Analogies and Analogue Thinking in Teaching Learning Interdisciplinary Mathematical Models

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Abstract: - Reasoning by analogies involves identifying a system of common relations between two situations and generating additional deductions based on coincidences. These coincidences can also include similarities of concrete properties between the situations being analysed, contributing to infer associations or mental constructions of learning through relational thinking. This paper outlines the proposal to teach the topic Ordinary Differential Equations (ODEs) with Constant Coefficients (CC) through the analysis of basic engineering models with a view to find the analogies that lie in the systems presented. The aim is to identify and compare variables in equations in their symbolic, numerical and graphical aspects. Although this may seem a very complex process, it is evident that the strategy of teaching a topic by analogies allows to promote competency-based teaching.

Key-Words: - Analogies, analogue thinking, ODEs, systems, competencies, models.

Received: March 27, 2024. Revised: February 5, 2025. Accepted: April 9, 2025. Published: June 11, 2025.

1 Introduction

The technique of representing the real world in mathematical terms has become an invaluable instrument to try to deepen the knowledge of nature. For engineers, looking for answers to technical problems involves diving into the real world to model engineering situations. The process of imitating reality through the language of mathematics is known as mathematical modelling.

Problem formulation in mathematical terms has several benefits. First of all, it requires us to establish our premises clearly. Real-life problems are complex and involve several different mental processes, possibly interrelated.

Before proceeding to mathematical treatment, one must determine which variables are significant and which can be ignored. For important variables, relationships are often postulated in the form of laws, formulae, theories, etc. These hypotheses constitute the idealizations of the model. These mathematical relationships often have similar behaviours and equations.

The process of building knowledge of an effective mathematical model requires skill, imagination and objective evaluation. Naturally, knowledge of several existing models that illustrate various aspects of modelling can help to get a better idea of the process.

At present, given the technological advancement, these models try to represent complex systems that, in Mathematics instruction, should be conveniently similar to each other. The search for analogies in mathematics allows the construction of mental structures that lead to a systemic thinking that produces new ideas.

According to some definitions of Mathematics, it is a deductive science that studies the properties of abstract entities, such as numbers, geometric figures or symbols and their relationships. An important branch of this is applied mathematics, which deals with the application of mathematics to the resolution of problems in other disciplines, such as physics, biology or economics. However, there are two worrying facts that are generally repeated in most countries, [1].

A system can be seen as a process that results in the transformation of a system, for example: A car can be seen as a system where the input is the pressure on the accelerator and the output, the movement of the car.

A dynamic system is a system whose state evolves over time, and is also controllable and observable from the surroundings and reacts to actions in the surrounding and stimuli of the system.

In the present work, a class of advanced mathematics at university level is designed, whose topic is second order Ordinary Differential Equations (ODEs), based on a modelling technique

and using computational resources and analogies that exist in second order engineering systems.

2 Analogies and Analogue Thinking

Analogue thinking is a form of comparative reflection that, through induction, relates two different situations that respond to the same logic. Although the situations are different, they share a similar pattern or structure. Through reasoning, these similarities are identified to better understand a situation that may be new or complex. In this way, a process of systemic reasoning is triggered from a known case to understand another one that is similar in certain aspects.

From an academic perspective, analogies are a form of expression in daily life, and this manifests itself in different areas, both in the context of everyday life and in the classroom, and in any daily conversation, even in those of higher conceptual or expression level.

The most important influence on analogy research in the cognitive science tradition has been concerned with the representation of knowledge within physics and computational systems, [2].

The essence of analogue thinking is to detect that two specific and different situations or domains are organized by systems of relationships and comparable roles, [3], [4].

At all levels, people often use analogies that help them understand certain ideas. This resource can be used in the teaching-learning process of science.

Thus, the use of analogies allows to enhance the understanding of the contents of the sciences to provide solutions more easily. Teaching should pay attention to the use of analogies to achieve processes of generalization and abstraction in students' cognitive structures. In this way, tending to an analogue thinking can foster the creation of the foundations on which students build the conceptual basis of new knowledge.

Nowadays, analogies are related to research on mental models. A mental model is an internal representation of information that corresponds, analogously, to the state of things being represented. Mental models are structural analogues of the world, [5].

A key aspect of structured knowledge is that elements can be flexibly bound into the roles of relations. For example, "dog bit man" and "man bit dog" have the same elements and the same relation, but the role bindings have been reversed, radically altering the meaning. How the mind and brain accomplish role binding is thus a central problem to be solved by any psychological theory of structured knowledge, including any theory of analogy, [2].

The aim is to ease the construction of knowledge step by step through the application of examples of everyday life and its relationship with the analysis of the engineering topics of the physical principles used by the differential calculus, the instantaneous and average variations of a physical system, but above all the interpretation of results and the approach to new situations.

Analogies and metaphors are instruments for suggesting new ideas and making them intelligible, [6]. In the same vein, analogies are a tool that teachers can use, together with experimentation and demonstration, to increase the intelligibility and plausibility of explanations.

Metaphors and analogies are an essential aspect in the process of mental formation of the representations of the world around us and the inferences that can be established between them. A metaphor is effective when there is an interaction between the term and the analogue components. For example, in the metaphor 'a virus is an invisible intruder', the term 'virus' must be neither similar nor too different to the analogue 'invisible intruder' for the metaphor to be effective, [6].

3 Preliminary Analysis of the Class

Analysis of the location of the analogy in the educational sequence has, however, attracted much greater attention from researchers. In some cases, the said location has been understood as the placing of the analogy in the general sequence of a lesson, [7].

Engineering systems that respond in their modelling to first or second order ordinary differential equations (ODEs) with constant coefficients (CC) have similar equations for physical dynamic models in different disciplines.

The preliminary design of the class starts from the knowledge that hydraulic, electrical, mechanical, chemical and other systems have analogue second order ODEs with CC.

The general structure of second order ODEs with CC, with a, b and $c$real numbers, is of the form [8], [9]:

$$a\,y''(t) + b\,y'(t) + c\,y(t) = F(t) \quad (1)$$

If $a = 0$ and $b \neq 0$, the ODE is a first order one, whereas if $a \neq 0$, the ODE is a second order one.

In this work, second order ODEs with CC are analysed, where in principle the variables $\{y, t\}$have no significance from an engineering point of view:

they are abstract variables that do not represent a known phenomenon in the cognitive structure of the student.

In the course of the class, the topic is addressed giving meaning to the variables $\{y, t\}$ using the similarity that ODEs with CC represented by the equation (1) have for different engineering systems.

The objective is to try to take advantage of analogue thinking as a means to conceptualise the topic and give meaning to the variables, distinguishing the similarities and differences they have.

For students of the Mechanical Engineering (ME) programme, when tackling second order ODEs with CC, it is convenient to start the development of the subject through a mechanical system constituted by a body whose mass is subject to a spring.

A very simplified version of a car wheel suspension system is considered, as depicted in Figure 1, which responds to equation (2). The movement P at the point $x_i$is supposed to be the input of the system and the vertical movement $x_o$of the body is the output. The displacement x_o is measured from the equilibrium position in the absence of the inputx_i. The equation that models the system for forced vibrations is:

$$mx''(t) + c\,x'(t) + k\,x(t) = F(t) \qquad (2)$$

The system includes a spring and a damper. The spring has a rigidity of the material with which it was designed and this determines the stiffness constant of the spring$k$.

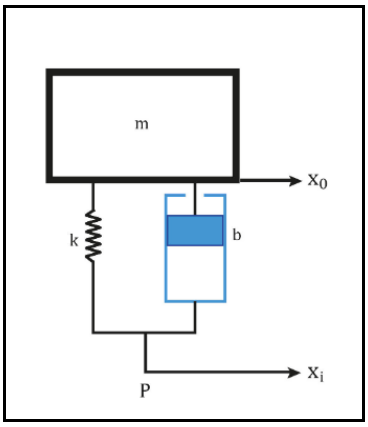

Fig. 1: Suspension system in equilibrium

The damper is the one who exerts friction in the suspension system and softens to a greater or lesser extent the swings of the spring. This ability to counteract the movement of the spring is characterised by the damping constant (c) between the mass (m) and the spring.

Figure 2 shows the suspension system simplified and then subjected to an external force (F) that forces the system to oscillate.

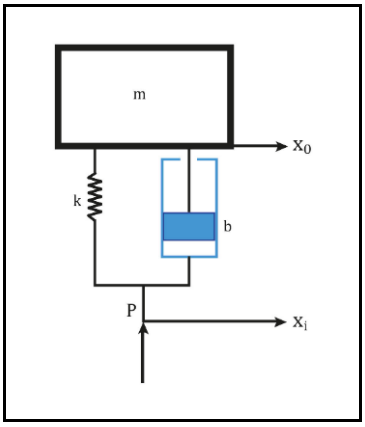

Fig. 2: Excited suspension system

The constant coefficients m, c, k are real numbers. x is the dependent variable, and represents the elongation experienced by the spring, and t is the independent variable, and refers to time.

In the need to achieve new instances of learning, and with the aim of facilitating the construction of knowledge from a known model, the aim is to relate this model with others that are similar and involve other branches of engineering.

Equation (2) represents a system that is initially known, attractive and already studied by ME students, but from the perspective of classical physics and not from the point of view of ODEs with CC.

This system is related to many examples within ME, such as the one that represents the device called mass-spring oscillator. Common examples of vibrating systems synchronised with their controllers are sound system speakers, cyclists who use their bicycles on railroad tracks, and ocean tides controlled by the gravitational influence of the Moon, [8].

What happens if these students are asked to analyse an electrical system?

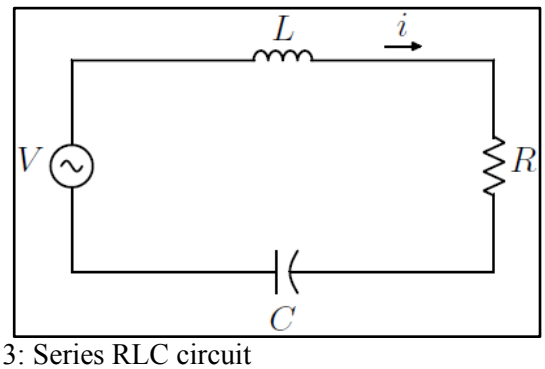

Fig. 3: Series RLC circuit

At first, it can be said that the electrical systems are not interesting for ME students, nor is it for Chemical Engineering (ChE) or Civil Engineering (CivE) students. However, they are of interest, and also useful for students of Electrical Engineering (EE).

If $i(t)$ denotes the current in the electrical circuit in series LRC, then the voltage drops in the inductor, resistor and capacitor are as shown in Figure 3. By Kirchhoff's second law, the sum of these voltages is equal to the voltage $V(t)$ applied to the circuit, [9].

For electrical circuits, equation (3) describes the mathematical behaviour of the circuit in Figure 3, and has a structure analogous to equation (1).

$$L\,q''(t) + R\,q'(t) + C^{-1}q(t) = V(t) \quad (3)$$

The nomenclature used in circuit analysis is similar to that used to describe mass-spring systems. This means that equation (3) is not only similar to equation (1), but also similar to equation (2) [9].

In equation (3), L is the inductance in the inductor, R is the electrical resistance; C is the capacity of the capacitor and V is the voltage applied. q is called electric charge and is the dependent variable of the system, and t, the independent variable called time [8], [9]. The modelling of the equations and their resolution in both cases are similar.

Appealing to the structure projection theory [10], it can be said that there are two situations where the objects, which are the constant parameters that we observe in the equations (2) and (3), though conceptually different, are analogous in terms of the resolution of each of the equations, [11], [12].

It can be seen from the point of view of analogue thinking that each of these coefficients, although conceptually different, shares a common structure of relationships, in terms of the algebraic resolution of the equation.

In Table 1, parameters that are similar between equation (2) and (3) are listed.

Table 1. Analogies between algebraic parameters of mechanical and electrical systems

Analogous variables in the	
Mechanical system	Electrical system
m	L
c	R
k	C^{-1}
F	V

Analogue thinking consists of transfer of knowledge, from a known domain, called base analogue; to a novel domain, which is to be understood, called target analogue. This type of thinking appears in a wide variety of cognitive activities, such as: problem solving, argumentation, knowledge transfer, science teaching, creative thinking, categorization of objects, understanding of physical principles, transfer of perceptive-motor patterns, scientific discovery, [13].

If analogue reasoning is conceived as the transfer of part of the knowledge from a known domain to a new domain, then it involves at least two differential processes: the process responsible for recovering the analogue, and the process responsible for the application of that relevant information. The correspondence that is established between the base domain and the target domain is called extrapolation.

For ME students, the system described by equation (2) means a base analogue, while for EE students, it would be a target analogue. However, if the situation is analysed in the opposite direction, for EE students the equation (3) would be their base analogue and for ME students it would be their target analogue.

The common structure for both equations describes the same model from ODEs with CC but with other symbols. These symbols may sometimes have no meaning as in the case of equation (1), and in others, as in equations (2) and (3), the meaning is subject to the interpretation of the interests and preferences that students may have according to the case studied in a particular type of engineering.

For ME students it is a more familiar situation to model and work with the dynamic system that represents equation (2), which can be the starting point to infer or incorporate in the knowledge structure a new information such as the model that shows equation (3), which would be a less familiar situation.

The analysis of these analogous systems in their mathematical structure represent constructions that students elaborate to respond to their need to interpret natural phenomena or scientific concepts, and to provide explanations, descriptions or predictions. Although they are personal constructions, they also have a universal character, and in the teaching practice, it has been noticed that students tend to resist changes. These changes sometimes generate persistent behaviours when learning new contents.

This shared structure allows for the application of the principle of systematicity, which prioritizes correspondence between relationships that are linked by higher order relations, [10], [11].

Applying analogue reasoning, it is possible to

reach a conclusion, based on previously established premises and establishing a comparison between them, in order to obtain a result which is a product of an analogy. With this type of reasoning, it is possible to reach different conclusions that are used in everyday life and are applicable to the teaching of Mathematics for engineering applications, [12].

Solving a problem by analogy involves transferring a solution from a known problem (the base problem) to a new problem whose solution is unknown (the target problem). In learning environments, the solution to a base problem serves as scaffolding for the application of a general method until the student has obtained the required fluency to apply this general method directly, [14], [15].

It is intended to carry out an alternative teaching methodology, which is evaluated at the end of the experience. If this experience is effective, it will be observed that students can apply the conceptualisation of a particular dynamic system to different mathematical and physical models, which benefit their professional development in later stages.

Through the use of analogies and the development of analogue thinking, students are expected able to design alternative and analogous models that facilitate the analysis and manipulation of variables. Systemic analogue thinking requires a mental representation that allows to manipulate conceptual entities, which can be extrapolated in the application of results by identifying the same behavioural rule in the different cases studied.

The main target of our study was to assess whether a set of activities promoting a connection between situation models and equation structures would help students adapt the learned solutions to novel problems introducing structural variations. As in the case of diagrams [16], we reasoned that simulations of situation models could help participants in deciding what operations to use and in evaluating whether those operations are conceptually sound. As predicted, the groups that were asked to ground equation simulation of situation models structures in the situation models of the problems outperformed a control group not required to carry out such simulations.

An analysis of the relation between participants' solution strategies and their relative probabilities of leading to correct equations can shed light on the specific ways through which our intervention might have helped participants in adapting the learned solutions to novel problems, [13].

Through the proposed strategies, students can achieve analogue connections through reasoning, accessing to higher levels of significance in the thought stages.

The relationships between the various variables are potentially approachable by the students, but they can generate cognitive conflicts since it could be difficult for them to explain the passage to the different records of semiotic representations: numerical, graphic, symbolic and verbal; which is why, they are part of the argumentative flow of very few students among the student population.

It is intended that through the use of analogies, students can manipulate the various sets of variables in the different stages of the learning process.

4 Learning space. Class design

Today's technological possibilities of representation and calculation require the development of common sense, the critical review of the results emanating from computer programmes, sensitivity to different alternatives, the capacity for analysis, and the rational selection of proposals such as the detection of the relevant variables in a problem.

The need to develop new styles of work and the implementation of different methodologies becomes apparent, and so does the need to rethink the interaction between the different areas.

Because of this need, inter- and multidisciplinary work is considered in this proposal as an unavoidable professional attitude in accordance to the new trends of the world of sciences and work.

Situation model simulations of certain algebra problems can be carried out either physically (e.g., via manipulating physical objects) or mentally. In domains as diverse as text comprehension [17], metaphor comprehension, and arithmetic, physical and imagined simulations yielded comparable effects, [18]. A second objective of this study was to assess whether imagined simulations are as effective as physical simulations in helping participants transfer solutions to the target problems with structural variations, [19].

The class seeks to promote scientific experimentation within the disciplines that make up engineering programmes. The purpose of the students' work is to address problems that require mathematical modelling, apply different methods of solution appropriately and identify the tools developed in each method, select the algorithm to apply and thus provide the future professional with the necessary skills to interpret and propose solutions to different alternatives and make decisions based on the solutions found.

The implementation of the teaching of ODEs

with CC through analogue thinking is carried out in a specific area, the Computer and Multidisciplinary Laboratory of Basic Sciences.

In the training of an Engineer, essential basic knowledge in the area of Mathematics must be contemplated to be able to approach and solve dynamic models that allow the student to migrate toward a form of self-managed learning that will be useful in his future professional life.

An interdisciplinary approach has gained relevance in modern educational programs as it is considered an important and challenging educational solution for the future. Compared to traditional educational approaches, interdisciplinary teaching and learning allows students to combine learning from different subjects simultaneously to find new and deeper ways of thinking about problems and their solutions. An interdisciplinary approach transforms the way students typically learn, allowing them to synthesize multiple perspectives rather than receiving information delivered by an instructor according to rigid guidelines. In this approach, teachers collaborate to invent and apply more effective and innovative teaching tools, connecting one subject to another, [20].

Firstly, students are presented with a system of a very simplified version of the wheel suspension of a car whose mathematical model is represented by equation (2). Students solve this model with computational tools, and obtain an expression that defines the elongation function with respect to time, and plot this function, to finally come up with the respective solutions.

For this purpose, the students meet in groups of three with the recommended bibliographic material getting answers to the questions raised that were used to address the problems of the next session.

The proposed rubrics for modelling the system are transcribed below:

- What forces act on the car wheel suspension system?
- Explain what happens to the spring and the damper if an external force is applied.
- Propose the model that represents the given situation.
- What are the possible relationships that can take place between the coefficients of the components considering a null external force?
- How do the relationships found affect the overall behaviour of the system?
- Among the relationships found above, which would be most appropriate for a car suspension system?
- Solve the selected case and plot the system response by establishing different initial conditions and appropriately choosing parameters for a constant external force.

The solution of the equation (2) made by the students for a generic system and for a constant force is presented below

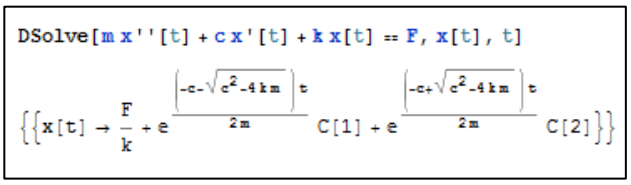

Fig. 4: ODEs solution for equation (2)

Equation (4) indicates the expression of elongation as a function of time.

$$x(t) = \frac{F}{k} + c_1 e^{\alpha_1} + c_2 e^{\alpha_2} \qquad (4)$$

Where the constants α_2 andα_1 are the ones described in the system (5)

$$\begin{cases} \alpha_1 = \dfrac{-c - \sqrt{c^2 - 4km}}{2m} \\ \alpha_2 = \dfrac{-c + \sqrt{c^2 - 4km}}{2m} \end{cases} \qquad (5)$$

Fig. 5 represents the graph of the model of equation (4) made by the students for the data $m = 1\,kg$, $c = 3\frac{Ns}{m}$, $k = 49\frac{N}{m}$ and $F = 20N$, for the initial conditions $x_o = 0$and $x'_o = 0$ is:

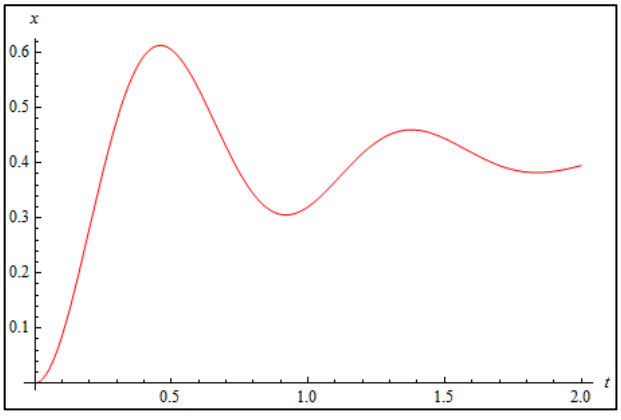

Fig. 5: Graph of the model

As the class is developed in the Computer Lab, we work with specific software, and after answering the previous questions, the analogy with the electrical system is made.

```
DSolve[L q''[t] + R q'[t] + Ca q[t] == V, q[t], t]

{{q[t] → V/Ca + e^((-R-√(-4 Ca L+R^2)) t)/(2 L) C[1] + e^((-R+√(-4 Ca L+R^2)) t)/(2 L) C[2]}}
```

Fig. 6: Solution of ODE to equation (3)

After carrying out the activity described above, the electrical system is analysed whose model is modelled by equation (3), whose solution is function $q(t)$. Figure 6 shows the expression of q for a constant value of the voltage that is carried out by the students with the software.

Equation (6) indicates the expression of the electric charge as a function of time.

$$q(t) = \frac{V}{C^{-1}} + c_1 e^{\beta_1} + c_2 e^{\beta_2} \qquad (6)$$

Where the constants β_2 andβ_1 are the ones described in the system (7)

$$\begin{cases} \beta_1 = \dfrac{-R - \sqrt{R^2 - 4C^{-1}L}}{2L} \\ \beta_2 = \dfrac{-R + \sqrt{R^2 - 4C^{-1}L}}{2L} \end{cases} \qquad (7)$$

Figure 7 shows the graph of the model of equation (6) made by the students for the data $L = 0{,}75\ H$, $R = 100\ ohm$,$C = 32 \times 10^{-6} F$ and $V = 100V$, with the initial conditions $q_o = 0$ and $q'_o = 0$ is:

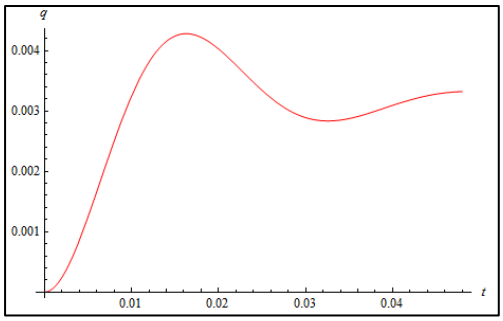

Fig. 7: Graph of the model

The analogies used in the case of the car damper and the electrical circuit will be applied in the future to explain new topics by comparing models that students are familiar with.

In the cases described above, the students can infer that although they studied two situations from different fields, the solution to the ODEs with CC associated with these systems −equations (4) and (6)− have the same structure, and even the representative graphs of the mechanical and electrical systems are analogous (Figure 4 and Figure 6).

Analogies permit us to draw new inferences about the target. Indeed, one major reason we use analogy is to learn something new about the target domain by recruiting our knowledge of a aconally similar base domain, [21], [22].

One key finding in analogy research is that people are highly selective in the inferences they make from analogies – we do not simply bring over everything we know about the base to the target.

According to the structure-mapping view, inference happens as a natural outcome of the structural alignment process.

Once the base and target have been aligned and their common relational structure found, if there are additional parts of the relational pattern in the base that are not present in the target, then this missing pattern will be brought over as a candidate inference.

Thus, one way to think about inference generation is as a process of relational pattern completion. The requirement that candidate inferences be connected to the common relational pattern effectively filters which inferences will be considered, [23].

After analysing these models, the students are proposed to analyse two situations of engineering application.

4.1 Hydraulic Systems: Two Interconnected Tanks in Series

A hydraulic system is a set of devices that, through the use of a flow of liquids, allows to generate a movement which can be used as a form of energy.

Hydraulic circuits are capable of producing many different combinations of movement and force. However, they are essentially the same, regardless of their application.

Students are proposed a flow system as observed in Figure 8, which shows two tanks interconnected in series by a pipe through which a fluid circulates.

This type of system is analogous to the fluid contained in a manometric tube that is at rest and suddenly moves, [24].

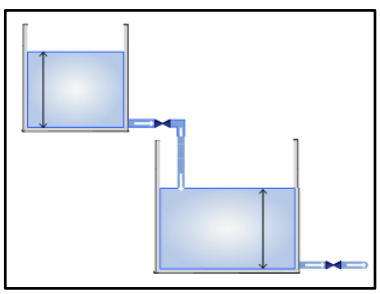

Fig. 8: Interconnected tanks

The model that governs the output flow of a fluid of two tanks interconnected in series is:

$$T_1\,T_2\,Q''(t) + (T_1 + T_2)Q'(t) + Q(t) = q_o \quad (8)$$

Being: q_o the input flow rate that is considered constant; T_1 and T_2time constants, and the flow function Q defined as a function of time t.

The software solution of the ODE (8) is shown in Figure 9.

```
DSolve[T1 T2 Q''[t] + (T1 + T2) Q'[t] + Q[t] == qo, Q[t], t]

{{Q[t] → qo + e^(-t/T2) C[1] + e^(-t/T1) C[2]}}
```

Fig. 9: ODE Solution to Equation (8)

Thus, $Q(t)$ is

$$Q(t) = q_o + c_1 e^{-\frac{t}{T2}} + c_2 e^{-\frac{t}{T1}} \quad (9)$$

The solution to equation (8) does not seem to present analogy with the models analysed in equations (2) and (3). However, the students observe that in (8) being 1 the coefficient that accompanies $Q(t)$it seems that differences between the equations are determined. But by making a careful comparison between (4) and (9), they observe that both solutions still present analogies.

4.2 Chemical Systems: Temperature Control System of a Fluid

The model presented aims to control the temperature of a liquid flowing through a perfectly insulated continuous stirred tank. Its governing equation is:

$$T''(t) + \alpha\,T'(t) + \beta\,T(t) = \theta(t) \quad (10)$$

Where the constants and α β are related by other parameters described in the system of equations (11)

$$\begin{cases} \alpha = 1 + F + R \\ \beta = R(B + F) \end{cases} \quad (11)$$

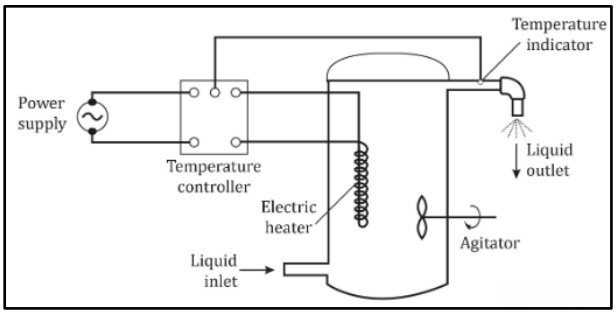

Fig. 10: Temperature control system

Such that: θ is the initial temperature; R is the heat capacity coefficient; F is the flow rate coefficient; B is the parameter of the tank temperature controller; and the function T describes the temperature to be controlled in relation to time$t$.

The operating scheme of the simple temperature control system of the equation model (10) appears in Figure 10.

For temperature control, the liquid is introduced into an electrical heating resistance of a given area, and a temperature sensitive element is placed in the output current, which can be assumed to be at the same temperature as the liquid inside the tank.

These two devices are connected to a temperature controller that supplies power to the heating resistance, [24].

The volume of liquid contained in the tank and the mass flow rate of liquid are constant.

The four cases that are proposed to students for analysis present the same analogue characteristics.

The result of the ODE with CC by applying the computational tool can be observed in Figure 11, and the same analogies as the previous models are detected.

```
alf = 1 + F + R

1 + F + R

bet = R (B + F)

(B + F) R

DSolve[T''[t] + alf T'[t] + bet T[t] == Q, T[t], t]

{{T[t] → Q/(BR + FR) + e^(1/2 (-1-F-R-√((1+F+R)^2-4 (BR+FR)))) t C[1] + e^(1/2 (-1-F-R+√((1+F+R)^2-4 (BR+FR)))) t C[2]}}
```

Fig. 11: ODE Solution to Equation (10)

5 Conclusion

Nowadays, engineering programmes undergo the application of a competency-based teaching approach, which aims to highlight the students' basic skills. This is the reason why it is necessary to reconstruct models using the analogies they present.

Such capabilities often require complex skills of a superior nature, which are usually labelled as innovation or creativity. In this sense, analogies could be an interesting tool to promote the skills and values that are required for this purpose.

In the teaching of ODEs, it is thought that analogies can become an elementary tool not only for learning this content, but also for the development of other classroom activities.

Analogue thinking encourages basic skills among students. These competencies are fundamental in engineering programmes to strengthen scientific modelling processes, highlighting the interdisciplinary character of teaching.

It is evident during classwork that it is important to assimilate and evaluate the dynamic models that

want to be taught, as well as to be able to elaborate such models and relate them to each other. This requires a series of strategies, skills and visions that have to be installed and stimulated from the basic cycle of the programme.

In the particular field of competences, the analogies that the teacher must develop allows them to work, in the classroom, on mechanisms of reasoning similar to those involved in the processes of scientific modelling.

Both learning and working from planned analogies with the teacher, as well as the review and questioning of known analogies or the reconstruction or deconstruction of new ones, involve processes and attitudes similar to those required by dynamic modelling processes.

However, it remains to be seen whether, in fact, the teaching proposal is capable of involving students to mobilise appropriate strategies and attitudes in carrying out other activities; and whether this has any impact on students' skills.

Finally, it is considered that analogue thinking among students on the topic of ODEs with CC maintains a productive link as a bridge for the approach of other mathematical topics.

Acknowledgement:
The authors would like to thank Eng. Franco Ruggiero and Lic. Elsa Dobolletta for their collaboration in translating this work into English.

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Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The authors have no conflicts of interest to declare.

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