

Advanced 3D Fatigue Life Prediction of Expansion Bellows in Fixed Tube-Sheet Heat Exchangers Using Continuum Damage Mechanics

DJEBBARA BENZERGA¹, FAYCAL SOTEHI¹, ADEL CHOUITER²

¹LSCMI, University of Sciences and Technology of Oran,
Mechanical Engineering Faculty,
B. P 1505 El M'Naouer 31000, Oran,
ALGERIA

²Institut des Sciences et des Techniques Appliquées (ISTA),
University Constantine 1,
ALGERIA

Abstract: - This paper presents a three-dimensional finite element framework coupled with continuum damage mechanics for predicting the fatigue life of expansion bellows in fixed tube sheet heat exchangers. The approach captures circumferential stress non-uniformity, local ovalization, and out-of-plane deformations induced by combined internal pressure and axial thermal displacement—effects inherently neglected in conventional axisymmetric analyses. A systematic comparison between 2D and 3D models is conducted, validated against ASME BPVC Appendix 26 fatigue equations. Results demonstrate that simplified 2D approaches overestimate fatigue life by up to 63%, yielding non-conservative designs. The proposed 3D damage model predicts 16% mean absolute error relative to ASME allowable cycles, with consistently conservative estimates and reduced scatter. Ultimately, this model offers a robust, physics-based approach for evaluating the structural integrity of critical petrochemical assets. It supports better design optimization and helps create smarter, data-driven inspection schedules.

Key-Words: - Expansion bellows; 3D finite element analysis; Continuum damage mechanics; Fatigue life; Heat exchanger; ASME validation; 2D–3D comparison.

Received: March 19, 2026. Revised: May 2, 2026. Accepted: June 2, 2026. Published: October 6, 2026.

1 Introduction

Expansion bellows are critical components in fixed tube-sheet heat exchangers, where they accommodate differential thermal expansion between the shell and tube bundle while maintaining mechanical integrity. Due to their thin-walled, highly convoluted geometry, bellows are subjected to complex stress states under combined loadings—internal pressure, axial thermal displacements, and repeated thermomechanical cycles. Premature failure during service is often caused by fatigue [1], [2]. Consequently, the ability to accurately predict fatigue life is crucial for ensuring both industrial reliability and safety.

Most industry standards, such as the ASME Boiler and Pressure Vessel Code Section VIII, Division 1, Appendix 26 [3], depend largely on analytical methods or 2D axisymmetric finite element models. Engineers favor these methods because they are computationally efficient and relatively simple to implement. However, recent numerical studies by [4], [5], [6], among others [7],

[8], [9] have refined stress predictions using finite element analysis, moving beyond the limitations of early thin-shell theory.

However, the axisymmetric assumption is strictly valid only for bellows under purely axial pressure and displacement loading. Real heat exchanger applications involve combined loading—including lateral deflection and thermal gradients—that induces circumferential stress variations, local ovalization, and out-of-plane deformations [10], [11], [12]. These three-dimensional phenomena cannot be captured by 2D axisymmetric models, resulting in non-conservative fatigue life estimates that systematically underestimate critical local stresses and mask damage initiation sites [13].

The problem is made worse by inconsistent validation practices throughout the literature. For instance, [14] claimed a "close match" between their analytical EJMA calculations and numerical simulations, but they did not provide quantitative metrics like percentage error or correlation coefficients. Because comparable studies [15], [16],

[17] also lack this systematic data, it is difficult to determine exactly when simplified approaches are valid or which design codes perform best under specific operating conditions.

To overcome these limitations, three-dimensional finite element models have been proposed, demonstrating the significant influence of 3D effects on stress distribution and fatigue damage initiation under combined thermomechanical loadings [15], [18]. In parallel, continuum damage mechanics (CDM) has emerged as a powerful tool for modeling how materials degrade progressively under cyclic loading [19], [20]. Unlike traditional methods that focus solely on stress, CDM-coupled FEA simulates the actual evolution of damage and crack initiation. This approach is thermodynamically consistent and leads to significantly more realistic fatigue life predictions.

The present work develops a three-dimensional finite element approach coupled with a continuum damage model to predict the fatigue life of expansion bellows in fixed tube-sheet heat exchangers. We conduct a systematic comparative analysis between axisymmetric 2D and full 3D models, validate numerical predictions against the fatigue equations of ASME BPVC Appendix 26, and quantify the error introduced by simplified modeling approaches.

The main contributions of this study are:

- Development of a detailed three-dimensional finite element model of expansion bellows accounting for geometric nonlinearity and contact mechanics
- Implementation of a continuum damage mechanics framework for physically-based fatigue life prediction
- Systematic comparative analysis between 2D axisymmetric and full 3D simulations, with quantified error metrics
- Validation of numerical results against ASME pressure vessel fatigue design rules
- Quantification of non-conservatism introduced by simplified 2D modeling approaches under combined loading conditions

The results demonstrate that simplified 2D analyses yield non-conservative estimates, while the proposed 3D damage-based framework provides more realistic and safer assessments suitable for industrial design and structural integrity evaluation.

2 Geometry and Mechanical Model

The expansion bellows considered in this study consist of a thin-walled single convolution attached to the shell of a fixed tube sheet heat exchanger. The main geometric features include the inner and outer radii, convolution height, wall thickness, and transition fillets, in accordance with industrial practices and ASME code recommendations [21].

The primary function of these bellows is to take up the axial differential displacement caused by the mismatch in thermal expansion between the tube bundle and the shell. By doing so, they safeguard the tube-to-tube-sheet welds and prevent leakage from occurring [22].

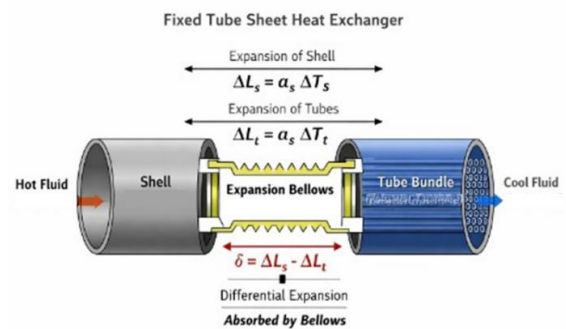


Fig. 1: Schematic representation of thermal expansion mismatch in a fixed tube sheet heat exchanger, showing ΔL_t (tube bundle expansion), ΔL_s (shell expansion), and δ absorbed by the expansion bellows.

Source: created by the authors

When heat is applied, both the tube bundle and the shell expand axially. We represent the expansion of the tube bundle as ΔL_t and that of the shell as ΔL_s . The difference between these two creates the differential displacement, defined as:

$$\delta = \Delta L_t - \Delta L_s$$

As shown in Figure 1, the bellows must absorb this total axial displacement δ . Since this displacement controls the deformation and stress state of the bellows, calculating δ accurately is vital. It directly dictates the cyclic strain magnitude on the convolutions, which is crucial for predicting fatigue life.

By incorporating this thermal displacement into the three-dimensional finite element model, the bellows' response to realistic operating conditions - including internal pressure and axial differential expansion - is captured. This approach ensures that the simulation reflects actual service conditions and

enables accurate prediction of localized stress, ovalization, and fatigue-induced damage.

In contrast to axisymmetric modeling, the developed three-dimensional model accounts for the full circumferential geometry, thereby enabling the analysis of non-uniform stress states and complex deformation modes that cannot be properly represented by two-dimensional analyses.

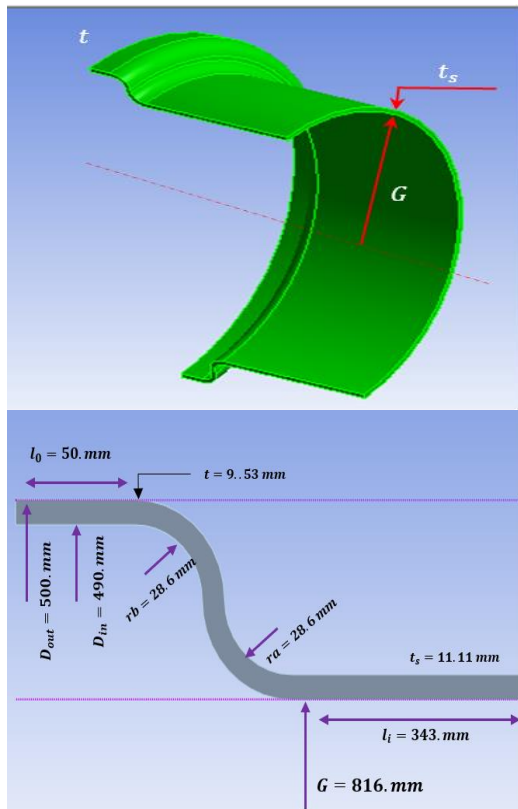


Fig. 2: 1/2 model of the expansion bellows used for simulation, exploiting axial symmetry. The main dimensions of the bellows are indicated.

Source: created by the authors

2.1 Finite Element Modeling

The numerical modeling of the expansion bellows is performed using the finite element analysis software ANSYS. In contrast to classical two-dimensional axisymmetric approaches, a full three-dimensional model is developed to represent the entire circumferential geometry and capture three-dimensional effects likely to influence the mechanical behavior of the bellows. The 3D model geometry includes the inner radius, thickness, convolution height, and fillet radii, conforming to industrial practices and design code recommendations (Figure 2).

The mesh is generated using three-dimensional 8-node solid elements, suitable for modeling thin-walled structures with curved geometry. These elements have compatible shape functions, ensuring

an adequate representation of displacement and stress fields.

Meshing is a critical step in finite element analysis, as it affects both the accuracy of numerical results and computational efficiency. We focused heavily on refining the mesh in critical areas, specifically around the bends and fillet radii. These are the zones where we expect the highest stress concentrations, so a finer mesh is necessary to capture them accurately (Figure 3).

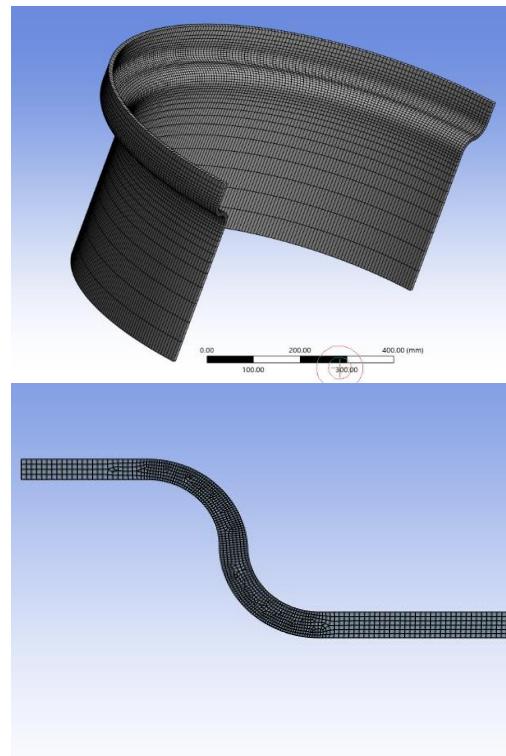


Fig. 3: Mesh of the expansion bellows: 2D (quadrilateral dominant 2036 element PLANE183) and 3D (21600 element SOLID186).

Source: created by the authors

A gradual transition between fine and coarse mesh areas is ensured to avoid excessive element distortion and guarantee the numerical stability of the model. The choice of the final mesh results from a convergence study, during which refinement was progressively increased until variations in maximum stresses became negligible.

The boundary conditions for the three-dimensional model are defined to accurately replicate experimental and service conditions. The following conditions are applied:

- (i) The shell end is fixed by a rigid support;
- (ii) The shell is subjected to internal pressure and its top and bottom walls are radially constrained;
- (iii) The bellows is subjected to a uniform internal pressure applied to the inner surface, as well as an

imposed axial displacement simulating the effect of thermal expansion.

This three-dimensional modeling accounts for circumferential stress non-uniformity, local ovalization, and out-of-plane deformations—phenomena that cannot be correctly represented by two-dimensional axisymmetric models.

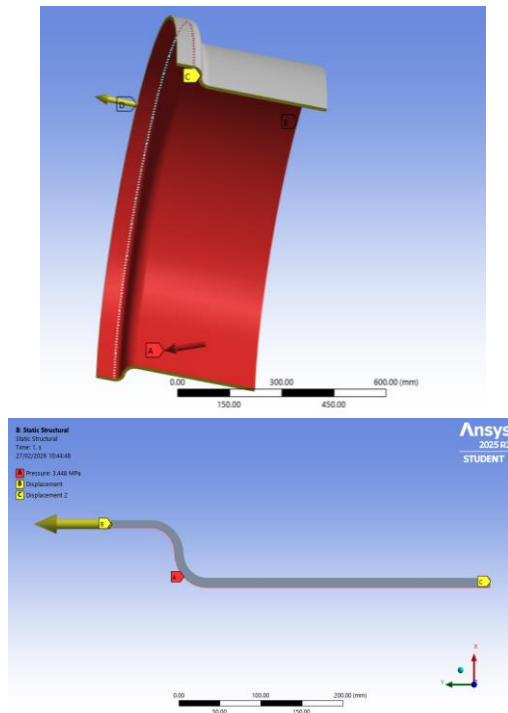


Fig. 4: Boundary conditions
Source: created by the authors

The boundary conditions and loads applied to the numerical model are defined to faithfully reproduce the operating conditions of the expansion bellows in a fixed tube sheet heat exchanger (Figure 4). The symmetry planes of the three-dimensional model are exploited to reduce the computational domain to half of the structure. On these planes, symmetry conditions are imposed, prohibiting displacements normal to the plane while allowing tangential displacements.

At the larger-diameter end, the bellows is allowed to move axially but is restricted radially. This setup simulates a connection that permits elongation while preventing radial shifting. In contrast, the smaller-diameter end is axially fixed but radially free. This mimics the anchoring of the bellows to the main exchanger structure while still permitting radial expansion.

The bellows is subjected to a uniform internal pressure, applied to the inner surface of the wall, representing the fluid pressure in service. Additionally, an imposed axial displacement is applied to simulate the effect of differential thermal

expansion between the exchanger components. The combination of these loads replicates a thermomechanical loading state representative of real operating conditions.

2.1.1 Material Properties

The material behavior is assumed to be isotropic elastoplastic, with mechanical and thermal properties taken from standard pressure vessel codes. Temperature-dependent effects are introduced through equivalent thermal displacement applied in the axial direction. The mechanical and thermal properties of the material as well as the geometric dimensions of the expansion bellows are derived from the ASME Section II code, Part D, and are summarized in Table 1. The values are expressed in both imperial units and SI units to ensure consistency with industry practices and normative recommendations. These data serve as a basis for the numerical validation of the proposed methodology.

Table 1. Geometrical and Material properties of the bellows

Parameter	Value (SI - imperial)
Material	SA 516 Grade 70
Damage parameters [24]	Allowable Stress 262 MPa 38 ksi
$D_{1c} = 0.22$, $\varepsilon_{pD} = 0\%$	Average Metal Temperature - Shell Side 54 °C 129.2 °F
$S = 2.55$	Average Metal Temperature - Tube Side 86 °C 186.8 °F
$\sigma_D^u = 153$. MPa	Elastic Modulus (E) 189 GPa 27400 ksi
$\sigma_y^u = 200$ MPa	Poisson's Ratio (ν) 0,30
$\sigma_u = 610$. MPa	
Geometric Parameter	Value (SI - imperial)
Outer Diameter of the External Cylinder	0,500 m 19.69 in
Inner Diameter of the Outer Cylinder	0,490 m 19.29 in
Geometric Parameter (G)	0,816 m 32.13 in
Length of the External Cylinder (l_o)	0,0508 m 2.00 in
Length of the Internal Cylinder (l_i)	0,343 m 13.5 in
Thickness of the Bellows (t)	9,53 mm .375 in
Thickness of the Shell (t_s)	11,11 mm .437 in
Internal and External Connection Radii ($r_a = r_b$)	28,6 mm 1.126 in

Source: [23]

where:

D_{1c} : Critical damage threshold in pure tension

S: Damage strength material parameter

ε_{pD} : Damage plastic strain threshold in pure tension

σ_u : Ultimate stress (MPa)

σ_y^u , σ_D^u : Yield stress (MPa) & Endurance limit (MPa) at microscale [25].

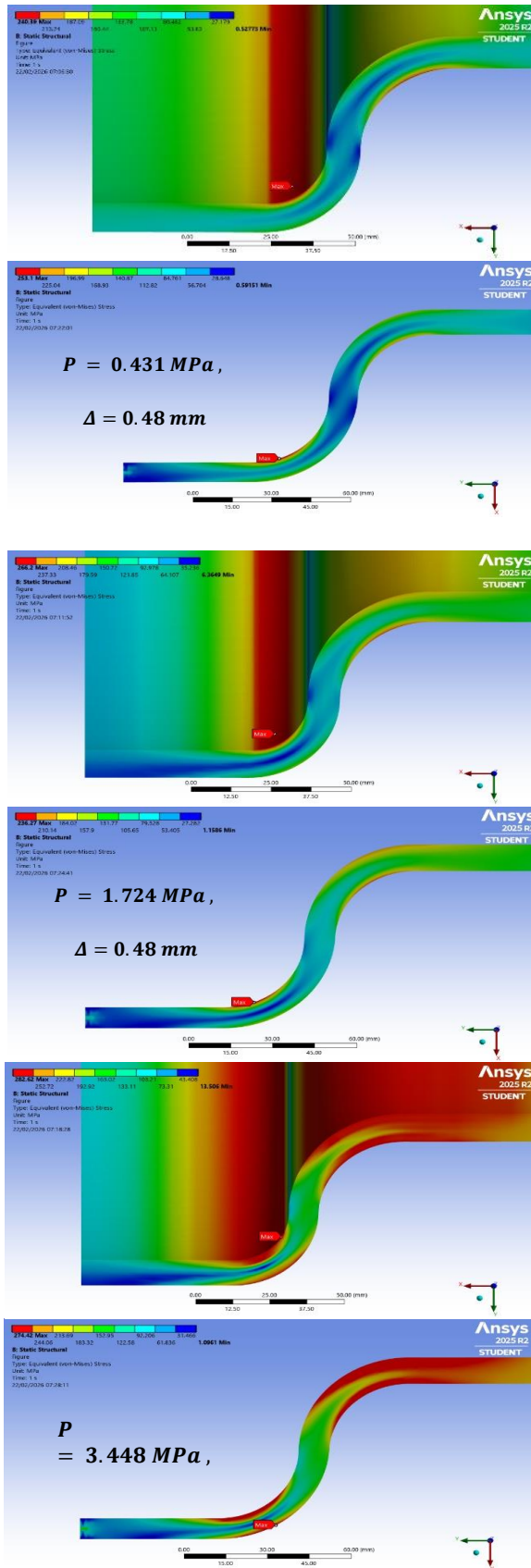


Fig. 5: Von Mises stress results: 2D and 3D models representing similar stress distribution with different max values

Source: created by the authors

2.1.2 Stress and Deformation Analysis: 2D versus 3D

The von Mises stress distribution obtained from the 3D model reveals significant circumferential variations, particularly in the knuckle regions. These variations are absent in the axisymmetric model, which predicts a uniform stress field along the circumference.

A direct comparison between 2D and 3D results demonstrates that the maximum stress values and their locations differ substantially, leading to different predictions of critical damage zones (Figure 5).

2.2 Stress Components Due to Pressure & Axial Displacement: ASME - EJMA

The design and fatigue evaluation of expansion bellows in pressure vessels are commonly performed following the guidelines of the EJMA - Expansion Joint Manufacturers Association [26] ASME Boiler and Pressure Vessel Code, Section VIII, Division 1, Appendix 26 [27]. This appendix provides detailed formulas for calculating the stresses induced by internal pressure and axial displacement, as well as the corresponding allowable number of cycles.

According to ASME BPVC.VIII.1-2021 Appendix 26, the stress components in unreinforced U-shaped bellows (Figure 6) are calculated as follows:

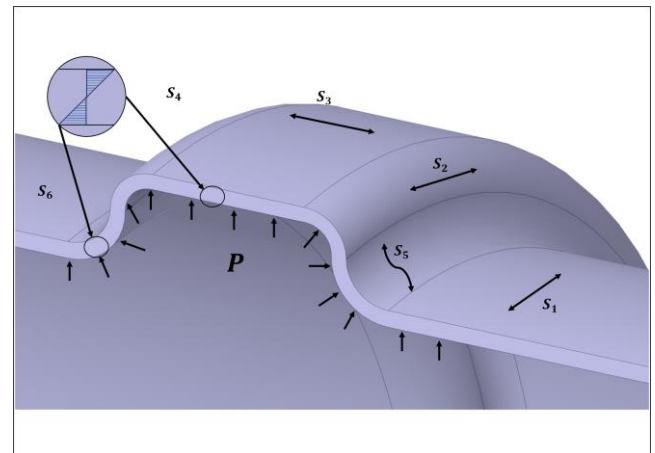


Fig. 6: Stress distribution across U-shaped bellows convolution: Axial, Bending, and Hoop stress characterization.

Source: [28].

- The meridional membrane stress due to pressure is given by:

$$S_3 = \frac{w}{2nt_p} P$$

- The meridional bending stress due to pressure is given by:

$$S_4 = \frac{1}{2n} \left(\frac{w}{t_p} \right)^2 C_p P$$

where:

- w = convolution depth (height)
- t_p = thickness of one ply
- n = number of plies
- P = internal pressure
- C_p = factor depending on bellows geometry

These meridional stresses must satisfy the combined stress criterion:

$$S_3 + S_4 \leq K_m S$$

with $K_m = 1.5$ for annealed bellows, and S the allowable stress at design temperature.

The stresses resulting from the total equivalent axial displacement range Δq per convolution are:

- Meridional membrane stress due to displacement:

$$S_5 = \frac{1}{2} \frac{E_b t_p^2}{w^3 C_f} \Delta q$$

- Meridional bending stress due to displacement:

$$S_6 = \frac{5}{3} \frac{E_b t_p}{w^2 C_d} \Delta q$$

where:

- E_b = modulus of elasticity of bellows material
- C_f, C_d = geometric factors depending on bellows shape
- Δq = equivalent axial displacement range per convolution

2.2.1 Total Stress Range and Fatigue Evaluation

For fatigue evaluation, the total stress range S_t combines pressure-induced and displacement-induced stresses. Following paragraph 26-6.6.2, we apply a 0.7 factor to the pressure components:

$$S_t = 0.7[S_3 + S_4] + [S_5 + S_6]$$

We then determine the allowable number of cycles (N_{alw}) using the fatigue equations from paragraph 26-6.6.3.2. These equations rely on a fatigue strength reduction factor (K_g) which falls between 1.0 and 4.0. This factor accounts for

geometric stress concentrations, weld geometry, and surface conditions. Depending on the stress level, the code defines two distinct regimes:

if $K_g \frac{E_o}{E_b} S_t \geq 448 \text{ MPa}$ (65,000 psi):

$$N_{alw} = \left(\frac{K_o}{K_g \frac{E_o}{E_b} S_t - S_o} \right)^2$$

- $K_o = 35\,850$ and $S_o = 264$, when S_t is expressed in MPa,

if $K_g \frac{E_o}{E_b} S_t < 448 \text{ MPa}$ (65,000 psi):

$$N_{alw} = \left(\frac{K_o}{K_g \frac{E_o}{E_b} S_t - S_o} \right)^2$$

- $K_o = 46\,200$ and $S_o = 211$, when S_t is expressed in MPa,

If $K_g \frac{E_o}{E_b} S_t \leq 257.2 \text{ MPa}$ (37,300 psi), then $N_{alw} = 10^6 \text{ cycles}$.

It is important to note that these formulas include built-in safety factors—2.6 on cycles and 1.25 on stresses. They represent the maximum allowable cycles for the specific operating conditions being evaluated.

2.2.2 Application to the Present Study

For the specific bellows geometry in this study, we define the geometric parameters as follows:

- Mean diameter $D_m = 0.49 \text{ m}$ (19.29 in)
- Convolution depth $w = 0.0655 \text{ m}$ (2.58 in)
- Thickness $t_p = 0.0111 \text{ m}$ (0.437 in)
- Shape factors: $C_p = 0.57$, $C_f = 1.59$, $C_d = 2.07$

By plugging these values into the stress formulas, we get the following simplified expressions:

$$S_3 = \frac{w}{2nt_p} P \approx 3p \text{ (in MPa)}$$

$$S_4 = \frac{1}{2n} \left(\frac{w}{t_p} \right)^2 C_p P \approx 17.73 P \cdot C_p \text{ (in MPa)}$$

$$S_5 = \frac{1}{2} \frac{E_b t_p^2}{w^3 C_f} \Delta q \approx \frac{\Delta q}{C_f} 40.7 \text{ (in MPa, with } \Delta q \text{ in mm)}$$

$$S_6 = \frac{5}{3} \frac{E_b t_p}{w^2 C_d} \Delta$$

$$\approx \frac{\Delta q}{C_d} 807.65 \text{ (in MPa, with } \Delta q \text{ in mm)}$$

Thus, the total stress range simplifies to:

$$\begin{aligned} S_t &= 0.7(3P + 10.1P) + (25.6 \Delta q + 390.2 \Delta q) \\ &= 12.2 P + 415.77 \Delta q \end{aligned}$$

This linear expression is used in the present validation to compute the ASME reference fatigue lives. A fatigue strength reduction factor $K_g = 2.0$ is adopted, accounting for the weld geometry and surface conditions typical of industrial bellows fabrication.

2.3 Fatigue Damage Model

The damage behavior of the expansion bellows is closely linked to the emergence of localized plastic deformations, induced by cyclic loadings of internal pressure and axial displacement. Due to its convoluted geometry, the bellows exhibits zones of stress and strain concentration, particularly at the roots and crowns of the folds. These regions are characterized by high levels of equivalent plastic strain, constituting preferred sites for the initiation of ductile damage [29].

Within the framework of continuum damage mechanics, the evolution of material degradation in the bellows is described using the Lemaitre model [30], [31]. The damage state is represented by a scalar variable D , which reflects the progressive loss of stiffness and load-bearing capacity of the material within a representative elementary volume. The influence of damage on the mechanical behavior is accounted for by the concept of effective stress, defined by:

$$\tilde{\sigma} = \frac{\sigma}{1 - D}$$

This allows for the representation of the progressive reduction in the load-bearing cross-section due to the growth of micro-voids.

The ductile damage of the bellows under cyclic loading is driven by the accumulation of equivalent plastic strain, resulting from repeated pressurization and depressurization cycles [32]. The adopted damage evolution law is expressed as:

$$\dot{D} = \frac{S}{1 - D} \left(\frac{\sigma_{eq}}{\sigma_0} \right)^m \dot{\varepsilon}_{eq}^p$$

In this equation, S and m represent material parameters. σ_{eq} stands for the von Mises equivalent stress, σ_0 is a reference stress, and $\dot{\varepsilon}_{eq}^p$ is the equivalent plastic strain rate. This formulation shows how critical local plasticity is to the evolution

of damage. Essentially, the areas of the bellows that experience the highest equivalent plastic strains will accumulate damage much faster than other regions.

Under cyclic loading, damage evolution occurs only when the strain state exceeds a threshold corresponding to the damage loading surface, while no evolution is observed during unloading phases. Failure of the bellows is assumed to occur when the damage variable reaches a critical value D_c , typically between 0.2 and 0.5 for ductile materials, marking the initiation of a macroscopic crack at the most highly stressed folds. The fatigue life of the expansion bellows is thus determined by integrating the damage evolution law over the entire loading history until this critical state is reached. This approach enables a physically-grounded life prediction, accounting for geometry effects, local plasticity, and the complexity of the cyclic loadings applied to the bellows.

2.3.1 Numerical Implementation in ANSYS

The mechanical behavior of expansion bellows subjected to pressure cycles and differential expansion is heavily dependent on material plasticity and the progression of damage. To accurately predict fatigue life, we employ an incremental algorithm based on Lemaitre's damaged elastic-plastic model, specifically adapted for multiaxial fatigue conditions.

In the ANSYS simulations, the equivalent plastic strain (EPEQ) is calculated incrementally from the plastic strain tensor. This value directly drives the evolution of the Lemaitre damage variable through an incremental formulation, enabling a fully coupled plasticity–damage analysis for fatigue life prediction.

a. Variables and Initialization

The simulation begins by initializing the state variables as follows:

$$\varepsilon_{eq}^p = 0, \quad D = 0, \quad \sigma = \mathbf{0}$$

where ε_{eq}^p is the equivalent plastic strain, D is the damage variable, and σ is the stress tensor. The material parameters include the shear modulus G , Young's modulus E , yield strength σ_y , isotropic hardening modulus H , and a fatigue-oriented damage progression coefficient.

b. Incremental Algorithm

For each load increment $n+1$, the algorithm executes the following steps:

Step 1: Retrieve Variables

Read the current strain tensor, previous plastic strain, damage value and cycle history:

$$\varepsilon^{(n+1)}, \quad \varepsilon_{eq}^{p(n)}, \quad D^n, \quad N_{Cycle}^n$$

where N_{Cycles} tracks the history for fatigue calculations.

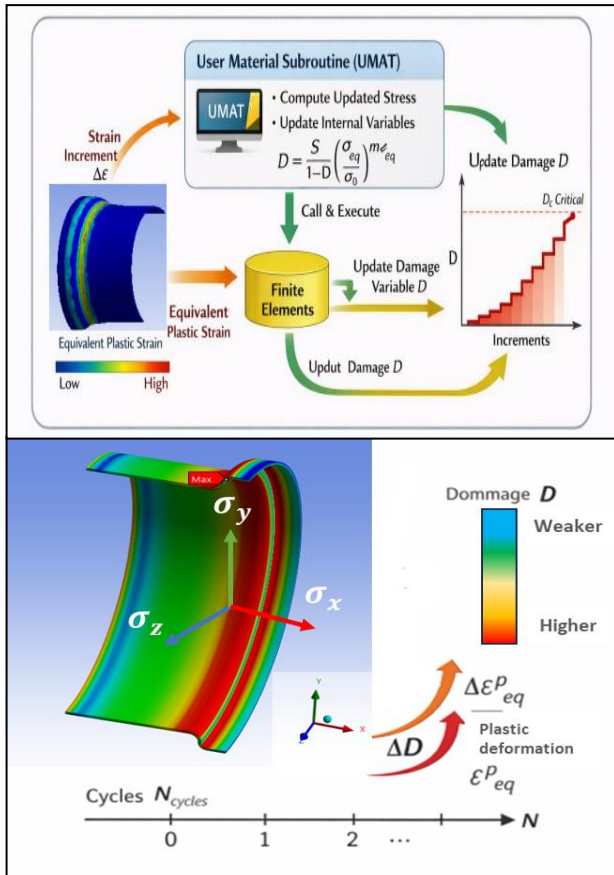


Fig. 7: Upper: Schematic representation of UMAT implementation and damage evolution. Lower: Strains and damage in the bellows
Source: created by the authors

Step 2: Trial stress calculation

Compute the trial stress assuming no new plastic strain has occurred:

$$\sigma_{trial} = (1 - D^n)C: (\varepsilon^{(n+1)} - \varepsilon_{eq}^{p(n)})$$

The deviatoric stress tensor:

$$s_{trial} = \sigma_{trial} - (1/3) tr(\sigma_{trial}) \mathbf{I}$$

This allows us to calculate the equivalent Von mises trial stress:

$$\sigma_{eq,trial} = \sqrt{\left(\frac{3}{2}\right) s_{trial}: s_{trial}}$$

Step 3: Check plastic state

Determine if the material has yielded and calculate the plastic increment (radial return method). The increment in equivalent plastic strain, $\Delta \varepsilon_{eq}^p$, is defined as:

$$\Delta \varepsilon_{eq}^p = \begin{cases} 0, & \text{if } \sigma_{eq,trial} \leq \sigma_y \\ \frac{\sigma_{eq,trial} - \sigma_y}{H + \frac{2}{3}G}, & \text{if } \sigma_{eq,trial} \geq \sigma_y \end{cases}$$

The equivalent plastic strain is then updated:

$$\varepsilon_{eq}^{p(n+1)} = \varepsilon_{eq}^{p(n)} + \Delta \varepsilon_{eq}^p$$

Step 4: Update fatigue-oriented damage

$$D^{(n+1)} = D^n + f_{fatigue}(\Delta \varepsilon_{eq}^p, N_{Cycles}^n),$$

$$0 \leq D^{(n+1)} \leq 1$$

Step 5: Final stress at increment n+1

$$\sigma^{(n+1)} = (1 - D^{(n+1)})C: (\varepsilon^{(n+1)} - \varepsilon_{eq}^{p(n+1)})$$

Step 6: Update state variables

$$\varepsilon_{eq}^{p(n+1)}, D^{(n+1)}, N_{Cycles}^{(n+1)}$$

Step 7: Proceed to the next increment

The algorithm repeats this loop until the analysis concludes or until the damage variable reaches a critical threshold (failure), see Figure 7.

This procedure explicitly accounts for multiaxial stress states (circumferential, radial, and axial) through the stress tensor σ . By tracking the evolution of fatigue-oriented damage, the model can accurately predict the bellows' actual service life.

The code can be implemented in ANSYS via a UMAT subroutine, adapting the fatigue function to the specific material used.

3 Results and Discussion

We systematically compared the axisymmetric 2D model against the full 3D model, focusing on stress distribution, the location of critical zones, and predicted fatigue life. Table 2 summarizes the key quantitative differences between the two approaches.

Because the 2D model assumes uniformity along the circumferential direction, it naturally underestimates peak stress levels. In contrast, the 3D model reveals stress amplification caused by local geometric imperfections and out-of-plane deformation modes (Figure 8).

The 3D von Mises stress maps clearly show pronounced stress concentrations in the knuckle regions of the convolutions, with noticeable variation around the circumference. These stress hotspots are not continuous along the ring, which suggests that crack initiation is driven by localized 3D effects rather than global axisymmetric behavior.

Table 2. Comparison between 2D and 3D numerical predictions

Criterion	2D Axisymmetric Model	Full 3D Model
Maximum von Mises stress	Lower, uniformly distributed	Higher, circumferentially non-uniform
Location of critical zone	Single knuckle region	Multiple localized zones along the circumference
Stress gradient	Radial-axial only	Radial-axial-circumferential
Damage initiation point	Fixed angular position	Variable angular position
Predicted fatigue life	Higher (non-conservative)	Lower (more realistic)

Source: created by the authors

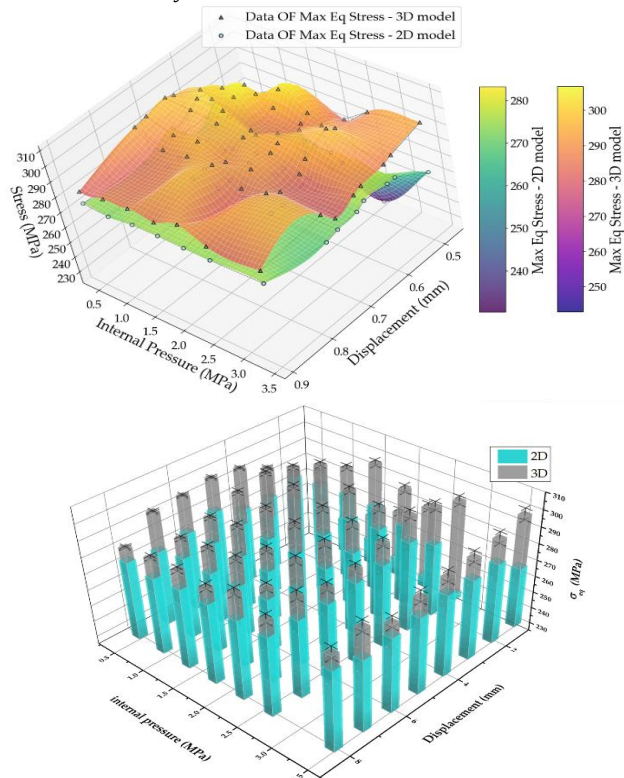


Fig. 8: Parametric and statistical comparative analysis of the influence of pressure and axial displacement on the bellows resistance: 2D vs 3D.
Source: created by the authors

The total deformation plots highlight ovalization of the bellows cross-section under combined internal pressure and axial displacement. This deformation mode, absent in 2D simulations, significantly alters the local stress state and accelerates damage evolution.

Damage distribution maps obtained from the continuum damage mechanics post-processing illustrate a strongly localized damage accumulation.

The damage variable increases rapidly in regions where high triaxiality and plastic strain coexist, confirming the relevance of a local coupling damage approach in three-dimensional analyses.

3.1 Fatigue Life Prediction

We define fatigue life as the number of loading cycles required to reach a critical damage threshold—the point where crack initiation occurs. The 3D damage-based approach predicts a shorter fatigue life compared to the 2D model (Figure 9, Figure 10, Figure 11 and Figure 12). This is a crucial finding: it highlights that using simplified axisymmetric analyses for design purposes can be non-conservative, as they may predict a longer service life than is actually safe.

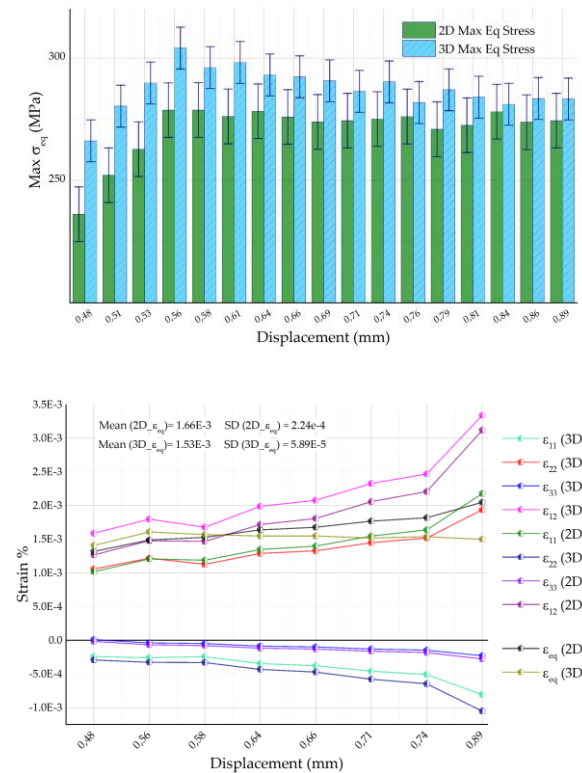


Fig. 9: Comparison of (a) maximum equivalent (von Mises) stress from 2D and 3D models at $P = 1.724$ MPa, and (b) the corresponding strain history for both models
Source: created by the authors

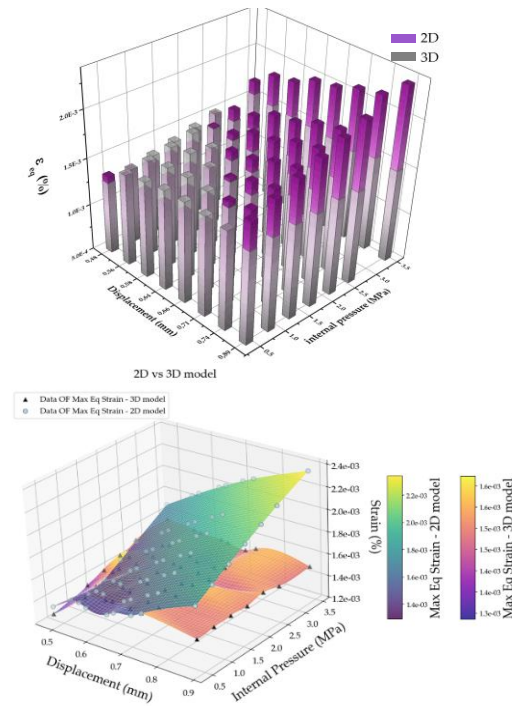


Fig. 10: Parametric and statistical analysis: 2D models show simplified linear trends, while 3D models capture realistic, interdependent strain-pressure-deflection relationships
Source: created by the authors

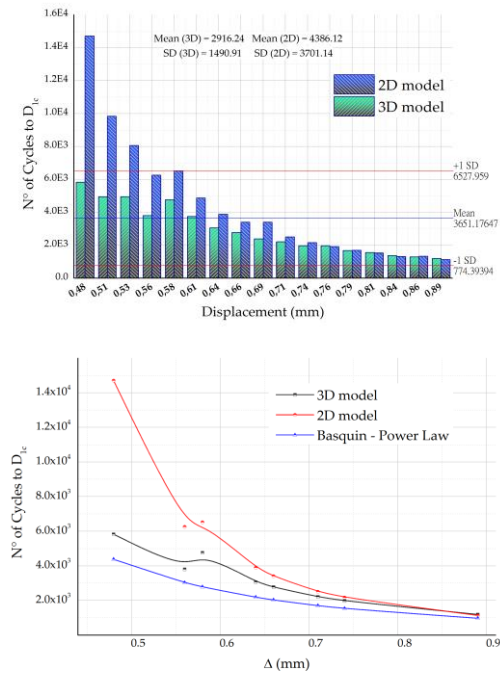


Fig. 12: Cyclic life prediction comparison. Upper: Bar chart comparing 3D and 2D model approaches. Lower: Damage model predictions versus Basquin power law
Source: created by the authors

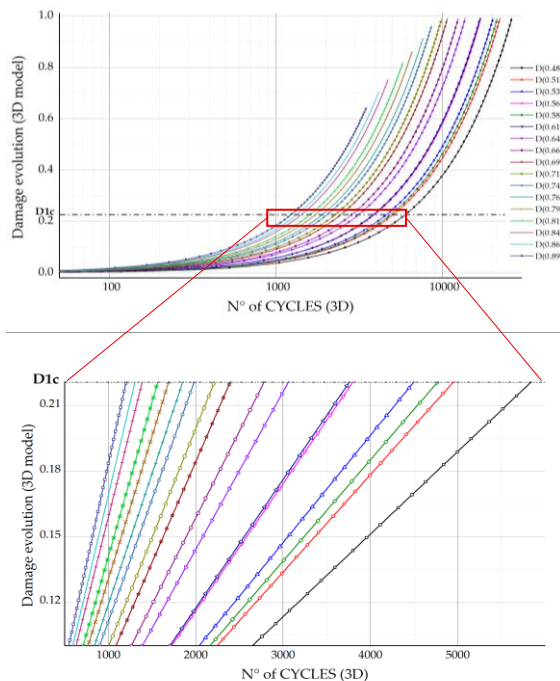


Fig. 11: at $P=1.724$ MPa the damage evolution function of cyclic life at different applied displacements, the slope tends to decrease, referring to a decreasing residual life
Source: created by the authors

3.2 Validation against ASME Code Provisions

To assess the reliability and conservatism of the proposed numerical models, a validation is performed by comparing the predicted fatigue lives with the allowable number of cycles calculated according to the ASME code formulas presented in Section 2.2. This comparison serves to evaluate both the 2D axisymmetric and 3D damage-based approaches against an established industrial standard.

Before examining fatigue lives, it is instructive to compare the stress ranges predicted by the ASME code with the maximum equivalent (von Mises) stresses obtained from the finite element models. Table 3 presents these values for a fixed internal pressure $P=1.724$ MPa and various axial displacements Δq . The numerical stresses corresponding to the first loading cycle (undamaged state) are extracted from the critical regions (knuckle zones).

Table 3. Comparison of stress ranges (MPa)
for $P=1.724$ MPa

Δq (mm)	ASME S_t	2D model		3D model	
		σ_{eq} (MPa)	Err	σ_{eq} (MPa)	Err
0.48	220.6	236.3	7%	266.2	21%
0.56	253.9	278.8	10%	304.1	20%
0.58	262.2	278.8	6%	296.2	13%
0.64	287.1	278.3	-3%	293.2	2%
0.66	295.4	276.0	-7%	292.4	-1%
0.71	316.2	274.5	-13%	286.4	-9%
0.74	328.7	275.2	-16%	290.3	-12%
0.89	391.1	274.5	-30%	283.3	-28%

Source: created by the authors

The ASME stress range increases linearly with displacement, whereas the numerical stresses (both 2D and 3D) tend to saturate beyond $\Delta q \approx 0.6$ mm. This saturation is due to plasticity and stress redistribution, effects that are not captured by the elastic ASME formulation. The 3D model generally yields higher stresses than the 2D model at low displacements, reflecting the additional stress concentrations arising from circumferential non-uniformity and ovalization.

At higher displacements, the 3D stresses remain below the ASME values, indicating that the code's elastic estimate becomes increasingly conservative in terms of peak stress magnitude.

3.2.1 Fatigue Life Comparison

The fatigue lives predicted by the 2D and 3D damage models (using the Lemaitre criterion with a critical damage $D_{1c} = 0.22$ for damage initiation and $D_c = 0.9$ for final fracture) are now compared with the ASME allowable cycles. Table A1 – Appendix, lists the number of cycles to failure for the same load cases ($P=1.724$ MPa). The ASME values are given for both branches (<448 MPa and >448 MPa), Figure 13, and the appropriate branch is selected based on the computed $K_g \frac{E_o}{E_b} S_t$ with $K_g = 2.0$. The last columns indicate the relative error of the numerical models with respect to the correct ASME reference.

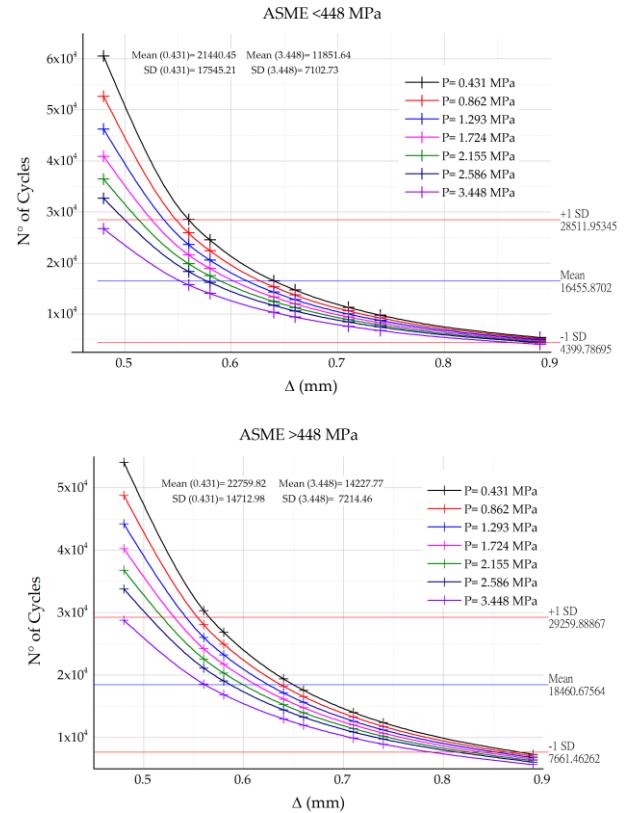


Fig. 13: The allowable number of cycles of the bellows, N_{alw} , given by ASME BPVC 2021 Section VIII Div. 1.

Source: created by the authors

The results reveal a clear and consistent trend:

- The **2D axisymmetric model** systematically overestimates the fatigue life compared to the ASME code for most displacement levels (positive errors up to 63%). This confirms the non-conservative nature of simplified 2D analyses, as they fail to capture three-dimensional stress concentrations and out-of-plane deformation modes that accelerate damage.
- The 3D damage model yields fatigue lives that are generally lower than the ASME allowable cycles, with errors ranging from -35% to +4.6%. At low displacements ($\Delta q \leq 0.58$ mm), the 3D predictions are more conservative (shorter life), while at intermediate displacements (0.64-0.74 mm) they approach the ASME values closely. For the highest displacement (0.89 mm), the 3D model again gives a more conservative estimate. This behavior indicates that the 3D approach provides a safer design margin, aligning with the objective of integrity assessment.

In terms of accuracy, the mean absolute error for the 3D model is about 16%, compared to 22% for the 2D model. Furthermore, the scatter (standard deviation) is smaller for the 3D predictions. This demonstrates that the 3D approach offers improved consistency and reliability.

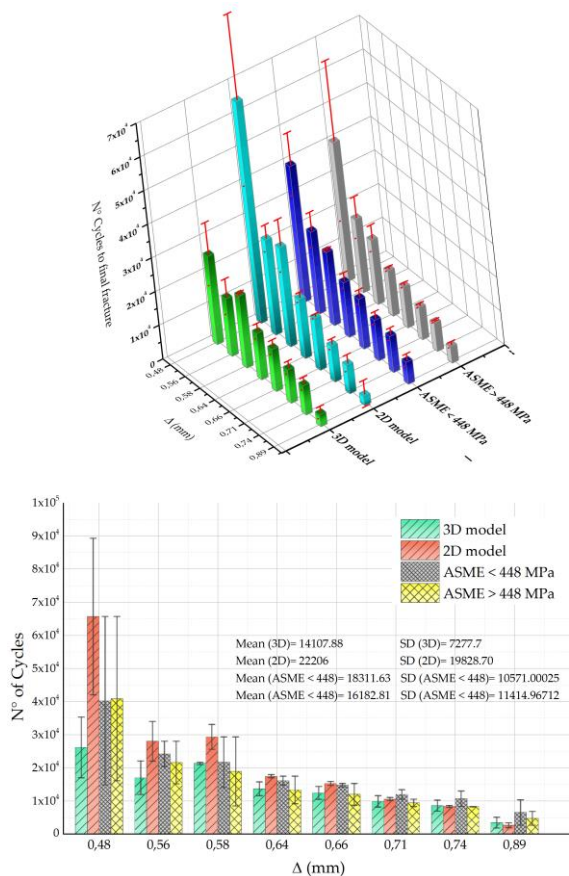


Fig. 14. Model validation: comparison of fatigue lives from 2D and 3D CDM-based simulations against ASME BPVC 2021 Section VIII Div. 1 - 26-6.6.3.2 Fatigue Equation for $P = 1.724$ MPa.

Source: created by the authors

Figure 14 presents a graphical comparison of the fatigue lives obtained from the three approaches. The 3D damage model predictions consistently lie below or close to the ASME reference curve, whereas the 2D model results frequently exceed it. This visual representation reinforces the quantitative findings and highlights the importance of three-dimensional effects in fatigue life estimation.

3.3 Discussion of Accuracy and Conservatism

The stress distribution from the three-dimensional model reveals significant circumferential stress variations along the bellows convolutions. While the

axisymmetric model smooths over these details, the 3D analysis highlights specific stress concentrations associated with geometric nonlinearities and out-of-plane deformation modes. These stress concentrations significantly influence fatigue damage accumulation.

A quantitative comparison shows that the fatigue life predicted by the two-dimensional model may be overestimated by up to 63% compared to the three-dimensional simulation. This discrepancy highlights the limitations of simplified modelling approaches for fatigue assessment of expansion bellows operating under cyclic loading conditions.

The discrepancies between numerical predictions and ASME code values arise from several fundamental sources:

- **Stress computation methodology:** ASME employs linear-elastic formulae, while the finite element models incorporate elastoplastic behavior and progressive damage. Plasticity reduces peak stresses but leads to accumulated damage per cycle, which is explicitly modeled in the damage mechanics' approach.
- **Fatigue curve basis:** The ASME equation is an empirical fit to experimental data with built-in safety factors (2.6 on cycles, 1.25 on stresses). The damage model, conversely, is based on physical mechanisms (plastic strain-driven damage) with material parameters that may not exactly match the ASME database. The observed conservatism of the 3D model relative to ASME is therefore desirable, as it provides an additional safety margin beyond the code requirements.
- **Geometric effects captured:** The 3D model captures local ovalization, circumferential stress variations, and out-of-plane deformations that accelerate damage initiation. These effects are absent in the 2D model and only indirectly accounted for in the ASME code through the fatigue strength reduction factor K_g . The value $K_g = 2.0$ used in this study appears appropriate, as the 3D results generally fall within the scatter band expected for this factor.
- **Failure criterion definition:** The damage model defines failure when $D_{1c} = 0.22$, corresponding to macrocrack initiation, whereas the ASME code defines allowable cycles with a safety margin relative to actual failure. The chosen critical damage value may influence the comparison; a lower D_c would yield even more conservative lives, while a higher value would approach the ASME reference more closely but potentially reduce the safety margin.

- **Statistical distribution:** The 3D model is much tighter around the mean value compared to the 2D model. The standard deviation is 7.28×10^3 cycles for the 3D model versus 1.98×10^4 cycles for the 2D model. This reduced scatter indicates that the 3D approach provides more consistent predictions, which is a distinct advantage for design purposes.

Even with these methodological differences, the overall agreement between the 3D damage model and the ASME code is satisfactory. This is especially true given that the code itself incorporates substantial safety factors. Interestingly, the 3D model often predicts lower life spans than the ASME allowable cycles. This implies it provides an additional safety margin—a highly desirable feature for critical components in petrochemical plants where the consequences of failure are severe.

4 Conclusion

This study presented an advanced three-dimensional finite element methodology for predicting the fatigue life of expansion bellows used in fixed tube-sheet heat exchangers.

The numerical results demonstrate that simplified axisymmetric models may significantly overestimate fatigue life due to their inability to capture circumferential stress variations and complex deformation modes.

The proposed three-dimensional modelling approach provides more accurate and conservative fatigue life predictions and therefore represents a valuable tool for the structural integrity assessment of heat exchanger components. The following conclusions are drawn:

- Three-dimensional effects are critical.
- 2D models are non-conservative, with errors reaching +63%. Reliance on 2D analyses for design may lead to unsafe life estimates.
- The 3D damage framework provides conservative predictions, (errors from –35% to +4.6%), offering an additional safety margin beyond code requirements. The reduced scatter compared to 2D results demonstrates improved reliability.
- Validation against ASME code confirms reliability. Good agreement with ASME BPVC Appendix 26 fatigue equations validates the methodology's physical relevance. The 3D model directly simulates stress concentration effects that the code accounts for through empirical safety factors.

- **Industrial Relevance.** This framework is well-suited for the final design validation of safety-critical heat exchangers. It enables the creation of optimized geometries and supports informed, data-driven inspection planning.

Future Work: Moving forward, we will focus on investigating thermo-mechanical coupling effects and validating the proposed methodology through experimental testing.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

The authors wrote, reviewed and edited the content as needed and they have not utilised artificial intelligence (AI) tools. The authors take full responsibility for the content of the publication.

References

- [1] B. Shi, Z. Luo, L. Li, L. Song, Z. Wang, and Q. Li, "Thin-Walled Structures An analytical model for the mechanical properties of semi-circular expansion bellows," *Thin-Walled Struct.*, vol. 219, no. PA, p. 114246, 2026, doi: 10.1016/j.tws.2025.114246.
- [2] ASME, *ASME Code for Pressure Piping, B31.3-2014*, vol. 3. Two Park Avenue - New York, NY 10016 USA: The American Society of Mechanical Engineers, 2015, [Online]. <https://www.asme.org/codes-standards/find-codes-standards> (Accessed Date: May 10, 2026).
- [3] ASME, *2021 ASME Boiler and Pressure Vessel Code, Section VIII, Division 1: Rules for Construction of Pressure Vessels*. New York, USA: The American Society of Mechanical Engineers, 2021.
- [4] S. C. S. P. K. Krovvidi, "Theoretical, numerical and experimental estimation of fatigue life of bellows for nuclear systems," *Int. J. Des. Eng.*, vol. 13, no. 2, 2024, doi: 10.1504/ijde.2024.146343.
- [5] S. C. S. P. K. Krovvidi, S. Goyal, and A. K. Bhaduri, "Experimental and Numerical Investigation of High-Temperature Low-Cycle Fatigue and Creep-Fatigue Life of Bellows," *J. Mater. Eng. Perform.*, vol. 30, no. 4, pp. 2742–2750, 2021, doi: 10.1007/s11665-021-05611-9.
- [6] A. I. Aistov, Y. V. Skvortsov, S. A. Chernyakin, and S. N. Perov, "Fatigue life prediction for expansion bellows," in *AIP Conference Proceedings*, 2017. doi: 10.1063/1.4972594.
- [7] Y. Ma, P. Li, T. Su, Q. Yuan, J. Wang, and

- L. Xia, "Fatigue life and experimental study of high-temperature bellows based on the improved Manson-Coffin equation," *Int. J. Press. Vessel. Pip.*, vol. 209, no. May, p. 105216, 2024, doi: 10.1016/j.ijpvp.2024.105216.
- [8] Fu, P.; Yu, G.; Yang, S.; Zhou, M.; Qi, S.; Meng, Q.; Jia, S.; Zhao, J., "Influence of Structural Parameters on the Mechanical Performance of Multi-Layer U-Shaped Metal Bellows," *Metals (Basel)*, vol. 14, no. 12, 2024, doi: 10.3390/met14121431.
- [9] E. Pavithra and V. S. S. Kumar, "Experimental investigation and numerical analysis on fatigue life of bellows," *Mater. Today Proc.*, vol. 5, no. 9, pp. 18848–18856, 2018, doi: 10.1016/j.matpr.2018.06.232.
- [10] A. Magesh, U. Ganesan, and R. Sundararajan, "Key parameters that affect the fatigue life of metal bellows-type expansion joint: Another look," *Mater. Today Commun.*, vol. 38, no. February, p. 108400, 2024, doi: 10.1016/j.mtcomm.2024.108400.
- [11] J. Zhang, Y. Ma, T. Su, Q. Yuan, Z. Song, and H. Pang, "Improved SWT model for predicting multiaxial fatigue life of bellows under combined movement load," *Int. J. Press. Vessel. Pip.*, vol. 208, no. January, p. 105146, 2024, doi: 10.1016/j.ijpvp.2024.105146.
- [12] A. Chouiter, D. Benzerga, A. Haddi, and T. Tamine, "Prediction of cycle life of expansion bellows for fixed tube sheet heat exchanger," *Frat. ed Integrita Strutt.*, vol. 13, no. 47, pp. 30–38, 2019, doi: 10.3221/IGF-ESIS.47.03.
- [13] A. S. Palve, S. Shinde, and K. S. Bhole, "On Comparison of Expansion Bellow Design Approaches," in *Lecture Notes on Multidisciplinary Industrial Engineering*, vol. Part F256, 2021. doi: 10.1007/978-981-15-6017-0_9.
- [14] S. K. Makke, "Estimation of Fatigue Life for U-Shaped Metal Expansion Bellows," *IOSR J. Mech. Civ. Eng.*, vol. 17, no. 03, 2017, doi: 10.9790/1684-17010031621.
- [15] S. D. Wankhede and S. H. Gawande, "Design and analysis aspect of metal expansion bellows: A review," *Forces Mech.*, vol. 13, no. October, p. 100244, 2023, doi: 10.1016/j.finmec.2023.100244.
- [16] N. D. Pagar and S. H. Gawande, "Multi-response design optimisation of convolution stresses of metal bellows using integrated PCA- GRA approach," *Aust. J. Mech. Eng.*, vol. 00, no. 00, pp. 1–21, 2020, doi: 10.1080/14484846.2020.1725347.
- [17] D. N. Pagar and S. H. Gawande, "Experimental Investigations On Meridional And Circumferential Stresses Of Bellows Due To Internal Pressure," in *Proceedings of the ASME 2019 Gas Turbine India*, ASME, Ed., Tamil Nadu, India: GTIndia2019, 2019, pp. 1–12. doi: 10.1115/GTINDIA2019-2771
- [18] S. H. Gawande, N. D. Pagar, V. B. Wagh, and A. A. Keste, "Numerical Investigations on Characteristics of Stresses in U-Shaped Metal Expansion Bellows," vol. 2015, 2015, doi: 10.1155/2015/957925.
- [19] F. Sotehi, D. Benzerga, A. Haddi, A. Chouiter, and B. Abderrahmane, "Numerical analysis of cracking at the nozzle junction in a distillation column," *Res. Eng. Struct. Mater.*, pp. 1–19, 2025, doi: <http://dx.doi.org/10.17515/resm2025-663me0207rs>.
- [20] K. Isik, Y. Yoshida, L. Chen, T. Clausmeyer, and A. Erman Tekkaya, "Modelling of the blanking process of high-carbon steel using Lemaitre damage model," *Comptes Rendus - Mec.*, vol. 346, no. 8, pp. 770–778, 2018, doi: 10.1016/j.crme.2018.05.003.
- [21] C. B. Oland and M. D. Lower, "Pressure-Testing And Overpressure Protection Requirements For Section Viii Division 1 And Division 2 Pressure Vessels," in *American Society of Mechanical Engineers, Pressure Vessels and Piping Division (Publication) PVP*, 2024. doi: 10.1115/PVP2024-123439.
- [22] S. H. Gawande and N. D. Pagar, "A Combined Numerical and Experimental Investigation on the Effect of Dynamics Characteristics of Metal Expansion Bellows", doi: 10.1007/s42417-018-0053-0.
- [23] ASME BPVC Committee on Materials, "ASME Boiler & Pressure Vessel Code, Section II Materials, Part D Properties (Customary)," *ASME Boil. Press. Vessel Code*, vol. II, 2021.
- [24] J. Lemaitre and J. A. Lemaitre, "Continuous Damage Mechanics Model for Ductile Fracture," *J. Eng. Mater. Technol.*, vol. 107, no. 1, 1985, doi: 10.1115/1.3225775i.
- [25] J. Lemaitre and I. Doghri, "Damage 90: a post processor for crack initiation," *Comput. Methods Appl. Mech. Eng.*, vol. 115, no. 3–4, pp. 197–232, 1994, doi: 10.1016/0045-7825(94)90060-4.

- [26] I. Expansion Joint Manufacturers Association, *Standards of the Expansion Joint Manufacturers Association, Inc.* Expansion Joint Manufacturers Association, Inc., 2015, [Online]. <https://www.ejma.org> (Accessed Date: May 10, 2026).
- [27] The American Society of Mechanical Engineers (ASME), *2019 ASME Boiler & Pressure Vessel Code*, 2019th ed. New York, NY: The American Society of Mechanical Engineers, 2019, [Online]. <https://www.asme.org/codes-standards> (Accessed Date: May 10, 2026)
- [28] N. D. Pagar and S. H. Gawande, "Parametric design analysis of meridional deflection stresses in metal expansion bellows using gray relational grade," *J. Brazilian Soc. Mech. Sci. Eng.*, vol. 42, no. 5, pp. 1–21, 2020, doi: 10.1007/s40430-020-02327-0.
- [29] J. D. Dobis and D. N. French, *Damage mechanisms affecting fixed equipment in Refining industry*, Third edit., no. 490. Washington, DC: API Publishing Services, 2020.
- [30] J. Lemaitre and J. Dufailly, "Damage Measurements," 1987. doi: 10.1016/0013-7944(87)90059-2.
- [31] J. Lemaitre and R. Desmorat, *Engineering Damage Mechanics*. Cachan, France: Springer Berlin Heidelberg New York Library, 2005. ISBN: 978-3-540-21503-5.
- [32] C. Garion and B. Skoczen, "Influence of micro-damage on reliability of cryogenic bellows in the lhc interconnections," *AIP Conf. Proc.*, vol. 986, no. February 2005, pp. 100–107, 2008, doi: 10.1063/1.2900331.
- [33] J. Lemaitre and R. Desmorat, "Isotropic and Anisotropic Damage Law of Evolution," *Handb. Mater. Behav. Model.*, no. December 2001, pp. 513–524, 2001, doi: 10.1016/b978-012443341-0/50057-0.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

All authors contributed to writing the manuscript, discussed the findings, and approved the final version.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

Creative Commons Attribution License 4.0 (Attribution 4.0 International, CC BY 4.0)

This article is published under the terms of the Creative Commons Attribution License 4.0

https://creativecommons.org/licenses/by/4.0/deed.en_US

APPENDIX

Table A1. Comparison of fatigue life (cycles) for P=1.724 MPa

Δq (mm)	3D model	2D model	ASME Cycles ($K_g \frac{E_o}{E_b} S_t < 448$)	ASME Cycles ($K_g \frac{E_o}{E_b} S_t > 448$)	Correct ASME	Error 3D (%)	Error 2D (%)
0.48	26,200	65,700	40,300	40,900	40,300	-35.0	+63.0
0.56	17,000	28,100	24,200	21,600	24,200	-29.8	+16.1
0.58	21,400	29,400	21,700	19,000	21,700	-1.4	+35.5
0.64	13,700	17,500	16,200	13,400	16,200	-15.4	+8.0
0.66	12,500	15,300	14,800	12,000	14,800	-15.5	+3.4
0.71	9,910	10,600	12,000	9,470	9,470	+4.6	+11.9
0.74	8,620	8,390	10,700	8,300	8,300	+3.9	+1.1
0.89	3,510	2,720	6,540	4,790	4,790	-26.7	-43.2

Note: The correct ASME branch is determined by comparing $K_g \frac{E_o}{E_b} S_t$ with 448 MPa. For $\Delta q=0.71$ mm and 0.74 mm, $K_g \frac{E_o}{E_b} S_t$ slightly exceeds 448 MPa, so the >448 branch applies; for $\Delta q=0.89$ mm, the >448 branch is clearly applicable

Source: created by the authors