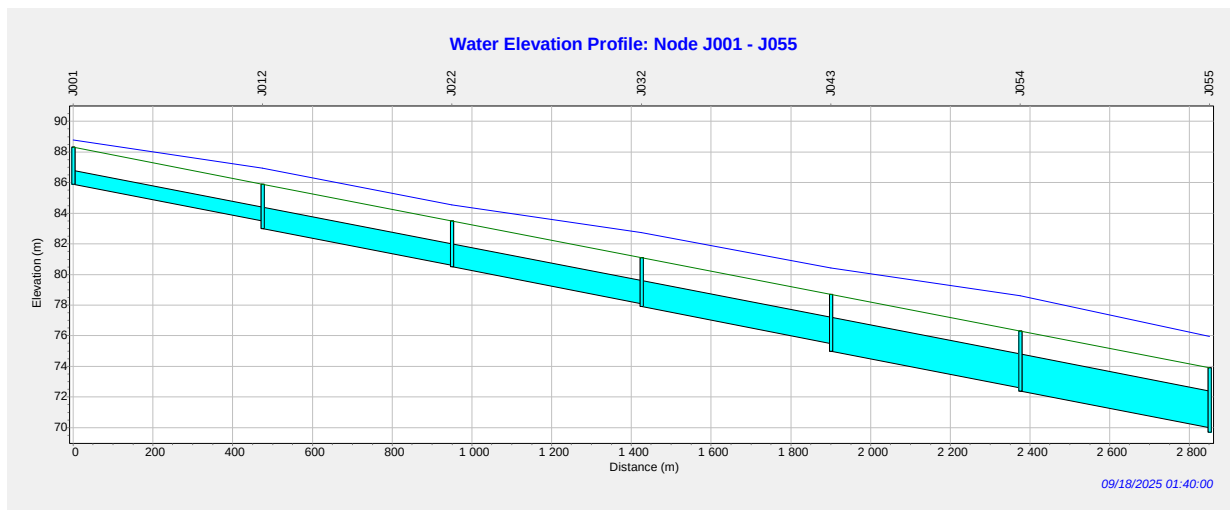


Fig. 5: Part of the profile of the network, following the application of a Chicago hyetograph with 2 years return period

Source: created by the authors using SWMM.

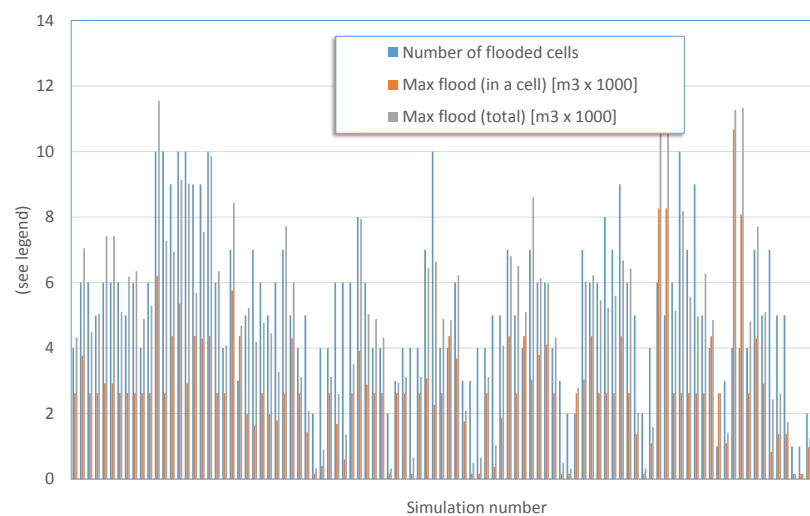


Fig. 6: Results of the 100 simulations. For each simulation, the three parameters that indicate “criticality” are shown

Source: created by the authors