

Empirical Development of Entropy Generation in Bingham Plastic Fluid with Peristaltic Propulsion

AHMED ZEESHAN^{1,2}, AMAD UR REHMAN¹, NIKOS MASTORAKIS³, SHAFIQ UR RAHMAN⁴

¹Department of Mathematics and Statistics, FOS,
International Islamic University,
H-10, Islamabad
PAKISTAN

²INTERBIT,
Sofia,
BULGARIA

³Technical University of Sofia,
Sofia,
BULGARIA

⁴Department of Mathematics,
University of Chakwal, Chakwal,
PAKISTAN

Abstract: - This study investigates entropy generation in Peristaltic transport of a non-compressible Bingham plastic nanofluid through a symmetric channel, accounting for the impact of buoyancy forces and thermodynamic irreversibility. By applying the long wavelength and low Reynolds number approximations, the governing nonlinear ordinary differential equations are derived and solved numerically using MATLAB's in-built bvp4c solver. A parametric sensitivity analysis is performed using Response Surface Methodology (RSM), where the Bingham number (B_n), Brinkman number (B_r), and Grashof number (G_r) are considered as input parameters, the entropy generation number (E_G) is used as the output answer. The RSM model demonstrates excellent predictive capability, with a coefficient of determination (R^2) of 99.95% and an adjusted R^2 of 99.90%. Analysis of Variance (ANOVA) validates the statistical significance of the model and reveals that the Brinkman number (B_r) is the most influential parameter affecting entropy generation.

Key-Words: - Entropy Generation, Bingham nanofluid, Peristaltic flow, Sensitivity Analysis, Response surface methodology.

Received: March 15, 2026. Revised: April 28, 2026. Accepted: May 23, 2026. Published: October 6, 2026.

1 Introduction

Peristaltic flow refers to the transport of fluid caused by repeated contractions and expansions along the walls of a deformable channel. Peristaltic flow is characteristic of many biological systems, including the digestive system, where it aids in the movement of contents through muscular action. Peristaltic mechanisms are applied in pumps for the transportation of sensitive or contamination-prone fluids in real-world applications, since the fluid is kept isolated from moving components. The properties of this flow are determined by a number of factors, namely wave shape, fluid characteristics, and the character of the channel boundaries. For the

first time, in [1] authors discussed the electro-magneto flow through a peristaltic mechanism in an asymmetric channel, including the impacts of thermal boundary conditions, slip, thermal radiation, and viscous dissipation, which are modeled and simulated mathematically. In [2] authors examined an electrokinetic peristaltic movement within a magnetized nanofluid at a microchannel along Joule heating, and entropy production is assessed. The way metachronal waves behave rheologically in a hydromagnetic Carreau-Yasuda reactive nanofluid that is pushed along a curved conduit by cilia is discussed in [3]. The two-phase nanofluid's peristaltic movement through flexible, wavy barriers

through Ohmic heating and thermal radiation as discussed in [4]. The numerical analysis of the Bingham plastic fluid flow on a heated spinning disk was evaluated in [5]. In [6] authors investigated a two-dimensional symmetric channel that contains a mixed convective peristaltic flow of an incompressible viscoplastic fluid. The entropy created during the peristaltic flow of a nanofluid in a flexible-walled conduit [7]. Authors [8] discussed the effects of convective circumstances and mixed convection are considered. These scholars have studied peristaltic entropy formation flow like in [9], [10], [11], [12] examined the inviscid fluid's peristaltic movement in an asymmetric conduit with a long wavelength and an approximate Reynolds number of zero.

Next, RSM is employed to create correlations between the outcome responses and the input variables, providing greater insight into the behavior of the system. In [13] authors discussed the ANN model and performed a sensitivity analysis for the best magnetic viscous fluid thermal transfer over a diminishing wedge. Author in [14] that using RSM, a sensitivity study has been conducted to determine how nanoparticles affect the formation of entropy. Some academics apply the sensitivity analysis to a variety of thermal engineering challenges. In [15] applied RSM to the sensitivity analysis of permeable heat exchangers in a number of research. The mathematical model for entropy generation in peristaltic flow under long wavelength and creeping flow assumptions was developed by [16].

This study investigates entropy production in the Peristaltic transport of a Bingham plastic fluid in a symmetric geometry, with the impact of buoyancy forces. The study uniquely considers non-Newtonian effects to assess thermodynamic irreversibility. A sensitivity analysis using RSM is performed, revealing the influence of key parameters. The model shows excellent predictive accuracy, with $R^2 = 99.95\%$ and $\text{Adj } R^2 = 99.90\%$, confirming the strength and reliability of the regression model.

2 Problem Formulation

Examine the peristaltic movement of non-compressible viscoplastic fluids in a 2-D homogeneous conduit with breadth $2\alpha_1$. The wall of the conduit is described through a mathematical equation:

$$H(\tilde{X}, \tilde{t}) = A + B \cos(k\tilde{X} - \omega\tilde{t}) \quad (1)$$

Here $k = \frac{2\pi}{\lambda}$.

Here, λ is the wavelength and B represents the wave amplitude, ω denotes the speed of the peristaltic wave, and $\tilde{t}$ is time. The coordinates $(\tilde{X}, \tilde{t})$ refer to a fixed rectangular coordinate system. A schematic representation of the flow is shown in Figure 1.

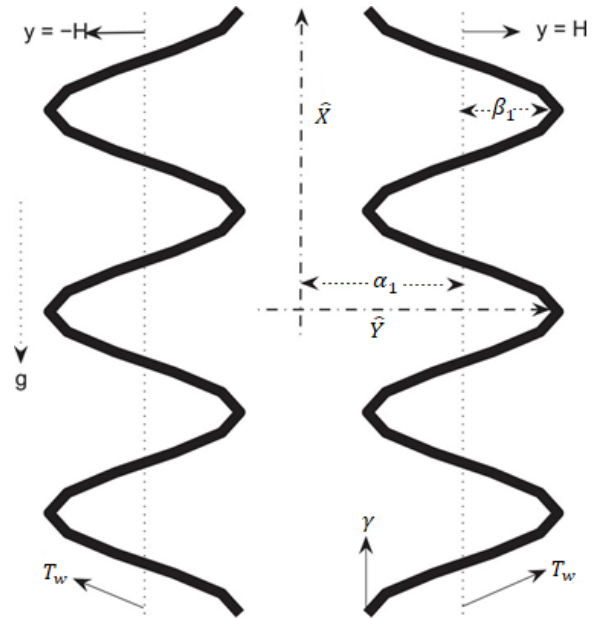


Fig. 1: Geometry of the Problem

Source: created by the authors

Here T_w is the constant temperature of the wall. The continuity, momentum, and energy equations of the flow in $(\tilde{X}, \tilde{Y})$ for the problem is [4] as follows:

$$\frac{\partial \tilde{U}}{\partial \tilde{X}} + \frac{\partial \tilde{V}}{\partial \tilde{Y}} = 0 \quad (2)$$

$$\rho_f \left(\frac{\partial}{\partial \tilde{t}} + \tilde{U} \frac{\partial}{\partial \tilde{X}} + \tilde{V} \frac{\partial}{\partial \tilde{Y}} \right) \tilde{U} \quad (3)$$

$$= -\frac{\partial \tilde{P}}{\partial \tilde{X}} + \frac{\partial}{\partial \tilde{X}} \tilde{\tau}_{XX} + \frac{\partial}{\partial \tilde{Y}} \tilde{\tau}_{XY} \\ + (1 - C_0) \rho_{f_0} g \alpha (\tilde{T} - \tilde{T}_0) \\ + (\rho_f - \rho_{f_0}) g \beta^* (\tilde{C} - \tilde{C}_0)$$

$$\rho_f \left(\frac{\partial}{\partial \tilde{t}} + \tilde{U} \frac{\partial}{\partial \tilde{X}} + \tilde{V} \frac{\partial}{\partial \tilde{Y}} \right) \tilde{V} \quad (4)$$

$$= -\frac{\partial \tilde{P}}{\partial \tilde{Y}} + \frac{\partial}{\partial \tilde{X}} \tilde{\tau}_{YX} + \frac{\partial}{\partial \tilde{Y}} \tilde{\tau}_{YY}$$

$$\begin{aligned}
 (\rho c)_f \left(\frac{\partial}{\partial \tilde{t}} + \tilde{U} \frac{\partial}{\partial \tilde{X}} + \tilde{V} \frac{\partial}{\partial \tilde{Y}} \right) \tilde{T} & \quad (5) \\
 &= k \left(\frac{\partial^2 \tilde{T}}{\partial \tilde{X}^2} + \frac{\partial^2 \tilde{T}}{\partial \tilde{Y}^2} \right) \\
 &+ (\rho c)_p D_B \left(\frac{\partial \tilde{T}}{\partial \tilde{X}} \frac{\partial \tilde{C}}{\partial \tilde{X}} \right. \\
 &\quad \left. + \frac{\partial \tilde{T}}{\partial \tilde{Y}} \frac{\partial \tilde{C}}{\partial \tilde{Y}} \right) \\
 &+ \frac{D_T}{T_0} \left(\left(\frac{\partial \tilde{T}}{\partial \tilde{X}} \right)^2 + \left(\frac{\partial \tilde{T}}{\partial \tilde{Y}} \right)^2 \right) \\
 &\quad \left(\frac{\partial}{\partial \tilde{t}} + \tilde{U} \frac{\partial}{\partial \tilde{X}} + \tilde{V} \frac{\partial}{\partial \tilde{Y}} \right) \tilde{C} \\
 &= D_B \left(\frac{\partial^2 \tilde{C}}{\partial \tilde{X}^2} + \frac{\partial^2 \tilde{C}}{\partial \tilde{Y}^2} \right) \\
 &+ \frac{D_T}{T_0} \left(\frac{\partial^2 \tilde{T}}{\partial \tilde{X}^2} + \frac{\partial^2 \tilde{T}}{\partial \tilde{Y}^2} \right)
 \end{aligned} \quad (6)$$

Here g , $\tilde{P}$, C_p , ρ , α , k , and $\tilde{T}$ are gravitational force, hydrodynamic pressure, specific heat, density of fluid, coefficient of thermal expansion, thermal conductivity, and temperature respectively. $\tilde{V} = [\tilde{U}, \tilde{V}]$ is the velocity tensor, $\tilde{\tau}_{\tilde{X}\tilde{X}}$, $\tilde{\tau}_{\tilde{X}\tilde{Y}}$ and $\tilde{\tau}_{\tilde{Y}\tilde{Y}}$ are elements of the extra stress tensor $\tilde{\tau}$, $\nabla = \left[\frac{\partial}{\partial \tilde{X}}, \frac{\partial}{\partial \tilde{Y}} \right]$ and σ is the Cauchy stress tensor, which is defined as:

$$\sigma = -\tilde{P}\tilde{I} + \tilde{\tau} \quad (7)$$

where

$$2(\eta D + \tau_0 \tilde{D}) \quad (8)$$

In the above relation η and τ_0 are dynamic shear viscosity and stress yield, respectively. The rate of deformation tensor is $D = \frac{1}{2}(\nabla \tilde{V} + (\nabla \tilde{V})^*)$ and the tensor $\tilde{D}$ is defined as:

$$\tilde{D} = D / \sqrt{\varepsilon_1 + 2trD^2} \quad (9)$$

The presence of an essential constant ε_1 is the way to prevent the unboundedness at $D \rightarrow 0$.

By eq. (8) and eq. (9), the extra stress tensor components, expressed in the laboratory frame, are

$$\left. \begin{aligned}
 \tilde{\tau}_{\tilde{X}\tilde{X}} &= 2\eta D_{\tilde{X}\tilde{X}} + \frac{2\tau_0 D_{\tilde{X}\tilde{X}}}{\sqrt{\varepsilon_1 + 2trD^2}}, \\
 \tilde{\tau}_{\tilde{X}\tilde{Y}} &= 2\eta D_{\tilde{X}\tilde{Y}} + \frac{2\tau_0 D_{\tilde{X}\tilde{Y}}}{\sqrt{\varepsilon_1 + 2trD^2}}, \\
 \tilde{\tau}_{\tilde{Y}\tilde{Y}} &= 2\eta D_{\tilde{Y}\tilde{Y}} + \frac{2\tau_0 D_{\tilde{Y}\tilde{Y}}}{\sqrt{\varepsilon_1 + 2trD^2}}
 \end{aligned} \right\} \quad (10)$$

The transformation between the laboratory frame $[\tilde{X}, \tilde{Y}]$ and wave frame $[\tilde{x}, \tilde{y}]$ are

$$\begin{aligned}
 \tilde{x} &= \tilde{X} - \gamma \tilde{t}, \quad \tilde{y} = \tilde{Y}, \quad \tilde{p}(\tilde{x}, \tilde{y}) \\
 &= \tilde{P}(\tilde{X}, \tilde{Y}, \tilde{t}), \quad \tilde{u} \\
 &= \tilde{U} - \gamma, \quad \tilde{v} = \tilde{V},
 \end{aligned} \quad (11)$$

where $\tilde{u}$ and $\tilde{v}$, $\tilde{p}$, are velocity components and pressure in the wave frame $(\tilde{x}, \tilde{y})$, respectively.

Now, to use the eq (11) to transform the eqs (1)-(6), and (10) in wave frame and introducing the following non-dimensional variables:

$$\begin{aligned}
 x &= \frac{\tilde{x}}{\lambda}, \quad y = \frac{\tilde{y}}{\alpha_1}, \quad u = \frac{\tilde{u}}{\gamma}, \quad v = \frac{\tilde{v}}{\gamma}, \quad t \\
 &= \frac{\pi \gamma \tilde{t}}{\delta}, \quad p = \frac{\delta \alpha_1 \tilde{p}}{\eta \gamma}, \quad h \\
 &= \frac{\tilde{h}}{\alpha_1}, \\
 S &= \frac{\alpha_1}{\gamma \eta} \tilde{S}, \quad \theta = \frac{\tilde{T} - \tilde{T}_w}{\tilde{T}_w},
 \end{aligned} \quad (12)$$

to acquire

$$h(x) = 1 + \phi \cos 2\pi x, \quad (13)$$

$$\delta \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (14)$$

$$\begin{aligned}
 \text{Re} \delta \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) u & \quad (15) \\
 &= -\frac{\partial p}{\partial x} + \delta \frac{\partial}{\partial x} \tau_{xx} + \frac{\partial}{\partial y} \tau_{xy} \\
 &+ Gr_t \theta - Gr_c,
 \end{aligned}$$

$$\begin{aligned}
 \text{Re} \delta^2 \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) v & \quad (16) \\
 &= -\frac{\partial p}{\partial y} + \delta^2 \frac{\partial}{\partial x} \tau_{xy} \\
 &+ \delta \frac{\partial}{\partial y} \tau_{yy},
 \end{aligned}$$

$$\begin{aligned}
 \text{Re} \delta^2 \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \theta & \quad (17) \\
 &= \frac{1}{\text{Pr}} \left(\delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \\
 &+ Nb \left(\delta^2 \frac{\partial \phi}{\partial x} \frac{\partial \theta}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial \theta}{\partial y} \right) \\
 &+ Nt \left(\left(\delta \frac{\partial \theta}{\partial y} \right)^2 + \left(\frac{\partial \theta}{\partial y} \right)^2 \right)
 \end{aligned}$$

$$\begin{aligned}
 \text{Re} Sc \left(u \delta \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} \right) & \quad (18) \\
 &= \left(\delta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) \\
 &+ \frac{Nt}{Nb} \left(\delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right)
 \end{aligned}$$

$$\begin{aligned} \tau_{xx} &= 2\delta \frac{\partial u}{\partial x} \\ &+ \frac{2\delta Bn \left(\frac{\partial u}{\partial x}\right)}{\sqrt{\epsilon + 2\delta^2 \left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \delta \frac{\partial v}{\partial x}\right)^2}} \end{aligned} \quad (19)$$

$$\begin{aligned} \tau_{xy} &= \left(\frac{\partial u}{\partial y} + \delta \frac{\partial v}{\partial x}\right) \\ &+ \frac{Bn \left(\frac{\partial u}{\partial y} + \delta \frac{\partial v}{\partial x}\right)}{\sqrt{\epsilon + 2\delta^2 \left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \delta \frac{\partial v}{\partial x}\right)^2}} \end{aligned} \quad (20)$$

$$\begin{aligned} \tau_{yy} &= 2\delta \frac{\partial v}{\partial y} \\ &+ \frac{2Bn \left(\frac{\partial v}{\partial y}\right)}{\sqrt{\epsilon + 2\delta^2 \left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \delta \frac{\partial v}{\partial x}\right)^2}} \end{aligned} \quad (21)$$

The dimensionless numbers, amplitude ratio (ϕ), wave number (δ), Reynold number (Re), Prandtl number (Pr), Bingham number (Bn), Eckert number (Ec), Grashof number (Gr) and ϵ are defined as

$$\begin{aligned} \phi &= \frac{\alpha_1}{\beta_1} (< 1), \quad \delta = \frac{\alpha_1}{\lambda}, \quad Re = \frac{\rho\gamma\alpha_1}{\eta}, \quad Gr = \frac{\rho g a \alpha_1^2 (T - T_w)}{\eta\gamma}, \quad Bn = \frac{\alpha_1 \tau_0}{\eta\gamma}, \quad Pr = \frac{C_p \eta}{k}, \quad Ec = \frac{\gamma^2}{C_p T_w}, \quad Br = Pr \cdot Ec, \quad \epsilon = \frac{\epsilon_1 \alpha_1^2}{\gamma^2} \quad (0 < \epsilon \ll 1). \end{aligned} \quad (22)$$

and

$$u = \psi_y, \quad v = -\delta\psi_x \quad (23)$$

Eq (14), the continuity equation is identically satisfied, while the eq (15) to eq (21) take the following form

$$\begin{aligned} Re\delta \left(\psi_y \frac{\partial}{\partial x} - \delta\psi_x \frac{\partial}{\partial y}\right) \psi_y &= -\frac{\partial p}{\partial x} + \delta \frac{\partial}{\partial x} \tau_{xx} \\ &+ \frac{\partial}{\partial y} \tau_{xy} + Gr\theta, \end{aligned} \quad (24)$$

$$\begin{aligned} -Re\delta^3 \left(\psi_y \frac{\partial}{\partial x} - \delta\psi_x \frac{\partial}{\partial y}\right) \psi_x &= -\frac{\partial p}{\partial y} + \delta^2 \frac{\partial}{\partial x} \tau_{xy} \\ &+ \delta \frac{\partial}{\partial y} \tau_{yy} \end{aligned} \quad (25)$$

$$\begin{aligned} Re\delta^2 \left(\psi_y \frac{\partial}{\partial x} - \delta\psi_x \frac{\partial}{\partial y}\right) \theta &= \frac{1}{Pr} \left(\delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) \\ &+ Nb \left(\delta^2 \frac{\partial \phi}{\partial x} \frac{\partial \theta}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial \theta}{\partial y}\right) \\ &+ Nt \left(\left(\delta \frac{\partial \theta}{\partial y}\right)^2 + \left(\frac{\partial \theta}{\partial y}\right)^2\right) \end{aligned} \quad (26)$$

$$\begin{aligned} Re Sc \left(\delta\psi_y \frac{\partial \phi}{\partial x} - \delta\psi_x \frac{\partial \phi}{\partial y}\right) &= \left(\delta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) \\ &+ \frac{Nt}{Nb} \left(\delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) \end{aligned} \quad (27)$$

$$\begin{aligned} \tau_{xx} &= 2\delta\psi_{xy} \\ &+ \frac{2\delta Bn \psi_{xy}}{\sqrt{\epsilon + 2\delta^2 (\psi_{xy})^2 + 2\delta^2 (\psi_{xy})^2 + (\psi_{yy})^2}} \end{aligned} \quad (28)$$

$$\begin{aligned} \tau_{xy} &= (\psi_{yy} - \delta^2 \psi_{xx}) \\ &+ \frac{Bn (\psi_{yy} - \delta^2 \psi_{xx})}{\sqrt{\epsilon + 2\delta^2 (\psi_{xy})^2 + 2\delta^2 (\psi_{xy})^2 + (\psi_{yy})^2}} \end{aligned} \quad (29)$$

$$\begin{aligned} \tau_{yy} &= -2\delta\psi_{xy} \\ &+ \frac{-2\delta Bn \psi_{xy}}{\sqrt{\epsilon + 2\delta^2 (\psi_{xy})^2 + 2\delta^2 (\psi_{xy})^2 + (\psi_{yy})^2}} \end{aligned} \quad (30)$$

On the employing the approximation of long wavelength and low Reynold number, eqs (23) to (30) reduce to

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \tau_{xy} + Gr\theta, \quad (31)$$

$$0 = -\frac{\partial p}{\partial y}, \quad (32)$$

$$0 = \frac{\partial^2 \theta}{\partial y^2} + Nb \frac{\partial \phi}{\partial y} \frac{\partial \theta}{\partial y} + Nt \left(\frac{\partial \theta}{\partial y} \right)^2, \quad (33)$$

$$0 = \frac{\partial^2 \phi}{\partial y^2} + \frac{Nt}{Nb} \frac{\partial^2 \theta}{\partial y^2} \quad (34)$$

$$\tau_{xx} = 0, \quad \tau_{xy} = \psi_{yy} + \frac{Bn\psi_{yy}}{\sqrt{\epsilon + (\psi_{yy})^2}}, \quad \tau_{yy} = 0. \quad (35)$$

For the eliminating the pressure, differentiating the eq (31) and eq (32) w.r.t y and x respectively, one can write

$$0 = \frac{\partial^2}{\partial y^2} \tau_{xy} + Gr \frac{\partial \theta}{\partial y}. \quad (36)$$

Substituting Eq. (35) into Eqs. (33), (34), and (36) yield the following nonlinear coupled differential equations.

$$0 = \frac{\partial^2}{\partial y^2} \left(\psi_{yy} + \frac{Bn\psi_{yy}}{\sqrt{\epsilon + (\psi_{yy})^2}} \right) + Gr \frac{\partial \theta}{\partial y} \quad (37)$$

Provided below are the non-dimensional volume flow rate and its boundary conditions:

$$\psi = -\frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -1, \quad \theta = 0, \quad \text{at } y = -h, \quad (38)$$

$$\psi = \frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -1, \quad \theta = 0, \quad \text{at } y = h, \quad (39)$$

$$q - 2 = F = \int_{-h}^h \frac{\partial \psi}{\partial y} dy = \psi(h) - \psi(-h) \quad (40)$$

In this context, F denotes the mean flow rate in the wave frame, while q corresponds to that in the fixed frame, both in a form that is not dimensional.

The expression of entropy generation defines the dimensionless entropy generation number for Bingham plastic fluid undergoing peristaltic flow in a symmetric channel, accounting for the combined effects of viscous dissipation, buoyancy force, and yield stress behavior [17].

$$E_G = \left(\frac{\partial \theta}{\partial y} \right)^2 + \frac{B_r}{\Omega} \left(\psi_{yy} + \frac{B_n \psi_{yy}}{\sqrt{\epsilon + (\psi_{yy})^2}} \right)^2 + \Gamma \left(\frac{\Lambda}{\Omega} \right)^2 \left(\frac{\partial \phi}{\partial y} \right)^2 + \xi \left(\frac{\partial \theta}{\partial y} \right) \left(\frac{\partial \phi}{\partial y} \right). \quad (41)$$

3 Problem Solution

In this study, entropy generation in the peristaltic flow of an incompressible Bingham plastic fluid through a symmetric channel is analyzed by incorporating the effects of buoyancy force and thermodynamic irreversibility. The governing flow equations are simplified using the long wavelength and low Reynolds number approximations, leading to a set of nonlinear ODEs that can be numerically solved with the MATLAB function `bvp4c`. The dimensionless entropy generation number is calculated based on the resulting velocity as well as temperature graphs. To examine the influence of key input parameters B_n , B_r , and G_r on entropy generation, a comprehensive sensitivity analysis is performed using RSM. The statistical significance of each parameter is evaluated through ANOVA, with the regression model showing a high degree of accuracy, indicated by a $R^2 = 99.95\%$ and an adjusted $R^2 = 99.90\%$. This integrated approach provides deep insight into the relative contribution of physical parameters to entropy generation and thermodynamic irreversibility.

3.1 Response Surface Methodology

RSM is an empirical technique for modeling and analyzing the impact of numerous input variables on a response variable. In this work, the parameters B_n , B_r , and G_r are considered the primary inputs and are labeled as A, B, and C, respectively. Under the full quadratic model of RSM, the expression for entropy generation E_G can be written as:

$$E_G = \alpha_0 + \alpha_1 A + \alpha_2 B + \alpha_3 C + \alpha_{11} A^2 + \alpha_{12} B^2 + \alpha_{13} C^2 + \alpha_{21} AB + \alpha_{22} AC + \alpha_{23} BC, \quad (42)$$

Here, the coefficients α_i , α_{ij} , for $i = j = 1, 2, 3$, are unknown constants determined through Response Surface Methodology. This study applies the CCD, a methodology proposed by [18]. To construct the quadratic model, RSM implies 20 experimental runs with 19 degrees of freedom, considering three levels of each parameter under the CCD framework. The response variable considered is E_G , and three input variables are carefully selected for simulation. These input parameters B_n , B_r , and G_r are varied within specific ranges, as summarized in Table 1 (Appendix).

3.2 ANOVA

ANOVA is a statistical tool for determining if the various input components influencing a performance outcome differ in statistically

significant ways. In this case, the output replies are E_G and the input parameters are B_n , B_r , and G_r . To get the polynomial of correlation for the replies (E_G) we employed multiple regressions. The MINITAB-19 is one of several statistical software programs available for data analysis in this study. The serial F-test, as seen in Table 2 (Appendix), regression analysis with 29 degrees of freedom and 20 tests are used. The surface design whose response will be saved is determined using ANOVA. The ANOVA, which is shown in Appendix in Table 3 and Table 4, is used to get all statistical estimates for the output response E_G . Each term has its own set of F and P values in equation (42). P-values demonstrate the data's validity, whereas F-values are used to compute the data's variance. Each term in the regression model has a P-value larger than zero 0.5 will be eliminated at the 95% significance level. P-values ≤ 0.05 in an ANOVA are considered statistically significant, whereas P-values > 0.05 are considered statistically insignificant. The mathematical equations (42) can be expressed as follows:

$$E_G = 12.4869 + 4.6700A + 9.1296B - 0.2724C + 1.365B^2 + 3.2409AB \quad (43)$$

As shown in Table 4 (Appendix), the adjusted coefficient of determination ($\text{Adj } R^2$) 99.90% and the E_G coefficient of determination (R^2) 99.95% are near to one another, indicating the correctness of the model. Next, a review of the model's correctness for the output parameter E_G , we turn our attention to the input parameters' sensitivity analysis, which is illustrated in the next section.

3.3 Sensitivity Analysis

In the RSM model, sensitivity is directly related to the rate of change of the response function with respect to its inputs. The regression coefficients for E_G are displayed in Table 4 (Appendix). The sensitivity function for various response parameters from the chosen inputs in the RSM model is inferred as follows using equation (43).

$$\frac{\partial}{\partial A}(E_G) = 4.6700 + 3.2409B \quad (44)$$

$$\frac{\partial}{\partial B}(E_G) = 9.1296 + 2.73B + 3.2409A \quad (45)$$

$$\frac{\partial}{\partial C}(E_G) = -0.2724 \quad (46)$$

4 Results and Discussion

This section presents a discussion of the results obtained from the sensitivity analysis and the evaluation of peristaltic flow efficiency measures, such as entropy generation, using the Bingham plastic model. Numerical solutions were computed using the MATLAB-provided bvp4c solver routine. This study aims to examine the Connection between input quantity and outcome. To accomplish it, expressions for entropy generation (E_G) were derived, and RSM was applied using CCD, as shown in equation (43). The sensitivity of the regression model to input factors A, B, and C is evaluated and illustrated in Figure 2 (Appendix). Sensitivity functions (Eqs. 44–46) describe the increasing or decreasing trends of the response variables with respect to A, B, and C. Figure 2 (Appendix) also visualizes the parameterized sensitivity analysis of entropy generation, with corresponding values at the medium level of B listed in Table 5 (Appendix). The results indicate that entropy generation increases with A and B (positive sensitivity), while it decreases with C (negative sensitivity), as shown in Figure 2 (Appendix).

In order to evaluate model dependability and spot any discrepancies in data representation, this study examines the relationship. Use plots of residuals to compare the input parameters and output responses. ANOVA was employed to support the residual analysis presented in Figure 3 (Appendix). The normal probability plot in Figure 3(a) (Appendix) shows a near-linear trend, confirming the normality of the data. Figure 3(b) (Appendix) displays a residual histogram that is nearly symmetric with minimal skewness, indicating a well-fitting model. The observation order plot in Figure 3(c) (Appendix) reveals a strong correlation between observed and predicted values, with residuals decreasing as the observation order increases, further supporting the model's reliability.

Figure 4 (Appendix) represents the surface contour plot of E_G . The entropy generation (E_G) as an objective of the A, B, and C represented in Figure 4 (Appendix). The influence that B and C have on E_G when $A=0$, $B=0$, $C=0$ is shown in Figure 4(a-c) (Appendix). It is noted that the minimum E_G exists when B is lower and C is greater. Conversely, the higher entropy is observed when B is low, and C is high.

4 Conclusion

A thorough study of the sensitivity of the Bingham plastic fluid model's peristaltic flow was conducted

through both numerical simulations and statistical methods. The developing D.E are numerically solved using Matlab, which is incorporated into bvp4c. The numeric values of E_G are calculated for various input values. Then, using the numerical values, create an empirical model for sensitivity analysis of output responses E_G . The main outcomes identified in this study are:

- The high coefficient of determination (R^2) reflects a good model fit.
 - The evaluation figures verify the validity of the model, the normality of the residuals, and its high prediction accuracy.
 - E_G is most sensitive to the Brinkman number (B), followed by the Bingham number (A) and the Grashof number.
 - The entropy generation E_G is minimized at lower Brinkman numbers and higher Grashof numbers.
- Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)
A.Z. concept, supervision, A.U.R. Results, writup, S.U.R. visualization, methodology, NM: supervision, review, evaluation
- Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself: Nil
- Conflict of Interest: No conflict of interest declare

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

The authors wrote, reviewed and edited the content as needed and they have not utilised artificial intelligence (AI) tools. The authors take full responsibility for the content of the publication.

References:

- [1] Sridhar, V., & Ramesh, K. (2025). Entropy generation analysis on EMHD peristaltic propulsion of blood-mediated gold-silver nanoparticles: Application to fatal diseases. *Waves in Random and Complex Media*, 35(1), 1002-1027.
- [2] Bhatti, M. M., Sheikholeslami, M., & Zeeshan, A. (2017). Entropy analysis on electro-kinetically modulated peristaltic propulsion of magnetized nanofluid flow through a microchannel. *Entropy*, 19(9), 481.
- [3] Abbas, Z., Rafiq, M. Y., & Arslan, M. S. (2025). Rheological characteristics of metachronal waves in Carreau–Yasuda nanofluid for heat and mass transfer in a curved configuration. *Multiscale and Multidisciplinary Modeling, Experiments and Design*, 8(3), 179.
- [4] Mishra, S. R., Panda, S., Baithalu, R., & Shah, N. A. (2025). Peristaltic transport of mixed convective two-phase nanofluid with Ohmic heating used in drug delivery systems: Adomian decomposition method. *Physics of Fluids*, 37(2).
- [5] Khan, M., Salahuddin, T., Awais, M., Altanji, M., Ayub, S., & Khan, Q. (2023). Calculating the entropy generation of a Bingham plastic fluid flow due to a heated rotating disk. *International Communications in Heat and Mass Transfer*, 143, 106721.
- [6] Z.Asghar and N. Ali, Mixed convective heat transfer analysis for the peristaltic transport of viscoplastic fluid: Perturbation and numerical analysis, *AIP advances*, 5(9) (2019): 095001.
- [7] Hayat, T., Nawaz, S., Alsaedi, A., & Rafiq, M. (2016). Analysis of entropy generation in mixed convective peristaltic flow of nanofluid. *Entropy*, 18(10), 355.
- [8] Nisar, Z., Ahmed, B., Ghoneim, M. E., & Elkotb, M. A. (2024). Thermal performance of mixed convective radiative peristaltic flow of Bingham nanofluid. *Sensors and Actuators A: Physical*, 373, 115399.
- [9] T. Hayat, Q. Hussain and N. Ali, Influence of partial slip on the peristaltic flow in a porous medium. *Physica A: Statistical Mechanics and its Applications*, 387(14) (2008): 3399-3409.
- [10] T. Hayat, N. Ali and S. Asghar, Hall effects on peristaltic flow of a Maxwell fluid in a porous medium. *Physics Letters A*. 363(5-6) (2007): 397-403.
- [11] R. Ellahi, M.M. Bhatti, A.Riaz and M. Sheikholeslami, Effects of magnetohydrodynamics on peristaltic flow of Jeffrey fluid in a rectangular duct through a porous medium, *Journal of Porous Media*, 17(2) (2014).
- [12] Zeeshan, A., Khan, M. I., Ellahi, R., & Asghar, Z. (2023). Artificial neural network simulation and sensitivity analysis for optimal thermal transport of magnetic viscous fluid over shrinking wedge via RSM. *International Journal of Numerical Methods for Heat & Fluid Flow*, 33(10), 3492-3518.
- [13] Asghar, Z., Hussain, D., Zeeshan, A., & Alsulami, H. (2023). Numerical and statistical analysis of transport performance of viscous flows over a wedge with heat transfer by using response surface methodology: sensitivity analysis. *The*

- European Physical Journal Plus, 138(10), 1-11.
- [14] Zeeshan, A., Asghar, Z., & Rehman, A. U. (2024). AI based optimal analysis of electro-osmotic peristaltic motion of non-Newtonian fluid with chemical reaction using artificial neural networks and response surface methodology. *International Journal of Numerical Methods for Heat & Fluid Flow*, 34(6), 2345-2375.
- [15] Rehman, A. U., Asghar, Z., Zeeshan, A., & Marin, M. (2024). Empirical modeling and sensitivity analysis of pressure rise per wavelength and frictional forces for the peristaltic flow of Bingham plastic fluids: application of response surface methodology. *Journal of Thermal Analysis and Calorimetry*, 149(17), 9619-9637.
- [16] B. Darbari, S. Rashidi and J. Abolfazli Esfahani, Sensitivity analysis of entropy generation in nanofluid flow inside a channel by response surface methodology, *Entropy*, 18(2), (2016): 52.
- [17] H. A. Sayed and M. Y. Abouzeid, Radially varying viscosity and entropy generation effect on the Newtonian nanofluid flow between two co-axial tubes with peristalsis, *Scientific Reports*, vol. 13, no. 1, pp. 11013, 2023.
- [18] Box and Wilson., "On the experimental attainment of optimum conditions," *J. R. Stat. Soc. Ser. B*, vol. 13, no. 1, pp. 1-45, 1995.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

Creative Commons Attribution License 4.0 (Attribution 4.0 International, CC BY 4.0)

This article is published under the terms of the Creative Commons Attribution License 4.0

https://creativecommons.org/licenses/by/4.0/deed.en_US

APPENDIX

Table 1. Design factors and their defined Ranges. (Self Assumption)

Type	Factor	Symbol	Levels		
			-1	0	1
Input Factors	B_n	A	0.5	1.0	1.5
	B_r	B	0.5	1.0	1.5
	G_r	C	0.5	1.0	1.5

Source: created by the authors

Table 2. Design of experiments and observed outcomes. (Self generated numerical solution)

Exp. No.	Point type	Coded values			Real values			Output responses
		A	B	C	B_n	B_r	G_r	
1	Factorial	-1	-1	-1	0.5	0.5	0.5	3.4751
2		1	-1	-1	1.5	0.5	0.5	6.5863
3		-1	1	-1	0.5	1.5	0.5	15.6874
4		1	1	-1	1.5	1.5	0.5	31.9962
5		-1	-1	1	0.5	0.5	1.5	3.3921
6		1	-1	1	1.5	0.5	1.5	6.4488
7		-1	1	1	0.5	1.5	1.5	14.9215
8		1	1	1	1.5	1.5	1.5	30.7080
9	Axial	-1	0	0	0.5	1	1	8.5372
10		1	0	0	1.5	1	1	16.9738
11		0	-1	0	1	0.5	1	4.8977
12		0	1	0	1	1.5	1	22.7826
13		0	0	-1	1	1	0.5	12.7179
14		0	0	1	1	1	1.5	12.2685
15-20	Centre	0	0	0	1	1	1	12.4945

Source: created by the authors

Table 3. Analysis of entropy generation using ANOVA. (results)

	Source	DF	Adj SS	Adj MS	F-Value	P-Value
E_G	Model	9	1148.98	127.664	2020.65	0.000
	Linear	3	1052.32	350.773	5551.98	0.000
	Square	3	12.17	4.057	64.21	0.000
	Interaction	3	84.49	28.164	445.77	0.000
	Error	10	0.63	0.063		
	Lack-of-Fit	5	0.63	0.126	*	*
	Pure error	5	0.00	0.000		
	Total	19	1149.61			

Source: created by the authors

Table 4. Regression analysis of entropy generation (E_G). (results)

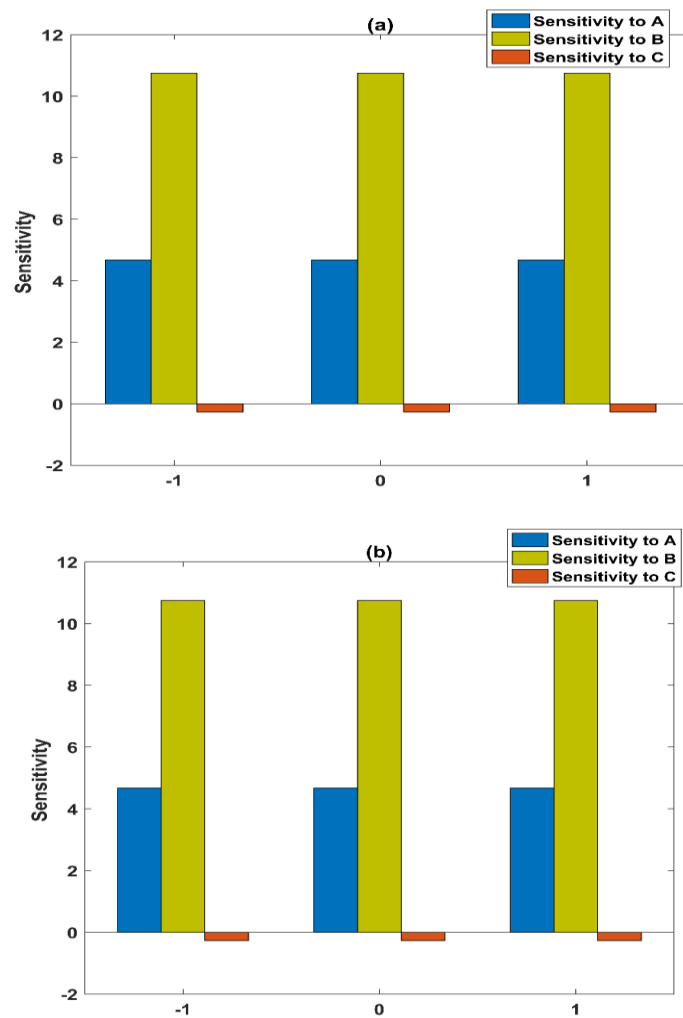
Terms	Coefficient	P-value	Significant
Constants	12.4869	0.000	Y
A	4.6700	0.000	Y
B	9.1296	0.000	Y
C	-0.2724	0.006	Y
A^2	0.280	0.095	N
B^2	1.365	0.000	N
C^2	0.018	0.910	N
AB	3.2409	0.000	N
AC	-0.0721	0.436	Y
BC	-0.2292	0.027	N
	$R^2=99.95\%$		$AdjR^2=99.90\%$

Source: created by the authors

Table 5. Sensitivity of E_G (results)

A	C	$\frac{\partial E_G}{\partial A}$	$\frac{\partial E_G}{\partial B}$	$\frac{\partial E_G}{\partial C}$
	-1	4.6700	10.7500	-0.2724
-1	0	4.6700	10.7500	-0.2724
	1	4.6700	10.7500	-0.2724
	-1	4.6700	10.7500	-0.2724
0	0	4.6700	10.7500	-0.2724
	1	4.6700	10.7500	-0.2724
	-1	4.6700	10.7500	-0.2724
1	0	4.6700	10.7500	-0.2724
	1	4.6700	10.7500	-0.2724

Source: created by the authors



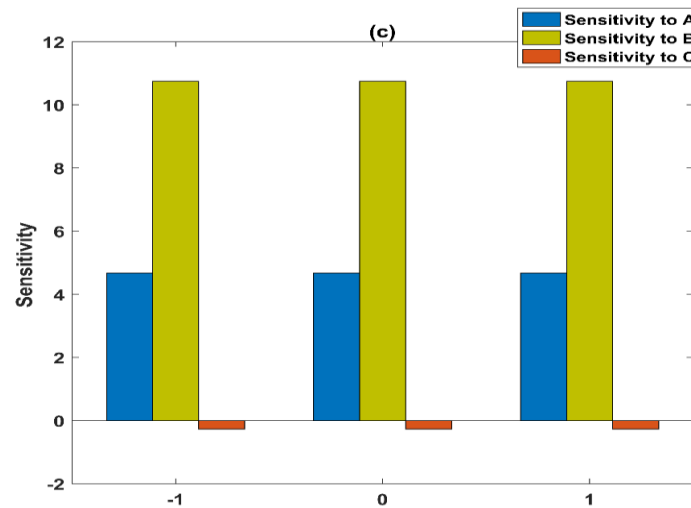
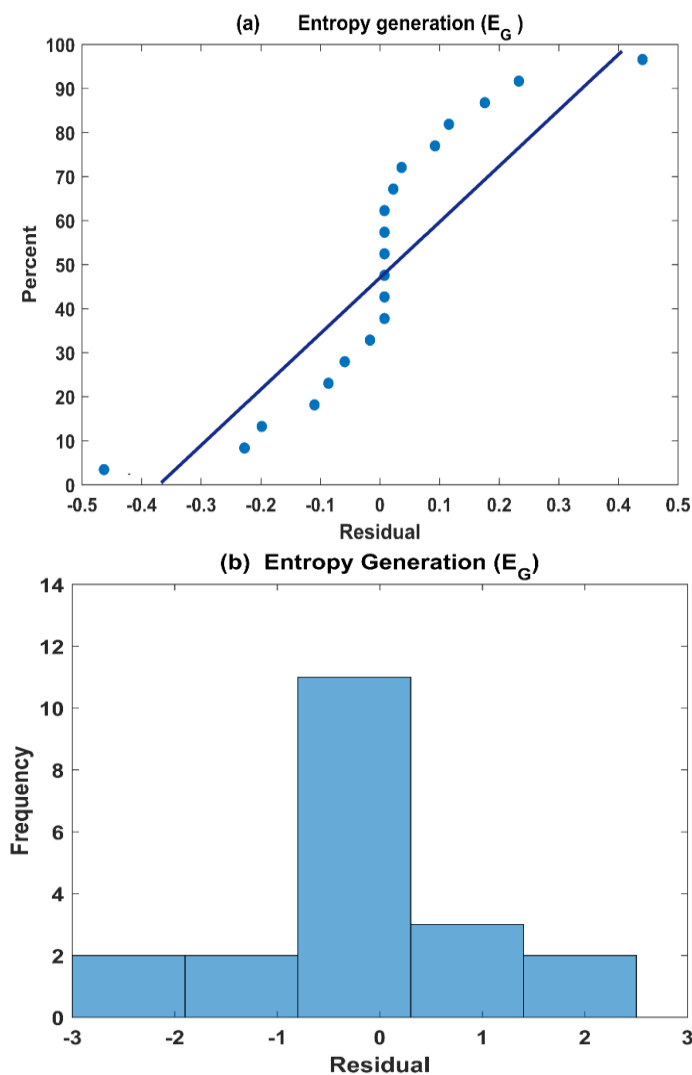


Fig. 2: Sensitivity for E_G for variation of B with a. A=-1, C=-1, 0,1 b. A=0, C=-1, 0,1 c. A=1, C=-1, 0, 1.
Source: created by the authors



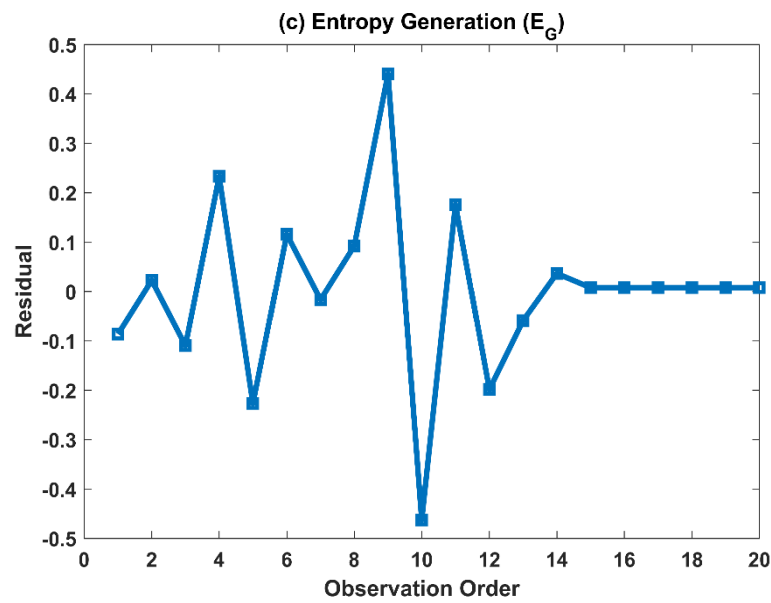
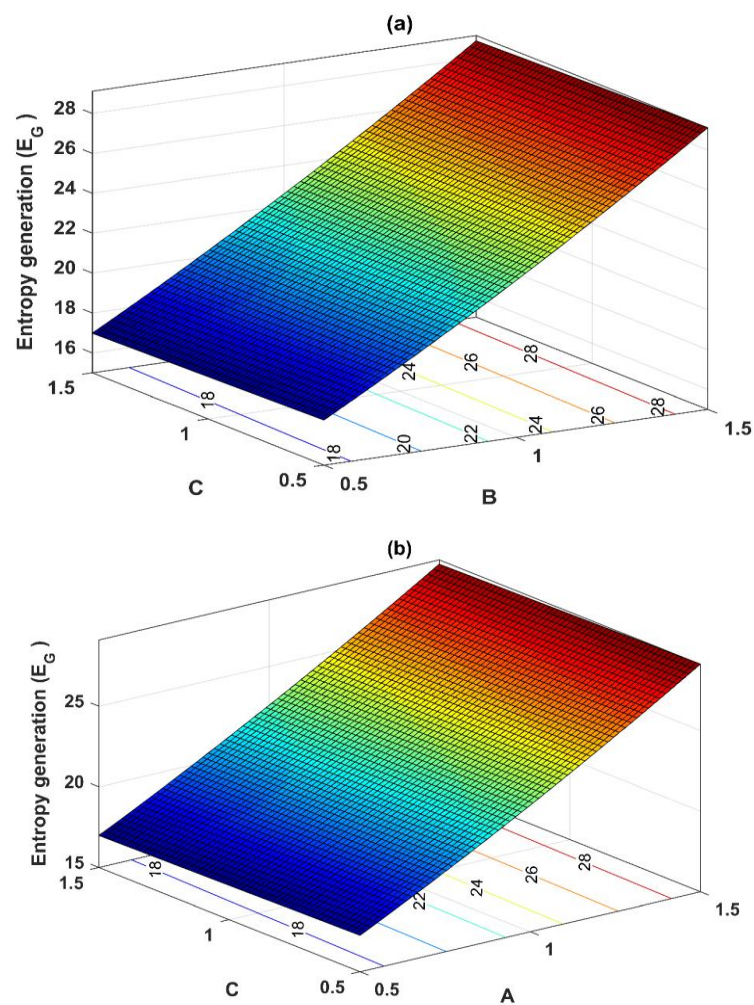


Fig. 3 (a) Normal Distribution, (b) Histogram, and (c) observation order against standardized Residual plot of E_G . (Results)

Source: created by the authors



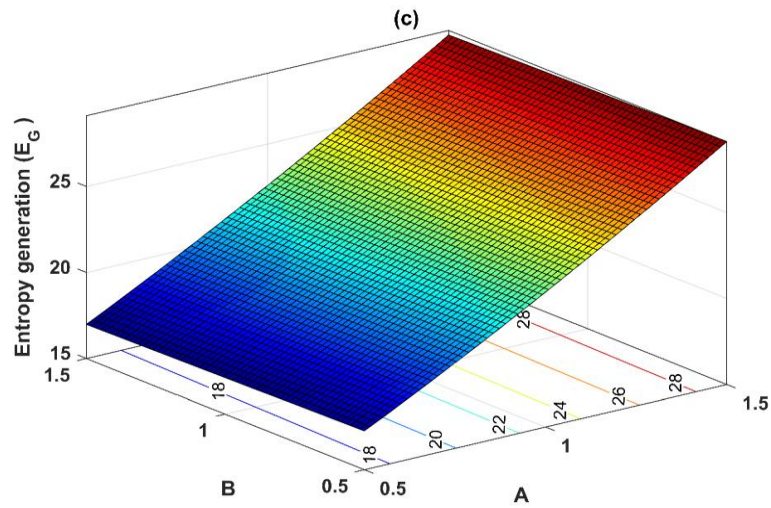


Fig. 4: Contour Maps of E_G : (a) $A=0$ (b) $B=0$, (c) $C=0$
Source: created by the authors