Engineering World
Open Access Journal
ISSN: 2692-5079

Robust Fuzzy Controller Design with Decay Rate for Nonlinear Perturbed Singular Systems

Wen-Jer Chang*, Che-Lun Su and Cheung-Chieh Ku

Abstract**—This paper deals with the robust fuzzy controller design problem for the perturbed nonlinear singular systems. A novel robust fuzzy control technique is investigated based on the state-derivative feedback approach. In this paper, the nonlinear perturbed singular systems are expressed by the uncertain Takagi-Sugeno fuzzy models and the so-called parallel distributed compensation method is applied to design the state-derivative feedback fuzzy controller. Considering the Takagi-Sugeno fuzzy perturbed singular systems, the Lyapunov stability theory is employed to derive sufficient stability conditions with decay rate. These sufficient stability conditions can be effectively transferred into the linear matrix inequality problem. At last, a numerical example is provided to verify the applicability and effectivity of the proposed robust fuzzy controller design method.**

I. Introduction

THE nonlinear systems have become an important field of research in control theory. Over the past few decades, the Takagi-Sugeno (T-S) fuzzy control [1-2] has become the most of the successful and active technology. According to the T-S fuzzy model, the Lyapunov theory can be used to analyze the stability of the nonlinear perturbed systems. In order to design a fuzzy controller for the T-S fuzzy models, a useful technique so-called Parallel Distributed Compensation (PDC) was developed in [1-3]. The main idea of the PDC method is to derive each fuzzy control rule to compensate each plant rule of the T-S fuzzy systems. In [4-5], some PDC-based fuzzy control approaches have been investigated for complex nonlinear systems represented by the T-S fuzzy models.

In the past few years, the singular systems [6-7] also known as descriptor systems have attracted the attention of many researchers. The singular system can model a lot of special case form in the state space and its physical significations are more complete than the nonsingular systems. Many efforts have been devoted to singular systems because the singular system is hard to be stable. Only when the system is regularity and impulse-free [8-9] then the singular system will be stabilized. Many researchers have paid much attention to the control of singular systems and many results have been successfully established [10] for continuous-time and discrete-time systems.

The perturbations usually exist in the practical systems. The perturbation impacts the system stability or performance, so it is important to consider perturbation effects for the control systems. By extending the control problem to the T-S fuzzy model, the perturbations can be considered in each subsystem and its stability analysis can be proposed by using Lyapunov theory [11]. More and more literature developed the control theory for the stability analysis of the T-S fuzzy perturbed systems. It is well known that the robust control theory can inhibition the perturbations of the system. In [12], the robust controller design for the T-S fuzzy perturbed singular systems has been studied. However, the controllers developed in [12] were designed based on the traditional state feedback approach.

It is known that the state derivative feedback method is very useful for the controller design of some particular systems [13]. The motivation of using the state derivative feedback is that some systems use the accelerometers to measure the motions. Because the state derivative feedback method may cause noise contamination in measured signals amplified by differentiation, and the realization of perfect derivatives of signals may be difficult in practice. That is why the state derivative feedback method has received little attention in the literature. In recent years, the state derivative feedback method is more and more popular when the papers used the state derivative feedback method to discuss the sensitivity of the parameter variation and input disturbance in the multivariable system [14].

In this paper, the nonlinear singular systems are expressed by the T-S fuzzy model with considering the internal perturbations. The state-derivative feedback fuzzy control approach investigated in this paper is developed based on the by PDC method. Following the Lyapunov stability theory, the sufficient conditions can be obtained to design the robust fuzzy controller. The sufficient conditions can be transferred into the Linear Matrix Inequality (LMI) problem, which can be efficiently solved by using the MATLAB LMI-toolbox. At last, a numerical example is provided to show the applicability and effectivity of the proposed robust fuzzy control method.

II. System Statements And Problem Descriptions

In this paper, a fuzzy controller with the state-derivative feedback and robust control theory for the T-S singular systems is proposed. Aclass of the complex nonlinear perturbed singular systems can be expressed by the following T-S fuzzy perturbed singular system:

Plant Rule i :

*W. J. Chang is with the Department of Marine Engineering, National Taiwan Ocean University, Taiwan, R.O.C. (corresponding author e-mail: wjchang@mail.ntou.edu.tw).

C. L. Su is with the Department of Marine Engineering, National Taiwan Ocean University, Taiwan, R.O.C. (e-mail: 10766010@mail.ntou.edu.tw).

C. C. Ku is with the Department of Marine Engineering, National Taiwan Ocean University, Taiwan, R.O.C. (e-mail: ccku@mail.ntou.edu.tw).

IF $\mu_{i1}(t)$ is M_{i1} and … and $\mu_{ik}(t)$ is M_{ik}
THEN $\mathbf{E}\dot{x}(t) = (\mathbf{A}_i + \Delta\mathbf{A}_i)x(t) + \mathbf{B}_i u(t)$ (1)

where $i = 1,2,\cdots,n$ and n is the rules number, M_{ik} are fuzzy sets, k is the number of premise variables, $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is control input vector. $\mathbf{A}_i \in \Re^{n\times n}$, $\mathbf{B}_i \in \Re^{n\times m}$ are the state, input, constant matrices, $\mathbf{E} \in \Re^{n\times n}$ is a constant matrix with $rank(\mathbf{E}) = r < n$.

The perturbations in the T-S fuzzy model (1) is considered as $\Delta\mathbf{A}_i = \mathbf{H}_i\Delta_i\mathbf{R}_i$, where Δ_i denote the time-varying uncertain matrices, $\mathbf{H}_i$, $\mathbf{R}_i$ denote the known matrices, which are the composition of the perturbations.

The overall T-S fuzzy perturbed singular model can be "blending" as follows:

$$\mathbf{E}\dot{x}(t) = \sum_{i=1}^{n} h_i(\mu(t))\{(\mathbf{A}_i + \Delta\mathbf{A}_i)x(t) + \mathbf{B}_i u(t)\} \quad (2)$$

where $\omega(t) = \prod_{j=1}^{k} \mathrm{M}_{ij}(\mu_j(t))$, $h_i(\mu(t)) = \omega(t) \Big/ \sum_{i=1}^{n} \omega(t)$, $h_i(\mu(t)) \geq 0$, $\sum_{i=1}^{n} h_i(\mu(t)) = 1$ and $M_{ij}(\mu_j(t))$ is the grade of the membership of $\mu_j(t)$.

One can apply the PDC concept to design the T-S fuzzy controller. The T-S fuzzy controller designed in this paper for the T-S fuzzy model (2) has the following form:

Controller Rule i :

IF $\mu_{i1}(t)$ is M_{i1} and … and $\mu_{ik}(t)$ is M_{ik}
THEN $u(t) = -\mathbf{F}_i\dot{x}(t)$ (3)

Then the overall fuzzy controller can be represented by

$$u(t) = -\sum_{j=1}^{n} h_j(\mu(t))\mathbf{F}_j\dot{x}(t) \quad (4)$$

Substituting the control input (4) into the T-S fuzzy perturbed singular model (2), the closed-loop state equation can be obtained as follows:

$$\mathbf{E}\dot{x}(t) = \sum_{i=1}^{n}\sum_{j=1}^{n} h_i(\mu(t))h_j(\mu(t))\{(\mathbf{A}_i + \Delta\mathbf{A}_i)x(t) - \mathbf{B}_i\mathbf{F}_j\dot{x}(t)\} \quad (5)$$

For the fuzzy controller design, one needs to find the constant matrix $\mathbf{F}_j \in \Re^{m\times n}$ and $(\mathbf{E} + \mathbf{B}_i\mathbf{F}_j)$ must be a full rank matrix. From [15], it is known that the matrix $(\mathbf{E} + \mathbf{B}_i\mathbf{F}_j)$ has a full rank when the following equation holds:

$$rank[\mathbf{E}, \mathbf{B}_i] = n \quad (6)$$

It is assumed that the condition (6) holds, then the closed-loop system (5) can be rewritten as follows:

$$\dot{x}(t) = \left(\mathbf{E} + \sum_{k=1}^{n}\sum_{l=1}^{n} h_k(\mu(t))h_l(\mu(t))\mathbf{B}_k\mathbf{F}_l\right)^{-1} \times \sum_{i=1}^{n} h_i(\mu(t))(\mathbf{A}_i + \Delta\mathbf{A}_i)x(t) \quad (7)$$

Remark 1 [16]

Recalling for any nonsymmetric matrix $\mathbf{Q}(\mathbf{Q} \neq \mathbf{Q}^{\mathrm{T}})$, $\mathbf{Q} \in \Re^{n\times n}$, if $\mathbf{Q} + \mathbf{Q}^{\mathrm{T}} < 0$ then $\mathbf{Q}$ has a full rank.

Considering the perturbations of the T-S fuzzy model (1), they are constructed as $\Delta\mathbf{A}_i(t) = \mathbf{H}_i\Delta_i(t)\mathbf{R}_i$. For these perturbations, the following lemma is useful for the design of proposed robust fuzzy controllers.

Lemma 1 [17]

Given real compatible dimension matrices $\mathbf{A}$, $\mathbf{H}$ and $\mathbf{R}$ for any matrix $\mathbf{X}>0$, $\xi>0$ with the conditions $\mathbf{\Delta}^{\mathrm{T}}(t)\mathbf{\Delta}(t) \leq \mathbf{I}$ and $\mathbf{X} - \xi\mathbf{\Delta}(t)\mathbf{\Delta}^{\mathrm{T}}(t) \geq 0$, one can find two results as follows:

$$\mathbf{H}\mathbf{\Delta}(t)\mathbf{R} + \mathbf{R}^{\mathrm{T}}\mathbf{\Delta}^{\mathrm{T}}(t)\mathbf{H}^{\mathrm{T}} \leq \xi\mathbf{H}\mathbf{H}^{\mathrm{T}} + \xi^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{R} \quad (8)$$

and

$$(\mathbf{A} + \mathbf{H}\mathbf{\Delta}(t)\mathbf{R})^{\mathrm{T}}\mathbf{X}^{-1}(\mathbf{A} + \mathbf{H}\mathbf{\Delta}(t)\mathbf{R}) \leq \mathbf{A}^{\mathrm{T}}(\mathbf{X} - \xi\mathbf{H}\mathbf{H}^{\mathrm{T}})^{-1}\mathbf{A} + \xi^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{R} \quad (9)$$

#

Definition 1 [18]

Considering the T-S fuzzy system (5), the decay rate is defined as the largest real constant γ, $\gamma > 0$, such that

$$\dot{V}(x(t)) < -2\gamma x^{\mathrm{T}}(t)\mathbf{P}x(t) \quad (10)$$

hold for a positive definite matrix $\mathbf{P}$ and all trajectories $x(t)$, $t > 0$.

The purpose of this paper is to combine the robust control theory and state-derivative feedback method to derive Lyapunov stability conditions. In the next section, the robust fuzzy controller design for the closed-loop T-S fuzzy singular perturbed system (5) is introduced.

III. State-Derivative Feedback Controller Design For T-S Fuzzy Perturbed Singular Systems

In this section, a robust fuzzy controller for the closed-loop T-S fuzzy singular perturbed system (5) is proposed.

Defining the Lyapunov function, one can obtain the stability conditions by the following theorem. Besides, Lemma 1 provides a method to convert the uncertain item $\boldsymbol{\Delta}_i$ in the model perturbations. Then, the sufficient conditions for the robust fuzzy controller design can be presented in Theorem 1.

Theorem 1

If there exists a positive definite matrix $\mathbf{Q}$, feedback gains $\mathbf{K}_l$ to satisfy the following stability conditions, then the closed-loop singular perturbed system (5) is asymptotically stable.

$$\begin{bmatrix} \Theta+2\xi\mathbf{H}\mathbf{H}^{\mathrm{T}} & * & * \\ \mathbf{R}_1 & -\xi\mathbf{I} & 0 \\ \mathbf{R}_2 & 0 & -\xi\mathbf{I} \end{bmatrix} < 0 \quad \text{for } i,k,l=1......n \tag{11}$$

where

$$\Theta = \mathbf{E}\mathbf{Q}\mathbf{A}_i^{\mathrm{T}} + \mathbf{B}_k\mathbf{K}_l\mathbf{A}_i^{\mathrm{T}} + \mathbf{A}_i\mathbf{Q}\mathbf{E}^{\mathrm{T}} + \mathbf{A}_i\mathbf{K}_l^{\mathrm{T}}\mathbf{B}_k^{\mathrm{T}}$$

Proof:

To analyze the stability of the closed-loop system, let us choose the Lyapunov function as the following form:

$$V\left(x(t)\right) = x^{\mathrm{T}}(t)\mathbf{P}x(t) \tag{12}$$

where $\mathbf{P}>0$. Taking differential of the Lyapunov function $V\left(x(t)\right)$, then one can get

$$\dot{V}\left(x(t)\right) = \dot{x}^{\mathrm{T}}(t)\mathbf{P}x(t) + x^{\mathrm{T}}(t)\mathbf{P}\dot{x}(t) \tag{13}$$

From (7), the (13) can be rewritten as follows:

$$\begin{aligned}\dot{V}\left(x(t)\right) = x^{\mathrm{T}}(t)\Bigg\{&\sum_{i=1}^{n} h_i\left(\mu(t)\right)\left(\mathbf{A}_i+\Delta\mathbf{A}_i\right)^{\mathrm{T}} \\ &\times\left(\mathbf{E}+\sum_{k=1}^{n}\sum_{l=1}^{n} h_k\left(\mu(t)\right)h_l\left(\mu(t)\right)\mathbf{B}_k\mathbf{F}_l\right)^{-\mathrm{T}}\mathbf{P} \\ &+\mathbf{P}\left(\mathbf{E}+\sum_{k=1}^{n}\sum_{l=1}^{n} h_k\left(\mu(t)\right)h_l\left(\mu(t)\right)\mathbf{B}_k\mathbf{F}_l\right)^{-1} \\ &\times\sum_{i=1}^{n} h_i\left(\mu(t)\right)\left(\mathbf{A}_i+\Delta\mathbf{A}_i\right)\Bigg\}x(t)\end{aligned} \tag{14}$$

Pre-multiplying $\wp$ and post-multiplying $\wp^{\mathrm{T}}$ on both sides of (14), one can obtain

$$\begin{aligned}\dot{V}\left(x(t)\right) = &\sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} h_i\left(\mu(t)\right)h_k\left(\mu(t)\right)h_l\left(\mu(t)\right)x^{\mathrm{T}}(t) \\ &\times\Big\{\left(\mathbf{E}+\mathbf{B}_k\mathbf{F}_l\right)\mathbf{P}^{-1}\left(\mathbf{A}_i+\Delta\mathbf{A}_i\right)^{\mathrm{T}} \\ &+\left(\mathbf{A}_i+\Delta\mathbf{A}_i\right)\mathbf{P}^{-1}\left(\mathbf{E}+\mathbf{B}_k\mathbf{F}_l\right)^{\mathrm{T}}\Big\}x(t)\end{aligned} \tag{15}$$

where $\wp = \left(\mathbf{E}+\sum_{k=1}^{n} h_k\left(\mu(t)\right)\sum_{l=1}^{n} h_l\left(\mu(t)\right)\mathbf{B}_k\mathbf{F}_l\right)\mathbf{P}^{-1}$.

Now, defining a new variable $\mathbf{Q}=\mathbf{P}^{-1}$, $\mathbf{Q}>0$, $\mathbf{K}_l=\mathbf{F}_l\mathbf{Q}$ and $\Delta\mathbf{A}_i=\mathbf{H}_i\Delta_i\mathbf{R}_i$, one can rewrite (15) as

$$\begin{aligned}\dot{V}\left(x(t)\right) = &\sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} h_i\left(\mu(t)\right)h_k\left(\mu(t)\right)h_l\left(\mu(t)\right)x^{\mathrm{T}}(t) \\ &\times\Big\{\Theta+\mathbf{E}\mathbf{Q}\mathbf{R}_i^{\mathrm{T}}\Delta_i^{\mathrm{T}}\mathbf{H}_i^{\mathrm{T}}+\mathbf{H}_i\Delta\mathbf{R}_i\mathbf{Q}\mathbf{E}^{\mathrm{T}} \\ &+\mathbf{B}_k\mathbf{K}_l\mathbf{R}_i^{\mathrm{T}}\Delta_i^{\mathrm{T}}\mathbf{H}_i^{\mathrm{T}}+\mathbf{H}_i\Delta_i\mathbf{R}_i\mathbf{K}_l^{\mathrm{T}}\mathbf{B}_k^{\mathrm{T}}\Big\}x(t)\end{aligned} \tag{16}$$

where $\Theta = \mathbf{E}\mathbf{Q}\mathbf{A}_i^{\mathrm{T}} + \mathbf{B}_k\mathbf{K}_l\mathbf{A}_i^{\mathrm{T}} + \mathbf{A}_i\mathbf{Q}\mathbf{E}^{\mathrm{T}} + \mathbf{A}_i\mathbf{K}_l^{\mathrm{T}}\mathbf{B}_k^{\mathrm{T}}$. Let $\mathbf{R}_1=\mathbf{R}_i\mathbf{Q}\mathbf{E}^{\mathrm{T}}$, $\mathbf{R}_2=\mathbf{R}_i\mathbf{K}_l^{\mathrm{T}}\mathbf{B}_k^{\mathrm{T}}$ and based on Lemma 1, the following inequality can be obtained from (16).

$$\begin{aligned}\dot{V}\left(x(t)\right) = &\sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} h_i\left(\mu(t)\right)h_k\left(\mu(t)\right)h_l\left(\mu(t)\right)x^{\mathrm{T}}(t) \\ &\times\Big\{\Theta+\mathbf{R}_1^{\mathrm{T}}\Delta_i^{\mathrm{T}}\mathbf{H}_i^{\mathrm{T}}+\mathbf{H}_i\Delta\mathbf{R}_1+\mathbf{R}_2^{\mathrm{T}}\Delta_i^{\mathrm{T}}\mathbf{H}_i^{\mathrm{T}}+\mathbf{H}_i\Delta_i\mathbf{R}_2\Big\}x(t) \\ \le &\sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} h_i\left(\mu(t)\right)h_k\left(\mu(t)\right)h_l\left(\mu(t)\right)x^{\mathrm{T}}(t) \\ &\times\Big\{\Theta+\xi\mathbf{H}_i\mathbf{H}_i^{\mathrm{T}}+\xi^{-1}\mathbf{R}_1^{\mathrm{T}}\mathbf{R}_1+\xi\mathbf{H}_i\mathbf{H}_i^{\mathrm{T}}+\xi^{-1}\mathbf{R}_2^{\mathrm{T}}\mathbf{R}_2\Big\}x(t)\end{aligned} \tag{17}$$

The nonlinear inequality can be converted into LMI form by using Schur complements. Then, the resulting LMI is

$$\begin{aligned}\dot{V}(t) \le &\sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} h_i\left(\mu(t)\right)h_k\left(\mu(t)\right)h_l\left(\mu(t)\right)x^{\mathrm{T}}(t) \\ &\times\begin{bmatrix} \Theta+2\xi\mathbf{H}_i\mathbf{H}_i^{\mathrm{T}} & * & * \\ \mathbf{R}_1 & -\xi\mathbf{I} & 0 \\ \mathbf{R}_2 & 0 & -\xi\mathbf{I} \end{bmatrix}x(t)\end{aligned} \tag{18}$$

It can be found that if the condition (11) is satisfied, then one can obtain $\dot{V}(t)<0$ from (18). Then, the proof of this theorem is completed.

#

Obviously, if the condition (11) in Theorem 1 can be satisfied, one can conclude that the closed-loop system (5) is asymptotically stable. In the next theorem, the decay rate constraint will be considered.

Theorem 2

If there exists a positive definite matrix $\mathbf{Q}$, feedback gains $\mathbf{K}_l$ and decay rate γ such that the following sufficient conditions are satisfied, then the closed-loop singular perturbed system (5) is asymptotically stable.

$$\begin{bmatrix} \Theta+2\xi\mathbf{H}_i\mathbf{H}_i^{\mathrm{T}} & * & * & * \\ \mathbf{R}_1 & -\xi\mathbf{I} & 0 & 0 \\ \mathbf{R}_2 & 0 & -\xi\mathbf{I} & 0 \\ (\mathbf{EQ}+\mathbf{B}_k\mathbf{K}_l)^{\mathrm{T}} & 0 & 0 & -(\mathbf{Q}/2\gamma) \end{bmatrix}<0 \tag{19}$$

for $i,k,l=1......n$

Proof:

By using the Schur complement, the condition (19) is equivalent to

$$\Theta+\xi\mathbf{H}_i\mathbf{H}_i^{\mathrm{T}}+\xi\mathbf{H}_i\mathbf{H}_i^{\mathrm{T}}+\xi^{-1}\mathbf{R}_1^{\mathrm{T}}\mathbf{R}_1+\xi^{-1}\mathbf{R}_2^{\mathrm{T}}\mathbf{R}_2$$
$$+(\mathbf{E}+\mathbf{B}_k\mathbf{F}_l)\mathbf{Q}(2\gamma\mathbf{Q}^{-1})\mathbf{Q}(\mathbf{E}+\mathbf{B}_k\mathbf{F}_l)^{\mathrm{T}}<0 \tag{20}$$

Defining a variable $\mathbf{P}^{-1}=\mathbf{Q}$, $\mathbf{Q}>0$, $\mathbf{K}_l$=$\mathbf{F}_l\mathbf{Q}$, then (20) can be rewritten as follows according to Lemma 1.

$$\left\{(\mathbf{E}+\mathbf{B}_k\mathbf{F}_l)\mathbf{P}^{-1}(\mathbf{A}_i+\Delta\mathbf{A}_i)^{\mathrm{T}}+(\mathbf{A}_i+\Delta\mathbf{A}_i)\mathbf{P}^{-1}(\mathbf{E}+\mathbf{B}_k\mathbf{F}_l)^{\mathrm{T}}\right.$$
$$\left.+(\mathbf{E}+\mathbf{B}_k\mathbf{F}_l)\mathbf{P}^{-1}(2\gamma\mathbf{P})\mathbf{P}^{-1}(\mathbf{E}+\mathbf{B}_k\mathbf{F}_l)^{\mathrm{T}}\right\}<0 \tag{21}$$

Pre-multiplying ς and post-multiplying ς^{T} on both sides of (21), one can obtain

$$\left\{(\mathbf{A}_i+\Delta\mathbf{A}_i)^{\mathrm{T}}(\mathbf{E}+\mathbf{B}_k\mathbf{F}_l)^{-\mathrm{T}}\mathbf{P}\right.$$
$$\left.+\mathbf{P}(\mathbf{E}+\mathbf{B}_k\mathbf{F}_l)^{-1}(\mathbf{A}_i+\Delta\mathbf{A}_i)+(2\gamma\mathbf{P})\right\}<0 \tag{22}$$

where $\varsigma=\mathbf{P}(\mathbf{E}+\mathbf{B}_k\mathbf{F}_l)^{-1}$.

By multiplying $x^{\mathrm{T}}(t)$ and $x(t)$ on the both sides of (22), then one can get

$$x^{\mathrm{T}}(t)\left\{(\mathbf{A}_i+\Delta\mathbf{A}_i)^{\mathrm{T}}(\mathbf{E}+\mathbf{B}_k\mathbf{F}_l)^{-\mathrm{T}}\mathbf{P}\right.$$
$$\left.+\mathbf{P}(\mathbf{E}+\mathbf{B}_k\mathbf{F}_l)^{-1}(\mathbf{A}_i+\Delta\mathbf{A}_i)\right\}x(t)<-2\gamma x^{\mathrm{T}}(t)\mathbf{P}x(t) \tag{23}$$

Note that (23) is equivalent to $\dot{V}(t)<-2\gamma V(x(t))$. Thus, if the condition (19) is held, then the system is asymptotically stable with the decay rate γ.

#

By solving sufficient conditions of Theorem 2, a PDC based fuzzy controller (4) can be designed for the T-S fuzzy perturbed singular system (5) with the decay rate γ. In order to express the applicability of the proposed fuzzy control method, a numerical example is introduced in next section.

IV. A Numerical Example

In this section, Theorem 2 is used to verify the applicability for the proposed robust fuzzy control method. Let us consider a nonlinear singular perturbed system, which is represented by the T-S fuzzy perturbed singular model as follows:

$$\mathbf{E}\dot{x}(t)=\sum_{i=1}^{2}\sum_{j=1}^{2}h_i(\mu(t))h_j(\mu(t))\left\{(\mathbf{A}_i+\Delta\mathbf{A}_i)x(t)+\mathbf{B}_iu(t)\right\} \tag{24}$$

where

$$\mathbf{A}_1=\begin{bmatrix}0&-1&1\\1&0&0.5\\-1&0&-1\end{bmatrix},\ \mathbf{A}_2=\begin{bmatrix}0&-1&-1\\1&-0.5&0.5\\1&0&-1\end{bmatrix},$$

$$\mathbf{B}_1=\mathbf{B}_2=\begin{bmatrix}0\\1\\0.5\end{bmatrix},\ \Delta(t)=\begin{bmatrix}0&0&0\\0&sin(t)&0\\0&0&sin(t)\end{bmatrix},\ \mathbf{E}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},$$

$$\mathbf{R}_1=\mathbf{R}_2=\begin{bmatrix}0&0.03&0\\0&0.05&0\\0&0&0.01\end{bmatrix},\ \mathbf{H}_1=\mathbf{H}_2=\begin{bmatrix}0&0.01&0\\0&0.03&0\\0&0&0.01\end{bmatrix},$$

The feedback gains can be obtained as follows by using MATLAB LMI-toolbox to solve the conditions of (19). The decay rate $\gamma=5$ can be also solved from (19) of Theorem 2.

$$\mathbf{F}_1=\begin{bmatrix}-1.2460 & -0.5121 & 0.3793\end{bmatrix}$$

$$\mathbf{F}_2=\begin{bmatrix}-1.2460 & -0.5121 & 0.3793\end{bmatrix}$$

In the simulations, the initial condition is chosen as $x(0)=\begin{bmatrix}1 & 0 & -1\end{bmatrix}^{\mathrm{T}}$. The simulation responses are shown in Fig. 1 to Fig. 3. From the simulation results, it can be found that the T-S fuzzy singular perturbed system (5) can be controlled by the proposed state-derivative feedback fuzzy controller with decay rate constraint.

V. Conclusions

In this paper, a robust fuzzy controller with state-derivative feedback has been designed for the T-S fuzzy singular perturbed systems. Based on the Lyapunov theory, the stability conditions have been derived by considering the robust constraint and decay rate constraint. The advantage of this paper is that the stability conditions developed in this paper are simpler than that derived by using the state feedback method. From the simulation results, it can be found that the present method was embodied in the constrained fuzzy control for T-S fuzzy singular perturbed systems. In the future, it can be extended to the discrete-time cases.

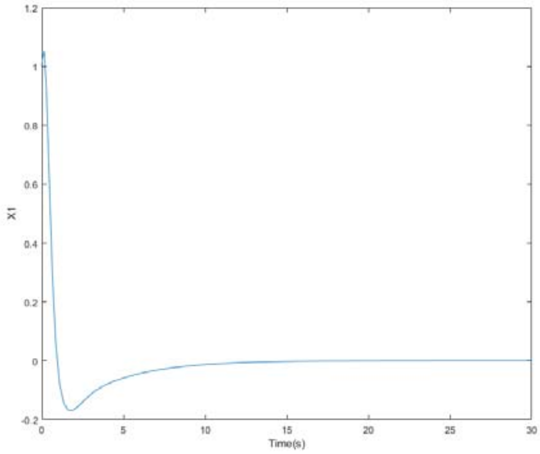

Fig. 1 The responses of state $x_1(t)$

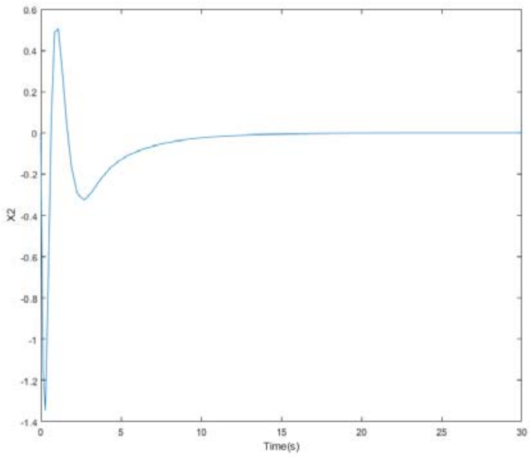

Fig. 2 The responses of state $x_2(t)$

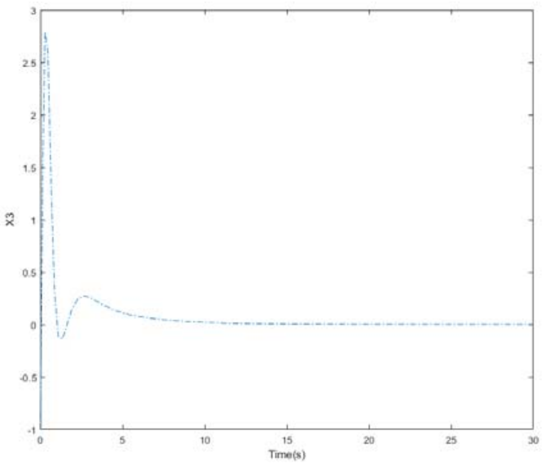

Fig. 3 The responses of state $x_3(t)$

ACKNOWLEDGMENT

This work was supported by the National Science Council of the Republic of China under Contract MOST108-2221-E-019-061.

REFERENCES

[1] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, John Wiley & Sons, Inc., New York, 2001.

[2] H. K. Lam, *Polynomial Fuzzy Model-Based Control Systems-Stability Analysis and Control Synthesis Using Membership Function Dependent Techniques*, Springer International Publishing Switzerland, 2016.

[3] W. J. Chang, C. C. Ku and C. H. Chang, “PDC and Non-PDC fuzzy control with relaxed stability conditions for continuous-time multiplicative noised fuzzy systems”, *Journal of the Franklin Institute - Engineering and Applied Mathematics*, vol. 349, no. 8, pp. 2664-2686, 2012.

[4] W. J. Chang, P. H. Chen and C. C. Ku, “Mixed sliding mode fuzzy control for discrete-time nonlinear stochastic systems subject to variance and passivity constraints”, *IET Control Theory and Applications*, vol. 9, no. 16, pp. 2369-2376, 2015.

[5] W. J. Chang, C. P. Kuo and C. C. Ku, “Intelligent fuzzy control with imperfect premise matching concept for complex nonlinear multiplicative noised systems”, *Neurocomputing*, vol. 154, pp. 276-283, 2015.

[6] S. Peng, H. Wang and C.C. Lim, “Network-based event-triggered control for singular systems with quantizations”, *IEEE Transactions on Industrial Electronics*, vol. 63, no. 2, pp. 1230-1238, 2015.

[7] Z. Weihai, Y. Zhao and L. Sheng, “Some remarks on stability of stochastic singular systems with state-dependent noise”, *Automatica*, vol. 51, pp. 273-277, 2015.

[8] Z. Feng, J. Lam, S. Xu and S. Zhou, “H_∞ control with transients for singular systems”, *Asian Journal of Control*, vol. 18, no. 3, pp. 817-827, 2016.

[9] C. Han, Z. Yi and Z. Y. Zhao, “Admissibility analysis for nonlinear singular system with time-delay via T-S fuzzy model”, *International Journal of Fuzzy Systems*, vol. 19, no. 1, pp. 207-214, 2015.

[10] F. Zhiguang, W. Li and J. Lam, “Dissipativity analysis for discrete singular systems with time-varying delay”, *ISA transactions*, vol. 64, pp. 86-91, 2016.

[11] W. J. Chang and B. J. Huang, “Robust fuzzy control subject to state variance and passivity constraints for perturbed nonlinear systems with multiplicative noises”, *ISA Transactions*, vol.53, no. 6, pp. 1787-1795, 2014.

[12] G. Zheng, D. Efimov, F. J. Bejarano and W. Perruquetti, “Robust stabilization for uncertain T–S fuzzy singular system”, *International Journal of Machine Learning and Cybernetics*, vol. 7, no. 5, pp. 699-706, 2016.

[13] H. Yueqiao, Y. Kao, and C. Gao, “Robust sliding mode control for uncertain discrete singular systems with time-varying delays and external disturbances”, *Automatica*, Vol. 75, pp. 210-216, 2017.

[14] A. Haraldsdottir, P. T. Kabamba and A. G. Ulsoy, “Sensitivity reduction by state derivative feedback,” *Journal of Dynamic Systems, Measurement, and Control*, vol. 110, no. 1, pp. 83-94, 1988.

[15] A. B. Gerstner, V. Mehrmann and N. K. Nichols, “Regularization of descriptor systems by derivative and proportional state feedback,” *SIAM Journal on Matrix Analysis and Applications*, vol. 13, no. 1, pp. 46–67, 1992.

[16] E. Assunção, M. C. M. Teixeira, F. A. Faria, “Robust state-derivative feedback LMI-based designs for multivariable linear systems,” *International Journal of Control*, vol. 80, no. 8, pp.1260-1270, 2007.

[17] W. J. Chang, C. C. Ku, and P. H. Huang, “Robust fuzzy control for uncertain stochastic time-delay Takagi-Sugeno fuzzy models for achieving passivity,” *Fuzzy Sets and Systems*, vol. 161, no. 15, pp. 2012-2032, 2010.

[18] S. Boyd, L. E. Ghaoui, E. Feron and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, 1994.