On the Diophantine Equation $a^x+(2a+12)^y=z^2$ where $a\equiv 8(\mathrm{mod}\,27)$

TASANAI RANGPUNG
Department of Mathematics, Faculty of Science,
Ubon Ratchathani Rajabhat University,
UbonRatchathani, 34000,
THAILAND

Abstract: - In this paper, we show that for all solutions of the Diophantine equation where x,y,z are non-negative integers are present a be a positive integer with $a\equiv 8(\mathrm{mod}\,27)$. It has exactly two non-negative integer infinite solutions $(x,y,z)=\left(1,0,\sqrt{a+1}\right)$ where $a=(9t\pm 3)^2-1$ and $(x,y,z)=\left(1,1,\sqrt{3a+12}\right)$ where $a=243t^2\pm 108t+8$. Moreover, we prove that $(x,y,z)=(3,2,36)$ is the unique non-negative integer solution and t is an integer.

Key-Words: - The Diophantine equation, non-negative integer, Congruence, infinite solutions, Mihăilescu's Theorem, integer solutions.

Received: May 15, 2025. Revised: July 7, 2025. Accepted: September 19, 2025. Published: December 31, 2025.

1 Introduction

In 2020, showed that the solutions are as follows: $\{(x,y,z,p)\}=\left\{\left(k,1,2^{nk}-1,2^{nk+1}-1\right)\cup\{(0,0,0,p)\}\right\}$ of the Diophantine equation $\left(4^n\right)^x-p^y=z^2$ where p is an odd prime, $n\in Z^+$ and x,y,z are non-negative integers, [1]. In 2006, in [2] showed that the equation $f(x)=Ax^3+3Bx^2+3Cx+D=0$

when $\Delta=-A^2D^2+6ABCD-4AC^3-4B^3D+3B^2C^2$ and $\Delta<0$ then one real root and one complex conjugate pair, [2]. In 2020, proved that $(x,y,z)=(1,0,3)$ is a unique non-negative integer solution of the Diophantine equation $8^x+n^y=z^2$ where x,y and z are non-negative integers, [3]. In 2021, proved that $(n,x,y,z)=(2,3,0,3)$ is the only non-negative integer solution of the Diophantine equation $n^x+13^y=z^2$ where $n\equiv 2(\mathrm{mod}\,30)$ and $n+1$ is not a square number, [4]. In 2021, showed that $(p,x,y,z)=(p,1,0,2)$ is unique solution of the Diophantine equation $3^x+p^y=z^2$ when $y=0$ and $p\equiv 2(\mathrm{mod}\,3)$ for each prime number p. Moreover, it has only solution $(p,x,y,z)=(2,0,3,3)$ if y is not divisible by 4, [5]. In 2022, proved the exponential Diophantine equation $5^x-2\cdot 3^y=z^2$, where x,y,z are non-negative integers that have no solution and proved that if x is an integer, then $x^2\equiv 0(\mathrm{mod}\,3)$ or $x^2\equiv 1(\mathrm{mod}\,3)$, [6]. In 2023, Tadee showed that $(n,x,y,z)=(2,3,0,3)$ is the unique non-negative integer solution of the Diophantine equation $n^x+10^y=z^2$ where are non-negative integers and $n\equiv 2(\mathrm{mod}\,30)$ with n is a positive integer, [7]. In 2023, showed that $(x,y,z)=(1,0,2)$ is a unique solution of the Diophantine equation $3^x+b^y=z^2$ when $b\equiv 5(\mathrm{mod}\,30)$, it has the solutions $(x,y,z)\in\left\{(1,0,2),\left(0,1,\sqrt{b+1}\right)\right\}$, [8]. In 2025, Rangpung showed that three infinite solutions are $(x,y,z)=\left(1,0,\sqrt{a+1}\right)$ when $a=64t^2\pm 32t+3$, $(x,y,z)=\left(1,1,\sqrt{3a}\right)$ when $a=192t^2\pm 48t+3$ and $(x,y,z)=\left(2,3,\sqrt{8a^3+a^2}\right)$ when $a=512t^2\pm 80t+3$ of the Diophantine equation $a^x+(2a)^y=z^2$ where $a\equiv 3(\mathrm{mod}\,16)$ with x,y,z are non-negative integers, [9].

In this paper, we find all solutions of the Diophantine equation $a^x+(2a+12)^y=z^2$ where and x,y,z are non-negative integers.

2 Preliminaries

Theorem 1. [10] (Mihăilescu's theorem) The Diophantine equation $a^x-b^y=1$ where a,b,x,y are positive integers has only one solution $(a,b,x,y)=(3,2,2,3)$ when $a,b,x,y>1$.

3 Main Results

Lemma 3. The Diophantine equation $a^x+(2a+12)^y=z^2$ where $a\equiv 8(\mathrm{mod}\,27)$ for any a is a positive integer with x is a positive odd number and y is a positive even number, then $(x,y,z)=(3,2,36)$ is a unique solution for all x,y,z are non-negative integers.

Proof. Suppose that x is a positive odd number and y is a positive even number, we get $x=2l_1+1$ and $y=2l_2;\ l_1,l_2\in Z^+$.

Case 1: Given $l_1=1$ and $l_2=1$ then $x=3$ and $y=2$,we have $a^3+(2a+12)^2=z^2$ then $z=\sqrt{a^3+4a^2+48a+144}$ and $a\equiv 8(\mathrm{mod}\,27)$ such that $a=27m+8$. Consider $a^3+4a^2+48a+144=0$ we see that by the Discriminant of a cube equation $\Delta=-A^2D^2+6ABCD-4AC^3-4B^3D+3B^2C^2$. We see that $A=1,B=\frac{4}{3}$,$C=16$, and $D=144$ it implies that $\Delta=-18,688<0$ then a has only one real solution. We get $a=27m+8$ with m is non-negative integers. Consider $m=0$ such that $a=8$ and substitute a into z. It implies that $z=\sqrt{1,296}=36$, therefore $(x,y,z)=(3,2,36)$. Consider $a^3+4a^2+48a+144>0$, we get $a^3\left[1+\frac{4}{a}+\frac{48}{a^2}+\frac{144}{a^3}\right]>0$. Since $\frac{4}{a},\frac{48}{a^2}$ and $\frac{144}{a^3}$ are proper fractions and improper fractions, which it is not integers.

Case 2: Given $l_1=1$ and $l_2>1$. Consider l_2 a positive even number such that $l_2=2q;\ q\in Z^+$. It implies that $a^3+(2a+12)^{4q}=z^2$, we get $(2a+12)^{4q}\left[\frac{a^3}{(2a+12)^{4q}}+1\right]=z^2$. Since $\frac{a^3}{(2a+12)^{4q}}$ is a proper fraction, which it is not an integer. Consider l_2 a positive odd number such that $l_2=2q+1;\ q\in Z^+$. It implies that $a^3+(2a+12)^{4q+2}=z^2$, we get $(2a+12)^{4q+2}\left[\frac{a^3}{(2a+12)^{4q+2}}+1\right]=z^2$. Since $\frac{a^3}{(2a+12)^{4q+2}}$ is a proper fraction, which it is not an integer.

Case 3: Given $l_1>1$ and $l_2=1$. Consider l_1 a positive even number such that $l_1=2q;\ q\in Z^+$. It implies that $a^{4q+1}+(2a+12)^2=z^2$, we get $a^{4q}\left[a+\left(\frac{2}{a^{2q-1}}+\frac{12}{a^{2q}}\right)^2\right]=z^2$. Since $\frac{12}{a^{2q}}$ is a proper fraction, which it is not an integer. Consider l_1 a positive odd number such that $l_1=2q+1;\ q\in\square^+$. It implies that $a^{4q+3}+(2a+12)^2=z^2$, we get $a^{4q+2}\left[a+\left(\frac{2}{a^{2q}}+\frac{12}{a^{2q+1}}\right)^2\right]=z^2$. Since $\frac{2}{a^{2q}}$ and $\frac{12}{a^{2q+1}}$ are proper fraction, which it is not an integer.

Case 4: Given $l_1>1$ and $l_2>1$. Consider $l_1=2q_1$ and $l_2=2q_2;\ q_1,q_2\in Z^+$,
we get $a^{4q_1+1}+(2a+12)^{4q_2}=z^2$ such that
$(2a+12)^{4q_2}\left[\frac{a^{4q_1+1}}{(2a+12)^{4q_2}}+1\right]=z^2$.
Since $\frac{a^{4q_1+1}}{(2a+12)^{4q_2}}$ is a fraction, which it is not an integer.

Consider $l_1 = 2q_1 + 1$ and $l_2 = 2q_2 + 1; q_1, q_2 \in Z^+$, we get $a^{4q_1+3} + (2a+12)^{4q_2+2} = z^2$ such that $(2a+12)^{4q_2+2}\left\lfloor \frac{a^{4q_1+3}}{(2a+12)^{4q_2+2}} + 1 \right\rfloor = z^2$.

Since $\frac{a^{4q_1+3}}{(2a+12)^{4q_2+2}}$ is a fraction, which it is not an integer.

Consider $l_1 = 2q_1$ and $l_2 = 2q_2 + 1; q_1, q_2 \in Z^+$, we get $a^{4q_1+1} + (2a+12)^{4q_2+2} = z^2$ such that $(2a+12)^{4q_2+2}\left\lfloor \frac{a^{4q_1+1}}{(2a+12)^{4q_2+2}} + 1 \right\rfloor = z^2$.

Since $\frac{a^{4q_1+1}}{(2a+12)^{4q_2+2}}$ is a fraction, which it is not an integer.

Consider $l_1 = 2q_1 + 1$ and $l_2 = 2q_2; q_1, q_2 \in Z^+$, we get $a^{4q_1+3} + (2a+12)^{4q_2} = z^2$ such that $(2a+12)^{4q_2}\left\lfloor \frac{a^{4q_1+3}}{(2a+12)^{4q_2}} + 1 \right\rfloor = z^2$. Since $\frac{a^{4q_1+3}}{(2a+12)^{4q_2}}$ is a fraction, which it is not an integer.

Theorem 4. The Diophantine equation $a^x + (2a+12)^y = z^2$ where $a \equiv 8 \pmod{27}$ for all x, y, z are non-negative integers and a is a positive integer has exactly two non-negative integer infinite solutions, as follows:

1. $(x, y, z) = \left(1, 0, \sqrt{a+1}\right)$ where $a = (9t_1 + 3)^2 - 1$ and $a = (9t_2 - 3)^2 - 1$
2. $(x, y, z) = \left(1, 1, \sqrt{3a+12}\right)$ where $a = 243t_1^2 + 108t_1 + 8$ and $a = 243t_2^2 - 108t_2 + 8$

and $(x, y, z) = (3, 2, 36)$ is the unique non-negative integer solution, t_1 and t_2 are integers.

Proof. We consider the Diophantine equation $a^x + (2a+12)^y = z^2$ where $a \equiv 8 \pmod{27}$ and a is a positive integer in nine cases as follows.

Case 1: Given $x = 0$, and $y = 0$ obviously, it has no solution because $z^2 = 2$ it is impossible.

Case 2: Given $x > 1$ and $y = 0$, we obtain the Diophantine $z^2 - a^x = 1$ and $\min\{z, a, 2, x\} > 1$ such that $(z, a, 2, x) = (3, 2, 2, 3)$. Since $z > 8$ and $a \geq 8$, which contradicts by Theorem 1.

Case 3: Given $x = 0$ and $y = 1$. Thus $z^2 = 2a + 13$, a is a positive integer. Since $a \equiv 8 \pmod{27}$ and $3 \mid 27$ such that $a \equiv 8 \pmod{3}$, we have and $z^2 \equiv 2 \pmod{3}$, which is impossible by Lemma 2.

Case 4: Given $x = 0$ and $y > 1$, then the Diophantine $z^2 - (2a+12)^y = 1$ and $\min\{z, 2a+12, 2, y\} > 1$ such that $(z, 2a+12, 2, y) = (3, 2, 2, 3)$. Since $z > 28$ and $2a + 12 \geq 28$, which contradicts by Theorem1.

Case 5: Given $x = 1$ and $y = 0$. Thus $z = \sqrt{a+1}$, a is a positive integer. Since $a \equiv 8 \pmod{27}$ such that $a = 27m + 8$ and substitute a into z. It implies that $z = 3\sqrt{3m+1}$, we consider $k^2 = 3m + 1$ such that $k^2 \equiv 1 \pmod{3}$. Therefore, $3 \mid (k-1)$ or $3 \mid (k+1)$, we consider $3 \mid (k-1)$ then $m = 3t_1^2 + 2t_1$; t_1 is a integer. Substitute m into a, it implies that $a = (9t_1 + 3)^2 - 1$. We consider $3 \mid (k+1)$ then $m = 3t_2^2 - 2t_2$; t_2 is a integer and substitute m into a. It implies that $a = (9t_2 - 3)^2 - 1$.

Case 6: Given $x = 1$ and $y = 1$. Thus $z = \sqrt{3a+12}$, a is a positive integer. Since $a \equiv 8 \pmod{27}$ such that $a = 27m + 8$ and substitute a into z. It implies that $z = 3\sqrt{9m+4}$, we consider $k^2 = 9m + 4$ such that $k^2 \equiv 4 \pmod{9}$. Therefore, $9 \mid (k-2)$ or $9 \mid (k+2)$, we consider $9 \mid (k-2)$ then $m = 9t_1^2 + 4t_1$; t_1 is a integer and substitute m into a. It implies that $a = 243t_1^2 + 108t_1 + 8$. We consider $9 \mid (k+2)$ then $m = 9t_2^2 - 4t_2$; t_2 is a integer and substitute m into a. It implies that $a = 243t_2^2 - 108t_2 + 8$.

Case 7: Given $x = 1$ and $y > 1$. Consider y a positive even number such that $y = 2k$; k is a non-negative integer solution. It implies that

$(2a+12)^{2k}+a=z^2$, we get $(2a+12)^{2k}\left\lfloor 1+\frac{a}{(2a+12)^{2k}}\right\rfloor=z^2$. Since $\frac{a}{(2a+12)^{2k}}$ is a proper fraction, which it is not an integer. Consider y a positive odd number such that $y=2k+1$; k is non-negative integer solutions. It implies that $(2a+12)^{2k+1}+a=z^2$, we get $(2a+12)^{2k+1}\left\lfloor 1+\frac{a}{(2a+12)^{2k+1}}\right\rfloor=z^2$.

Since $\frac{a}{(2a+12)^{2k+1}}$ is a proper fraction, which it is not an integer.

Case 8: Given $x>1$ and $y=1$. Consider x a positive even number such that $x=2k$; k is a non-negative integer solution. It implies that $a^{2k}+2a+12=z^2$, we get $a^{2k}\left[1+\frac{2}{a^{2k-1}}+\frac{12}{a^{2k}}\right]=z^2$. Since $\frac{12}{a^{2k}}$ is a proper fraction, which it is not an integer. Consider x a positive odd number such that $x=2k+1$; k is a non-negative integer solution. It implies that $a^{2k+1}+2a+12=z^2$, we get $a\left[a^{2k}+2+\frac{12}{a}\right]=z^2$. Since $\frac{12}{a}$ is a proper fraction, which it is not an integer.

Case 9: Given $x>1$ and $y>1$ with $a\equiv 8(\bmod 27)$ and $3\mid 27$, we get $a\equiv 8(\bmod 3)$. It implies that $a^{2k_1}\equiv 1(\bmod 3)$ and $a^{2k_1+1}\equiv 2(\bmod 3)$. Furthermore, $(2a+12)^{2k_2}\equiv 1(\bmod 3)$ and $(2a+12)^{2k_2+1}\equiv 1(\bmod 3)$ when $k_1,k_2\in Z^+\cup\{0\}$

Subcase 9.1: Given x and y are positive even numbers. We get $a^{2k_1}+(2a+12)^{2k_2}\equiv 2(\bmod 3)$ and $z^2\equiv 2(\bmod 3)$ which is a contradiction by Lemma1.

Subcase 9.2: Given x is a positive even number and y a positive odd number.

We get $a^{2k_1}+(2a+12)^{2k_2+1}\equiv 2(\bmod 3)$ which is a contradiction by Lemma1.

Subcase 9.3: Given x and y are positive odd numbers. We get $a^{2k_1+1}+(2a+12)^{2k_2+1}\equiv 0(\bmod 3)$ and $a\equiv 3m+8$, so $(3m+8)^{2k_1+1}+(6m+28)^{2k_2+1}\equiv 0(\bmod 3)$. Since 3 indivisible $(3m+8)^{2k_1+1}$ and 3 indivisible $(6m+28)^{2k_2+1}$ which is a contradiction.

Subcase 9.4: Given x is positive odd number and y is positive even number. By Lemma3, $(x,y,z)=(3,2,36)$ there is a unique solution for all x,y,z are non-negative integers.

Note: The Table shows particular values of x and y at the point $(x,y)=(1,1)$ and values a between 1 to 25,000 of the Diophantine equation $a^x+(2a+12)^y=z^2$ where $a\equiv 8(\bmod 27)$.

$a^x+(2a+12)^y=z^2$	Solution of the equation
$8^x+28^y=z^2$	$(x,y,z,a)=(1,1,6,8)$
$143^x+298^y=z^2$	$(x,y,z,a)=(1,1,21,143)$
$359^x+730^y=z^2$	$(x,y,z,a)=(1,1,33,359)$
$764^x+1{,}540^y=z^2$	$(x,y,z,a)=(1,1,48,764)$
$1{,}196^x+2{,}404^y=z^2$	$(x,y,z,a)=(1,1,60,1{,}196)$
$1{,}871^x+3{,}754^y=z^2$	$(x,y,z,a)=(1,1,75,1{,}871)$
$2{,}519^x+5{,}050^y=z^2$	$(x,y,z,a)=(1,1,87,2{,}519)$
$3{,}464^x+6{,}940^y=z^2$	$(x,y,z,a)=(1,1,102,3{,}464)$
$4{,}328^x+8{,}668^y=z^2$	$(x,y,z,a)=(1,1,114,4{,}328)$
$5{,}543^x+11{,}098^y=z^2$	$(x,y,z,a)=(1,1,129,5{,}543)$
$6{,}623^x+13{,}258^y=z^2$	$(x,y,z,a)=(1,1,141,6{,}623)$
$8{,}108^x+16{,}228^y=z^2$	$(x,y,z,a)=(1,1,156,8{,}108)$
$9{,}404^x+18{,}820^y=z^2$	$(x,y,z,a)=(1,1,168,9{,}404)$
$11{,}159^x+22{,}330^y=z^2$	$(x,y,z,a)=(1,1,183,11{,}159)$
$12{,}671^x+25{,}354^y=z^2$	$(x,y,z,a)=(1,1,195,12{,}671)$
$14{,}696^x+29{,}404^y=z^2$	$(x,y,z,a)=(1,1,210,14{,}696)$
$16{,}424^x+32{,}860^y=z^2$	$(x,y,z,a)=(1,1,222,16{,}424)$
$18{,}719^x+37{,}450^y=z^2$	$(x,y,z,a)=(1,1,237,18{,}719)$
$20{,}663^x+41{,}338^y=z^2$	$(x,y,z,a)=(1,1,249,20{,}663)$
$23{,}228^x+46{,}468^y=z^2$	$(x,y,z,a)=(1,1,264,23{,}228)$

Note: The Table shows particular values of x and y at the point $(x,y)=(1,0)$ and value a between 1 to

7,600 of the Diophantine equation $a^x+(2a+12)^y=z^2$ where $a\equiv 8(\bmod 27)$.

$a^x+(2a+12)^y=z^2$	Solution of the equation
$8^x+28^y=z^2$	$(x,y,z,a)=(1,0,3,8)$
$35^x+82^y=z^2$	$(x,y,z,a)=(1,0,6,35)$
$143^x+298^y=z^2$	$(x,y,z,a)=(1,0,12,143)$
$224^x+460^y=z^2$	$(x,y,z,a)=(1,0,15,224)$
$440^x+892^y=z^2$	$(x,y,z,a)=(1,0,21,440)$
$575^x+1{,}162^y=z^2$	$(x,y,z,a)=(1,0,24,575)$
$899^x+1{,}810^y=z^2$	$(x,y,z,a)=(1,0,30,899)$
$1{,}088^x+2{,}188^y=z^2$	$(x,y,z,a)=(1,0,33,1{,}088)$
$1{,}520^x+3{,}052^y=z^2$	$(x,y,z,a)=(1,0,39,1{,}520)$
$1{,}763^x+3{,}538^y=z^2$	$(x,y,z,a)=(1,0,42,1{,}763)$
$2{,}023^x+4{,}618^y=z^2$	$(x,y,z,a)=(1,0,48,2{,}023)$
$2{,}600^x+5{,}212^y=z^2$	$(x,y,z,a)=(1,0,51,2{,}600)$
$3{,}248^x+6{,}508^y=z^2$	$(x,y,z,a)=(1,0,57,2{,}348)$
$3{,}599^x+7{,}210^y=z^2$	$(x,y,z,a)=(1,0,60,3{,}599)$
$4{,}355^x+8{,}722^y=z^2$	$(x,y,z,a)=(1,0,66,4{,}355)$
$4{,}760^x+9{,}532^y=z^2$	$(x,y,z,a)=(1,0,69,4{,}760)$
$5{,}624^x+11{,}250^y=z^2$	$(x,y,z,a)=(1,0,75,5{,}624)$
$6{,}083^x+11{,}250^y=z^2$	$(x,y,z,a)=(1,0,78,6{,}083)$
$7{,}055^x+11{,}250^y=z^2$	$(x,y,z,a)=(1,0,84,7{,}055)$
$7{,}568^x+11{,}250^y=z^2$	$(x,y,z,a)=(1,0,87,7{,}568)$

Example 5 $(x,y,z)=(3,2,36)$ is a unique solution when x,y,z are non-negative integers and a is a positive integer of the Diophantine equation $a^x+(2a+12)^y=z^2$ where $a\equiv 8(\bmod 27)$. Since $8\equiv 8(\bmod 27)$, by Lemma3 $(3,2,36)$, there is a unique solution (x,y,z) for the Diophantine equation $a^x+(2a+12)^y=z^2$ where $a\equiv 8(\bmod 27)$. When x,y,z are non-negative integers and a is a positive integer.

4 Conclusion

In this article, we find all solutions of the Diophantine equation $a^x+(2a+12)^y=z^2$ where $a\equiv 8(\bmod 27)$ and x,y,z are non-negative integers, as follows:

$(x,y,z)=(1,0,\sqrt{a+1})$ where $a=(9t\pm 3)^2-1$

$(x,y,z)=(1,1,\sqrt{3a+12})$ where $a=243t^2\pm 108t+8$

and $(x,y,z)=(3,2,36)$ is the unique non-negative integer solution, and t is an integer.

Acknowledgment:
This article is supported by the research from the Faculty of Science, UbonRatchatani Rajabhat University, Thailand.

Reference:

[1] Elshahed and Kamarulhaili, On the Diophantine equation $(4^n)^x-p^y=z^2$, *WSEAS transaction on Mathematics,* Vol.19, 2020, pp. 349-352. https://doi.org/10.37394/23206.2020.19.35.

[2] Blinn, How to solve a cubic equation. Part 1. The shape of the discriminant, *IEEE Computer Graphics and Applications,* Vol.19, 2006, pp. 84-93. https://doi.org/10.1109/MCG.2006.60.

[3] Orosram and Comemuang, On the Diophantine equation $8^x+n^y=z^2$, *WSEAS transaction on Mathematics,* Vol.19, 2020, pp. 520-522. https://doi.org/10.37394/23206.2020.19.56.

[4] Viriyapong, On the Diophantine Equation nx + 13y = z2 where n = 2 (mod 39) and n + 1 is not a Square Number, *WSEAS Transaction on Mathematics,* Vol.20, 2021, pp. 442-445. https://doi.org/10.37394/23206.2021.20.45.

[5] Tangjai and Chubthaisong, On the Diophantine equation $3^x+p^y=z^2$ where $p\equiv 2(\bmod 3)$, *WSEAS Transaction on Mathematics,* Vol.22, 2021, pp. 283-287. https://doi.org/10.37394/23206.2021.20.29.

[6] Thongnak, Chuayjan and Kaewong, On the Exponential Diophantine equation $5^x-2\cdot 3^y=z^2$, *Annal of Pure and Applied Mathematics,* Vol.25, No.2, 2022, pp. 109-112.

[7] Tadee, On the Diophantine equation $n^x+10^y=z^2$, *WSEAS Transaction on Mathematics,* Vol.22, 2023, pp. 150-153. https://doi.org/10.37394/23206.2023.22.19.

[8] Srimud and Tadee, On the Diophantine equation $3^x+b^y=z^2$, *International Journal*

of Mathematics and Computer Science, Vol.18, 2023, pp. 137-172.

[9] Rangpung, Infinite solution of the Diophantine equation $a^x+(2a)^y=z^2$ where $a\equiv 3(\mathrm{mod}16)$, *International Journal of Mathematics and Computer Science,* Vol.20, 2025, pp. 805-809.

[10] Mihăilescu, Primary cyclotomic units and a proof of Catalan's conjecture, *Journal für die reine and angewandte Mathematik,* Vol.572, 2004, pp. 167-195.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

Tasanai Rangpung design pattern the lemma3 and theorem4, including the tables and example5.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The author has no conflicts of interest to declare that is relevant to the content of this article.

Creative Commons Attribution License 4.0 (Attribution 4.0 International, CC BY 4.0)

This article is published under the terms of the Creative Commons Attribution License 4.0
https://creativecommons.org/licenses/by/4.0/deed.en_US