Confidence Intervals for the Percentile of the Zero-Inflated Rayleigh Distribution with an Application to Car Accident Mortality Data

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Abstract: - The zero-inflated Rayleigh distribution combines zero values and a standard Rayleigh distribution that represents positive values. Percentiles are widely used tools for measuring data, particularly in skewed continuous datasets. This study aims to construct confidence intervals (CIs) for the percentile of the zero-inflated Rayleigh distribution. The proposed approaches include the generalized confidence interval (GCI), normal approximation (NA), and the percentile bootstrap confidence interval (PBCI). The zero-inflation probability is estimated using the variance-stabilizing transformation (VST), Wilson's Score, and Hannig's methods. Their performances were evaluated using Monte Carlo simulations in R programming, comparing coverage probabilities (CPs) and average lengths (ALs). The results indicate that the GCI based on the VST outperformed the other methods. Additionally, the proposed CIs were applied to car accident mortality data in central Thailand to evaluate their practical efficacy.

Key-Words: - Rayleigh distribution, zero inflation, percentile, generalized confidence interval, normal approximation, percentile bootstrap confidence interval.

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1 Introduction

Initially introduced [1], the Rayleigh distribution is a well-known statistical model for lifetime data. It is a positive continuous distribution with a scale parameter. Many researchers have extended and applied it to lifetime data, building on its foundational uses. For example, the work in [2], examined a point estimator of the scale parameter of the Rayleigh distribution using lifetime data under integrated composite censoring at various experimental endpoints from different sample sizes. Similarly, in [3], the study estimated and constructed CIs for two populations of power Rayleigh distributions under the joint progressive censoring concept. In addition, the study in [4] introduced an extended Rayleigh model and applied it to a COVID-19 dataset.

In applications, real-world datasets often contain zero values; the work in [5] first introduced a model that handles excess zeros. Later, several researchers investigated this concept and extended it to other statistical distributions, including discrete and continuous data. In [6], the study proposed a zero-inflated Rayleigh distribution to address the issue of zero values contained in the model, which is more flexible for practical applications than the standard Rayleigh model. The model combines zero values with positive data from the Rayleigh samples. The zero values follow a binomial distribution with a zero-inflation probability. In line with the concept, several researchers have examined various zero-inflated distributions and applied them to real-world settings, including environmental, network, and clinical data. For example, the study in [7], established CIs for the mean and percentile parameters of zero-inflated lognormal data and applied them to call recording data. The work in [8], examined a zero-inflated Pareto distribution to construct generalized CIs and demonstrated their application to health risk data. According to [9], the study established CIs for the coefficient of variation, comparing the normal approximation (NA) and generalized confidence interval (GCI) using the variance-stabilizing transformation (VST), Wilson's Score method, and bootstrap method for the Birnbaum-Saunders distribution, which contains zeros. They also applied their proposed methods to assess wind-flow characteristics. Their findings showed that the GCI method performed best. The work in [10], examined various approaches within the framework of the zero-inflated gamma distribution. Their findings showed that the Box-Cox transformation methods yielded better results than the fiducial and MOVER approaches. They additionally conducted an empirical study using meteorological data.

For discrete model frameworks, such as the zero-inflated geometric distribution, the study in [11], investigated Wald and profile likelihood methods for interval estimation of the model's parameter. They concluded that the profile likelihood methods showed outstanding results. In empirical applications, they demonstrated their methods using datasets on agricultural technology. As shown in [12], the study investigated various approaches, including likelihood and bootstrap techniques, for point and interval estimation of the mean under the zero-inflated Poisson distribution and applied them to practical examples, including pension fund and veterinary datasets.

Percentiles are basic descriptive statistics that convey essential information, especially for skewed data. They represent the percentage or position of data points at or below a specific value. For complex or skewed data, the 50th percentile (median), a measure of central tendency, often differs significantly from the mean. Percentiles are also described in the variability measure using the interquartile range (IQR), the difference between the third and first quartiles represented by the 75th and 25th percentiles, respectively. Many datasets in practical settings exhibit skewness; describing data using percentiles is therefore suitable for conveying information about data characteristics in many circumstances, leading to their widespread use across various fields such as education, health care, insurance, and finance.

In statistical research on interval estimation, several researchers have developed CIs for percentiles in various distributions. For instance, the work in [13], investigated the Weibull and Birnbaum-Saunders distributions to construct lower percentile limits in multiple approaches and illustrated their application using biological and failure datasets. Additionally, the study in [14], proposed several methods for interval estimation of the ratio of percentiles in Birnbaum-Saunders distributions and applied them to environmental data.

To our best knowledge, no prior studies have explored CIs for the percentile of the zero-inflated Rayleigh distribution. Thus, this study aims to propose CIs using the generalized confidence interval (GCI), normal approximation (NA), and the percentile bootstrap confidence interval (PBCI). We will employ the variance-stabilizing transformation (VST), Wilson's Score, and Hannig's methods to estimate the parameter δ. The proposed CIs will be assessed through an empirical application using data on car accident mortality in central Thailand from 11–17 April 2021.

2 Methods

The concept of the zero-inflated model structure was initially introduced in [6]. This study, therefore, examines the zero-inflated Rayleigh distribution, which consists of two components. The first one is a zero-value component, and the other is a standard Rayleigh distribution indicating positive values.

Let the total sample size be the sum of the number of zero values, n_0, and the number of non-zero or positive values, n_1 derived from the Rayleigh distribution. The number of zero values follows a binomial distribution with a zero-inflation probability δ, where $0 < \delta < 1$.

Let $X = \{X_1, X_2, \ldots, X_n\}$ be a zero-inflated Rayleigh sample. According to [6], the cumulative density function (CDF) of X is given by:

$$G(x, \sigma, \delta) = \begin{cases} \delta, & x = 0, \\ \delta + (1 - \delta) F(x; \sigma), & x > 0. \end{cases} \tag{1}$$

where $F(x;\sigma)$ is the CDF of the Rayleigh distribution. Let θ be the pth percentile of the zero-inflated Rayleigh distribution. According to [6], the percentile of the zero-inflated Rayleigh distribution can be written as:

$$\theta = \begin{cases} 0, & p \le \delta, \\ \sigma\sqrt{-2\ln\left(\dfrac{1-p}{1-\delta}\right)}, & p > \delta. \end{cases} \tag{2}$$

The maximum likelihood estimators of σ and δ are:

$$\hat{\sigma} = \sqrt{\frac{1}{2n_1}\sum_{i=1}^{n_1} X_i^2} \text{ and } \hat{\delta} = \frac{n_0}{n}, \text{ respectively.}$$

2.1 The Generalized Confidence Interval (GCI) Method

Weerahandi originally proposed the generalized confidence interval (GCI), which broadens the conventional interval estimation and handles the complex issues of traditional methods that rely on parameters or non-normally distributed data, [15]. The basis of the GCI method is the concept of the generalized pivotal quantity (GPQ), a function of the observed data and the parameters, which satisfies two conditions.

Definition.

Let $\bar{x}=\left(x_1,x_2,\ldots,x_n\right)$ denote an observed data value from a random sample, $\overline{X}=\left(X_1,X_2,\ldots,X_n\right)$. Let/be a function of the observed data and parameters. It is defined as a GPQ if its probability distribution function does not depend on unknown parameters, and its observed values, denoted by $r(\overline{X},\bar{x},\sigma)$, are unaffected by nuisance parameters. The work in [16], introduced the concept of deriving a pivotal quantity for a scale parameter of a Rayleigh distribution. Later, Teawpongpan and Sangnawakij derived a GPQ for σ^2, whose probability density function does not depend on the parameters [17], as shown below:

$$R_{\sigma^2}=\frac{Y}{W}. \tag{3}$$

where $Y=\sum_{i=1}^{n_1} X_i^2$ and $W=\sum_{i=1}^{n_1} X_i^2/\sigma^2\sim Gamma\left(n_1,1/2\right)$, respectively. The various parameter estimation methods are provided in the following subsections.

2.1.1 The GCI Based on the VST Method

DasGupta introduced the variance-stabilizing transformation (VST), which stabilizes the dataset's variance to achieve data homogeneity and thereby improve the accuracy of statistical inference, [18]. Later, Wu and Hsieh applied the VST using the arcsine transformation, which is well-suited to complex or zero-inflated data. They derived a GPQ for δ to construct Cis, [19], as shown below:

$$R_\delta^{VST}=\sin^2\left[arc\sin\sqrt{\hat{\delta}-\frac{T}{2\sqrt{n}}}\right], \tag{4}$$

where

$T=2\sqrt{n}\left(arc\sin\sqrt{\hat{\delta}}-arc\sin\sqrt{\delta}\right)\sim N(0,1)$. The GPQs for σ^2 and δ, denoted by R_{σ^2} and $R_\delta^{GCI.VST}$, were provided in Equation (3) and Equation (4). The GPQ for θ can be written as follows:

$$R_\theta^{GCI.VST}=\sqrt{-2R_{\sigma^2}\ln\left(\frac{1-p}{1-R_\delta{}^{VST}}\right)}. \tag{5}$$

Therefore, the $100(1-\alpha)\,\%$ GCI based on the VST method for θ is

$$\begin{aligned}CI_\theta^{GCI.VST}&=\left(L_\theta^{GCI.VST},U_\theta^{GCI.VST}\right)\\&=\left(R_\theta^{GCI.VST}(\alpha/2),R_\theta^{GCI.VST}(1-\alpha/2)\right).\end{aligned} \tag{6}$$

where $R_\theta^{GCI.VST}(\alpha/2)$ and $R_\theta^{GCI.VST}(1-\alpha/2)$ are the $100(\alpha/2)\,th$ and $100(1-\alpha/2)\,th$ percentiles of $R_\theta^{GCI.VST}$.

2.1.2 GCI Based on the Wilson's Score Method

Wilson investigated a binomial distribution to estimate the parameter δ, [20]. Later, in [21] derived the GPQ by approximating it using the large-sample concept as n approaches infinity, as shown below:

$$R_\delta^W=\frac{n_0+Z_W^2/2}{n+Z_W^2}-\frac{Z_W}{n+Z_W^2}\sqrt{n_0\left(1-\frac{n_0}{n}\right)+\frac{Z_W^2}{4}}. \tag{7}$$

where Z_W was approximated as a standard normal distribution. The GPQs for all of the parameters, denoted by R_{σ^2} and $R_\delta^{GCI.W}$, were provided in Equation (3) and Equation (7); accordingly, the GPQ for θ can be written as follows:

$$R_\theta^{GCI.W}=\sqrt{-2R_{\sigma^2}\ln\left(\frac{1-p}{1-R_\delta{}^W}\right)}. \tag{8}$$

Therefore, the $100(1-\alpha)\,\%$ GCI based on the Wilson's Score method for θ is

$$\begin{aligned}CI_\theta^{GCI.W}&=\left(L_\theta^{GCI.W},U_\theta^{GCI.W}\right)\\&=\left(R_\theta^{GCI.W}(\alpha/2),R_\theta^{GCI.W}(1-\alpha/2)\right).\end{aligned} \tag{9}$$

where $R_\theta^{GCI.W}(\alpha/2)$ and $R_\theta^{GCI.W}(1-\alpha/2)$ are the $100(1-\alpha)^{th}$ and $100(1-\alpha/2)^{th}$ percentiles of $R_\theta^{GCI.W}$.

2.1.3 GCI Based on Hannig's Method

Hannig originally proposed the generalized fiducial quantity (GFQ) by integrating two beta distributions and recommended equal weighting, yielding the optimal GFQ of δ [22]. Subsequently, the authors in [21] refined and proposed the notion of GFQ for δ, which has the following form:

$$R_\delta^H\sim\frac{1}{2}beta\left(n_0,n_1+1\right)+\frac{1}{2}beta\left(n_0+1,n_1\right). \tag{10}$$

The GPQs for all of the parameters, denoted by R_{σ^2} and $R_\delta^{GCI.H}$, were provided in Equation (3) and Equation (10); accordingly, the GPQ for θ can be written as follows:

$$R_\theta^{GCI.H}=\sqrt{-2R_{\sigma^2}\ln\left(\frac{1-p}{1-R_\delta{}^H}\right)}. \tag{11}$$

Therefore, the $100(1-\alpha)\,\%$ GCI based on Hannig's method for θ is:

$$\begin{aligned}CI_\theta^{GCI.H}&=\left(L_\theta^{GCI.H},U_\theta^{GCI.H}\right)\\&=\left(R_\theta^{GCI.H}(\alpha/2),R_\theta^{GCI.H}(1-\alpha/2)\right).\end{aligned} \tag{12}$$

where $R_\theta^{GCI.H}(\alpha/2)$ and $R_\theta^{GCI.H}(1-\alpha/2)$ are the $100(1-\alpha)\,th$ and $100(1-\alpha/2)\,th$ percentiles of $R_\theta^{GCI.H}$.

Algorithm 1 provides the steps for constructing the CI for the percentile of the zero-inflated Rayleigh distribution using the GCI method.

Algorithm 1: The GCI method
Step 1: Input parameters n, σ, and δ.
Step 2: Generate a random sample $\{X_1, X_2, \ldots, X_n\}$ from the zero-inflated Rayleigh distribution and calculate $\hat{\sigma}$ and $\hat{\delta}$.
Step 3: At the J step.
Compute $Y = \sum_{i=1}^{n_1} X_i^2$ and generate $W \sim Gamma(n_1, 1/2)$
Calculate/using Equation (3).
Calculate R_{δ}^{VST} and $R_{\theta}^{GCI.VST}$ using Equation (4) and Equation (5).
Calculate R_{δ}^{W} and $R_{\theta}^{GCI.W}$ using Equation (7) and Equation (8).
Calculate R_{δ}^{H} and $R_{\theta}^{GCI.H}$ using Equation (10) and Equation (11).
Step 4: Repeat **step 3** for J = 5000 iterations.
Step 5: Calculate L_{θ}^{GCI} and U_{θ}^{GCI} using Equation (6), Equation (9), and Equation (12).

2.2 Normal Approximation

The normal approximation method, a statistical technique, is used to approximate the distribution of an asymptotically normal estimator. This process provides more accuracy with a larger sample size. The delta method approximates the distribution of a function of an estimator using a first-order Taylor expansion.

Let $\theta = g(\sigma, \delta) = \sigma\sqrt{-2\ln\left(\frac{1-p}{1-\delta}\right)}$. Suppose there is a differentiable function with $g'(\sigma, \delta) \neq 0$; then, consider the first-order Taylor expansion of $g(\hat{\sigma}, \hat{\delta})$ about σ and δ, and apply the delta method as described in [23]. Therefore, the asymptotic distribution of $g(\hat{\sigma}, \hat{\delta})$ can be calculated as follows:

$$g(\hat{\sigma}, \hat{\delta}) = g(\sigma, \delta) + \frac{\partial(\sigma, \delta)}{\partial\sigma}(\hat{\sigma} - \sigma) + \frac{\partial(\sigma, \delta)}{\partial\delta}(\hat{\delta} - \delta) + Remainder. \tag{13}$$

Using the Taylor series expansion, the probability of the $\sqrt{n}\,Remainder$ term converges to 0 as $n \to \infty$. The asymptotic distributions of the estimators for $\hat{\sigma}$ and $\hat{\delta}$ are given by $\hat{\sigma} \sim N\left(\sigma, \frac{\sigma^2}{4n_1}\right)$ and $\hat{\delta} \sim N\left(\delta, \frac{\delta(1-\delta)}{n}\right)$,

respectively, as demonstrated in [6]. Evaluate the first partial derivatives with respect to σ and δ, as obtained in Equation (13). Therefore, the estimator can be derived as follows:

$$g(\hat{\sigma}, \hat{\delta}) \approx \sigma\sqrt{-2\ln\left(\frac{1-p}{1-\delta}\right)} + \sqrt{-2\ln\left(\frac{1-p}{1-\delta}\right)}(\hat{\sigma} - \sigma) + \frac{\sigma}{(1-\delta)\sqrt{-2\ln\left(\frac{1-p}{1-\delta}\right)}}(\hat{\delta} - \delta). \tag{14}$$

According to Equation (14), the asymptotic mean and variance of the derived estimator can be expressed as follows:

$$E\left[g(\hat{\sigma}, \hat{\delta})\right] \approx \sigma\sqrt{-2\ln\left(\frac{1-p}{1-\delta}\right)} \quad \text{and}$$

$$V\left[g(\hat{\sigma}, \hat{\delta})\right] \approx \left(-2\ln\left(\frac{1-p}{1-\delta}\right)\right)\frac{\sigma^2}{4n_1} + \left(\frac{\sigma}{(1-\delta)\sqrt{-2\ln\left(\frac{1-p}{1-\delta}\right)}}\right)^2 \frac{\delta(1-\delta)}{n}.$$

Assume that the estimators $\hat{\sigma}$ and $\hat{\delta}$ are independent, and utilize the invariance properties of both. The maximum likelihood estimator of θ can be derived as follows:

$$\hat{\theta} = \hat{\sigma}\sqrt{-2\ln\left(\frac{1-p}{1-\hat{\delta}}\right)}. \tag{15}$$

The estimated variance of $\hat{\theta}$ can subsequently be calculated as:

$$\hat{V}[\theta] \approx \left(-2\ln\left(\frac{1-p}{1-\hat{\delta}}\right)\right)\frac{\hat{\sigma}^2}{4n_1} + \left(\frac{\hat{\sigma}}{(1-\hat{\delta})\sqrt{-2\ln\left(\frac{1-p}{1-\hat{\delta}}\right)}}\right)^2 \frac{\hat{\delta}(1-\hat{\delta})}{n}. \tag{16}$$

The NA method employs estimation techniques for the parameter δ based on the VST, Wilson's Score, and Hannig's methods, similar to the previous approach. Let $Z = \frac{\hat{\theta} - \theta}{\sqrt{\hat{V}(\hat{\theta})}} \sim N(0, 1)$ and apply the central limit theorem. Therefore, the $100(1-\alpha)\%$ NA confidence interval is:

$$CI_{\theta}^{NA}=\left(L_{\theta}^{NA}, U_{\theta}^{NA}\right) =\theta \pm Z_{\alpha/2}\sqrt{\widehat{V}(\widehat{\theta})} . \tag{17}$$

where $Z_{\alpha/2}$ is the $(\alpha/2)\,th$ quantile value of the standard normal distribution.

Algorithm 2 provides the steps for constructing the CI for the percentile of the zero-inflated Rayleigh distribution using the NA method.

Algorithm 2: The NA method
Step 1: Input parameters n, σ, and δ.
Step 2: Generate a random sample $\left\{X_1, X_2, \ldots, X_n\right\}$ from the zero-inflated Rayleigh distribution and calculate $\widehat{\sigma}$ and $\widehat{\delta}$.
Step 3: Calculate $\widehat{\theta}$ and $\widehat{V}(\widehat{\theta})$ using Equation (15) and Equation (16).
Step 4: Calculate L_{θ}^{PBCI} and U_{θ}^{PBCI} using Equation (17).
Step 5: Repeat **step 4** for M = 5000 iterations.

2.3 The Percentile Bootstrap Confidence Interval

The percentile bootstrap confidence interval (PBCI) method, as introduced in [24], is a resampling technique that generates bootstrap samples from the original data. The method is typically used to estimate the variability of a parameter when sample sizes are small or the statistical analysis is complex. Bootstrap samples, drawn with replacement, are used to compute bootstrap statistics and then construct bootstrap intervals. In this study, the PBCI method uses the estimation techniques for the parameter δ based on the VST, Wilson's Score, and Hannig's methods. From Equation (2), the bootstrap estimator of θ, denoted by θ^*, is consequently:

$$\theta^* =\sigma^* \sqrt{-2\ln\left(\frac{1-p}{1-\widehat{\delta}}\right)} . \tag{18}$$

The $100(1-\alpha)\,\%$ bootstrap confidence interval for θ is:

$$CI_{\theta}^{PBCI}=\left(L_{\theta}^{PBCI}, U_{\theta}^{PBCI}\right) \tag{19}$$
$$=(\theta^*(\alpha/2), \theta^*(1-\alpha/2)) .$$

where $\theta^*(\alpha/2)$ and $\theta^*(1-\alpha/2)$ are the $100(1-\alpha)\,th$ and $100(1-\alpha/2)\,th$ percentiles of θ^*.

Algorithm 3 provides the steps for constructing the CI for the percentile of the zero-inflated Rayleigh distribution using the PBCI method.

Algorithm 3: The PBCI method
Step 1: Input parameters n, σ, and δ.
Step 2: Generate a random sample $\left\{X_1, X_2, \ldots, X_n\right\}$ from the zero-inflated Rayleigh distribution and calculate $\widehat{\sigma}$ and $\widehat{\delta}$.
Step 3: At the B step.
Generate bootstrap samples $X_1^*, X_2^*, \ldots, X_n^*$ from the original sample in step 2.
Calculate the bootstrap estimator θ^* using Equation (18).
Step 4: Repeat **step 3** for B = 5000 iterations.
Step 5: Calculate L_{θ}^{PBCI} and U_{θ}^{PBCI} using Equation (19).

Algorithm 4 provides the steps for calculating a coverage probability (CP) and average length (AL) of CIs for the percentile of the zero-inflated Rayleigh distribution.

Algorithm 4: CP and AL calculation
Step 1: Construct CIs for θ.
Step 2: If $L_{\theta} \le \theta \le U_{\theta}$, set $p_{\theta}=1$; otherwise, set $p_{\theta}=0$.
Step 3: Compute $U_{\theta}-L_{\theta}$.
Step 4: Calculate the CP and AL by assessing the mean of p_{θ} and
$U_{\theta}-L_{\theta}$, respectively.

3 Results

A Monte Carlo simulation study was conducted in R programming to evaluate the performance of CIs for the percentile of the zero-inflated Rayleigh distribution. The criteria for assessing the performance of the proposed methods in the study are that their coverage probabilities (CPs) achieve or exceed the nominal level of 0.95 and that their average lengths (ALs) are the shortest (as provided in Algorithm 4).

Data were generated from a zero-inflated Rayleigh distribution. The simulation study combined sample sizes (n) of 20, 50, 100, 150, and 200; scale parameters (σ) of 0.5, 1.0, 1.5, and 2.0; and zero-inflation probabilities (δ) of 0.1, 0.3, and 0.5. For each combination, 5000 replications were set for the simulation study. Consequently, 5000 replications were used to evaluate the GCI methods, and 500 replications were used for the bootstrap methods.

Table 1 and Table 2 in Appendix represent the CPs and ALs of the proposed CIs (as shown in the appendix). According to the simulation evaluation, the results indicate that the CPs for the NA based on the VST method and the GCI based on the VST and Wilson's Score methods reached 0.95 in several cases. In contrast, the GCI based on Hannig's method and the NA based on Wilson's Score and Hannig's methods yielded CPs below 0.95 in almost all cases. The PBCI method improved considerably with larger sample sizes. For ALs, the GCI based on the VST method yielded shorter lengths than the NA method, and the remaining estimation techniques for the parameter δ were within the GCI. The PBCI method, based on the VST approach, provided the shortest lengths among the others. However, the CPs of the PBCI method were below 0.95 for all estimation techniques for parameter δ. Based on the two criteria, the GCI based on the VST method outperformed all combinations of sample sizes, scale parameters, and zero-inflation probabilities.

Figure 1, Figure 2 and Figure 3 (as shown in the appendix) depict the CPs and ALs across different sample sizes, scale parameters, and zero-inflation probabilities. Figure 1 (Appendix) shows that with larger sample sizes, all methods performed better, yielding shorter ALs. Figure 2 (Appendix) shows that all methods provided consistent CPs; however, they produced wider ALs with larger-scale parameters. Figure 3 (Appendix) shows that all methods performed well, with lower probabilities of zero inflation.

4 Empirical Application

Road traffic accidents are a critical public health issue, often resulting in injury, disability, or death. In Thailand, the government has a policy to promote road safety awareness, encourage responsible driving, and reduce accidents during holiday periods, particularly the high-risk "7 Dangerous Days" of the New Year and Songkran (Thai New Year). Consequently, the mortality data from these accidents is used as an indicator for government surveillance. This study analyzes mortality from car accidents in central Thailand from 11–17 April 2021, using data recorded in the Thailand Road Safety Collaboration, [25].

The histogram of the mortality data is shown in Figure 4 (Appendix). Some provinces reported zero fatalities during this period, while others reported deaths. We modelled positive values for both the Rayleigh and exponential distributions because they shared a scale parameter. The model selection was performed using AIC = -2ln(L) + 2k and BIC = -2ln(L) + kln(n), where k represents the number of parameters, n is the sample size (18 provinces in central Thailand), and ln(L) is the log-likelihood. The Rayleigh distribution provided a better fit, with lower AIC (23.8363, 25.1783) and BIC (24.0335, 25.3755) values than the exponential distribution.

From the data of car accident mortality in central Thailand (11-17 April 2021), the sample statistics are as follows: a total sample size (n) covering 18 provinces, an estimated proportion of zero inflation ($\hat{\delta}$) of 0.5 (with nine provinces reporting no deaths), an estimated scale parameter ($\hat{\sigma}$) of 1.0541, and an estimated 90th percentile of fatalities at 1.8912. Table 3 (Appendix) presents the 95% CIs for the percentile of car accident deaths; results from this data application show that the NA based on VST produced the shortest length (as shown in the appendix).

5 Discussion

Mortality due to road traffic accidents is used as an indicator to improve road safety. This study examined car accident mortality and constructed CIs for the mortality data percentile. To carry out safety evaluations and assess the accident risks, the estimated intervals provide ranges of projected road traffic fatalities. This statistical evidence is crucial support for policymakers in establishing solutions to boost public awareness, reduce risk factors associated with traffic deaths, and improve policies for road traffic accidents in specific risk areas or time periods.

To construct CIs for the percentiles of a zero-inflated Rayleigh distribution, this study focused on the generalized confidence intervals (GCI), normal approximation (NA), and percentile bootstrap confidence intervals (PBCI).

The simulation results reveal better performance for all methods with larger sample sizes, smaller scale parameters, and lower zero-inflation probabilities. The NA method consistently achieved or exceeded the nominal confidence level of 0.95 in almost all cases; however, it produced lengths wider than those of the GCI method. The PBCI method considerably improved with increasing sample sizes, especially for $n \geq 50$, but did not reach 0.95. The GCI method performed well, achieving 0.95 for several cases. The findings demonstrate stable results across all combinations, with the GGI based on the variance-stabilizing transformation (VST) method yielding the best results, consistently achieving or exceeding 0.95 while providing the shortest interval lengths. These findings align with those of prior studies in [9] and [13].

6 Conclusions

The GCI based on the VST method is recommended for constructing CIs for the percentile of the zero-inflated Rayleigh distribution. Future research could investigate the CIs for the difference in percentiles between the zero-inflated Rayleigh distributions.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed to the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflict of Interest

The authors have no conflicts of interest to declare.

APPENDIX

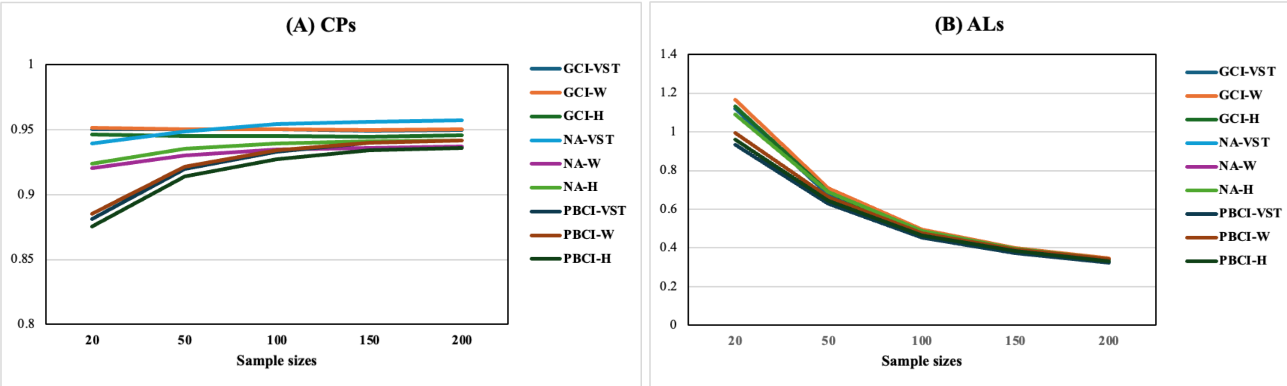

Fig. 1: Comparing the performances of the CIs for the percentile of the zero-inflated Rayleigh distribution across different sample sizes in terms of (A) coverage probabilities (CPs) and (B) average lengths (ALs)
Source: Created by the authors

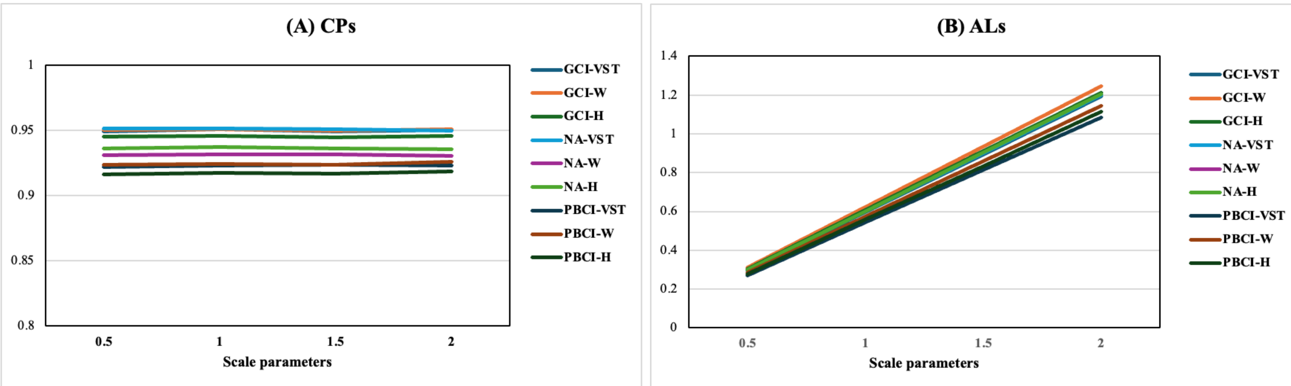

Fig. 2: Comparing the performances of the CIs for the percentile of the zero-inflated Rayleigh distribution across different scale parameters in terms of (A) coverage probabilities (CPs) and (B) average lengths (ALs)
Source: Created by the authors

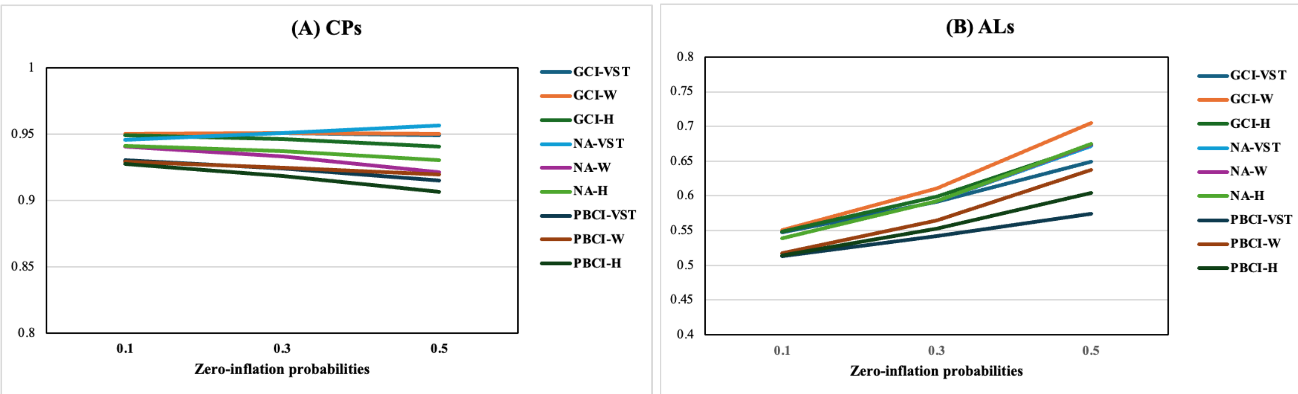

Fig. 3: Comparing the performances of the CIs for the percentile of the zero-inflated Rayleigh distribution across different zero-inflation probabilities in terms of (A) coverage probabilities (CPs) and (B) average lengths (ALs)
Source: Created by the authors

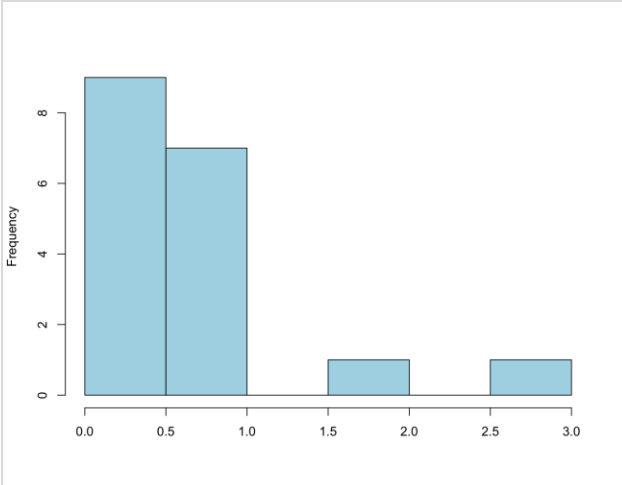

Fig. 4: Histogram of car accident mortality in central Thailand from 11-17 April 2021
Source: Created by the authors

Table 1. The coverage probabilities (CPs) for the percentile of the zero-inflated Rayleigh distribution

n	σ	δ	CPs								
			GCI			NA			PBCI		
			VST	W	H	VST	W	H	VST	W	H
20	0.5	0.1	0.9472	0.9494	0.9490	0.9434	0.9338	0.9342	0.9050	0.9014	0.8992
		0.3	0.9490	0.9498	0.9470	0.9366	0.9174	0.9234	0.8774	0.8804	0.8756
		0.5	**0.9530**	**0.9500**	0.9406	0.9388	0.9055	0.9105	0.8546	0.8670	0.8486
	1	0.1	**0.9516**	**0.9520**	**0.9524**	0.9396	0.9322	0.9320	0.9030	0.8984	0.8962
		0.3	**0.9524**	**0.9544**	0.9494	0.9400	0.9224	0.9240	0.8900	0.8910	0.8824
		0.5	0.9496	**0.9526**	0.9408	0.9398	0.9102	0.9170	0.8638	0.8766	0.8582
	1.5	0.1	**0.9500**	**0.9510**	**0.9506**	0.9390	0.9308	0.9302	0.8978	0.8940	0.8912
		0.3	0.9466	0.9482	0.9440	0.9382	0.9208	0.9236	0.8744	0.8752	0.8670
		0.5	0.9476	**0.9520**	0.9410	0.9416	0.9161	0.9174	0.8632	0.8768	0.8614
	2	0.1	0.9486	0.9494	0.9480	0.9358	0.9270	0.9292	0.8974	0.8936	0.8898
		0.3	0.9478	0.9472	0.9412	0.9414	0.9212	0.9256	0.8920	0.8954	0.8826
		0.5	**0.9506**	**0.9534**	0.9432	0.9402	0.9068	0.9182	0.8610	0.8814	0.8636
50	0.5	0.1	**0.9518**	**0.9534**	**0.9518**	0.9450	0.9394	0.9386	0.9224	0.9220	0.9210
		0.3	**0.9520**	**0.9522**	0.9486	0.9468	0.9294	0.9366	0.9210	0.9218	0.9168
		0.5	0.9498	**0.9532**	0.9454	**0.9520**	0.9150	0.9270	0.9158	0.9206	0.9014
	1	0.1	0.9492	0.9452	0.9452	0.9410	0.9398	0.9380	0.9208	0.9226	0.9214
		0.3	**0.9516**	**0.9528**	0.9476	0.9442	0.9262	0.9332	0.9216	0.9222	0.9168

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		0.5	**0.9506**	**0.9518**	0.9406	**0.9588**	0.9232	0.9336	0.9048	0.9154	0.9028
	1.5	0.1	**0.9514**	0.9484	0.9470	0.9488	0.9446	0.9432	0.9280	0.9284	0.9258
		0.3	**0.9518**	0.9498	0.9464	**0.9530**	0.9372	0.9400	0.9194	0.9196	0.9140
		0.5	0.9456	0.9462	0.9370	0.9478	0.9128	0.9242	0.9130	0.9138	0.9012
	2	0.1	**0.9518**	**0.9534**	**0.9518**	0.9450	0.9394	0.9386	0.9220	0.9220	0.9210
		0.3	**0.9520**	**0.9522**	0.9486	0.9468	0.9294	0.9366	0.9210	0.9218	0.9168
		0.5	0.9498	**0.9532**	0.9454	**0.9520**	0.9150	0.9270	0.9158	0.9206	0.9014
100	0.5	0.1	0.9492	0.9496	0.9490	0.9436	0.9422	0.9386	0.9376	0.9370	0.9360
		0.3	0.9472	0.9462	0.9416	**0.9534**	0.9350	0.9402	0.9304	0.9330	0.9288
		0.5	0.9486	0.9484	0.9378	**0.9646**	0.9248	0.9374	0.9260	0.9294	0.9164
	1	0.1	**0.9520**	**0.9536**	**0.9526**	0.9480	0.9422	0.9442	0.9386	0.9374	0.9358
		0.3	**0.9500**	0.9484	0.9446	**0.9562**	0.9398	0.9446	0.9308	0.9316	0.9254
		0.5	0.9490	**0.9504**	0.9400	**0.9648**	0.9206	0.9352	0.9280	0.9282	0.9160
	1.5	0.1	0.9486	**0.9512**	0.9492	0.9436	0.9422	0.9394	0.9436	0.9424	0.9406
		0.3	**0.9562**	**0.9516**	0.9488	**0.9518**	0.9334	0.9382	0.9346	0.9338	0.9290
		0.5	0.9456	0.9488	0.9410	0.9642	0.9244	0.9362	0.9296	0.9272	0.9168
	2	0.1	**0.9536**	**0.9538**	**0.9540**	0.9410	0.9368	0.9416	0.9366	0.9376	0.9360
		0.3	**0.9540**	**0.9532**	0.9496	**0.9556**	0.9402	0.9398	0.9314	0.9360	0.9304
		0.5	0.9496	**0.9506**	0.9416	0.9606	0.9308	0.9328	0.9290	0.9332	0.9210

n	σ	δ	CPs								
			GCI			**NA**			**PBCI**		
			VST	**W**	**H**	**VST**	**W**	**H**	**VST**	**W**	**H**
150	0.5	0.1	0.9496	**0.9506**	0.9482	**0.9528**	0.9476	0.9496	0.9410	0.9418	0.9408
		0.3	0.9476	**0.9500**	0.9460	**0.9576**	0.9392	0.9454	0.9394	0.9402	0.9326
		0.5	0.9488	**0.9516**	0.9418	**0.9656**	0.9270	0.9374	0.9416	0.9394	0.9296
	1	0.1	**0.9508**	**0.9510**	**0.9500**	**0.9512**	0.9474	0.9472	0.9376	0.9384	0.9364
		0.3	**0.9540**	**0.9544**	0.9490	**0.9568**	0.9394	0.9386	0.9392	0.9396	0.9334
		0.5	**0.9510**	0.9454	0.9350	**0.9638**	0.9262	0.9390	0.9402	0.9386	0.9284
	1.5	0.1	**0.9506**	0.9462	0.9472	0.9468	0.9426	0.9436	0.9382	0.9370	0.9368
		0.3	**0.9502**	0.9498	0.9454	**0.9550**	0.9366	0.9408	0.9430	0.9406	0.9360
		0.5	0.9470	**0.9512**	0.9396	0.9618	0.9236	0.9318	0.9362	0.9386	0.9272
	2	0.1	**0.9512**	**0.9506**	**0.9500**	**0.9504**	0.9456	0.9468	0.9422	0.9408	0.9382
		0.3	0.9462	0.9472	0.9418	**0.9564**	0.9396	0.9400	0.9376	0.9382	0.9362
		0.5	0.9496	0.9492	0.9408	0.9656	0.9296	0.9380	0.9330	0.9360	0.9262
200	0.5	0.1	0.9484	0.9486	0.9470	0.9496	0.9466	0.9464	0.9412	0.9428	0.9404
		0.3	**0.9514**	**0.9510**	0.9460	**0.9568**	0.9344	0.9396	0.9386	0.9414	0.9322
		0.5	0.9456	0.9492	0.9392	**0.9632**	0.9266	0.9356	0.9380	0.9408	0.9284
	1	0.1	0.9494	0.9474	0.9466	0.9470	0.9430	0.9436	0.9472	0.9456	0.9434
		0.3	**0.9540**	**0.9566**	**0.9516**	**0.9574**	0.9396	0.9444	0.9410	0.9408	0.9380
		0.5	0.9470	**0.9524**	0.9424	**0.9664**	0.9260	0.9420	0.9398	0.9378	0.9292

	1.5	0.1	0.9450	0.9472	0.9458	0.9498	0.9468	0.9494	0.9446	0.9420	0.9416
		0.3	**0.9544**	**0.9520**	0.9476	**0.9584**	0.9392	0.9456	0.9440	0.9422	0.9350
		0.5	0.9492	**0.9504**	0.9418	0.9654	0.9272	0.9384	0.9432	0.9430	0.9316
	2	0.1	**0.9552**	**0.9568**	**0.9558**	0.9480	0.9414	0.9442	0.9488	0.9478	0.9454
		0.3	**0.9502**	0.9488	0.9448	**0.9504**	0.9338	0.9396	0.9460	0.9446	0.9416
		0.5	0.9462	0.9434	0.9340	0.9622	0.9248	0.9350	0.9356	0.9380	0.9260

Source: Created by the authors

Table 2. The average lengths (ALs) for the percentile of the zero-inflated Rayleigh distribution

n	σ	δ	ALs								
			GCI			**NA**			**PBCI**		
			VST	**W**	**H**	**VST**	**W**	**H**	**VST**	**W**	**H**
20	0.5	0.1	0.5059	0.5083	0.5058	0.4870	0.4859	0.4856	0.4480	0.4509	0.4485
		0.3	0.5517	0.5675	0.5567	0.5375	0.5371	0.5361	0.4700	0.4903	0.4782
		0.5	0.6151	0.6645	0.6322	0.6095	0.6118	0.6140	0.4863	0.5507	0.5153
	1	0.1	1.0123	1.0167	1.0121	0.9750	0.9722	0.9721	0.8992	0.9058	0.9001
		0.3	1.1050	1.1365	1.1146	1.0723	1.0721	1.0697	0.9406	0.9810	0.9570
		0.5	1.2362	1.3355	1.2706	1.2234	1.2277	1.2319	0.9802	1.1086	1.0377
	1.5	0.1	1.5169	1.5228	1.5154	1.4651	1.4612	1.4606	1.3366	1.3464	1.3385
		0.3	1.6622	1.7095	1.6764	1.6109	1.6107	1.6076	1.4003	1.4617	1.4261
		0.5	1.8641	2.0125	1.9122	1.8293	1.8380	1.8468	1.4516	1.6426	1.5372
	2	0.1	2.0177	2.0266	2.0166	1.9503	1.9453	1.9445	1.7675	1.7809	1.7688
		0.3	2.2146	2.2773	2.2335	2.1428	2.1416	2.1386	1.8812	1.9606	1.9106
		0.5	2.4822	2.6791	2.5455	2.4428	2.4557	2.4687	1.9525	2.2080	2.0674
50	0.5	0.1	0.3119	0.3144	0.3128	0.3085	0.3081	0.3081	0.2957	0.2985	0.2967
		0.3	0.3354	0.3471	0.3406	0.339	0.3387	0.3386	0.3127	0.3254	0.3187
		0.5	0.3672	0.3993	0.3818	0.3839	0.3856	0.3847	0.3339	0.3683	0.3503
	1	0.1	0.6235	0.6286	0.6254	0.6184	0.6177	0.6177	0.5954	0.6006	0.5971
		0.3	0.6714	0.6951	0.6821	0.6774	0.677	0.6767	0.6278	0.6531	0.6393

		0.5	0.7315	0.7958	0.7607	0.7702	0.7735	0.7713	0.6631	0.7319	0.6959
	1.5	0.1	0.9360	0.9434	0.9387	0.9263	0.9252	0.9252	0.8915	0.8990	0.8937
		0.3	1.0053	1.0402	1.0203	1.0186	1.0181	1.0176	0.9413	0.9794	0.9592
		0.5	1.0994	1.1956	1.1432	1.1532	1.1579	1.1549	0.9954	1.0991	1.0449
	2	0.1	1.2474	1.2576	1.2511	1.2338	1.2325	1.2322	1.1830	1.1939	1.1869
		0.3	1.3414	1.3884	1.3622	1.3559	1.3549	1.3545	1.2510	1.3018	1.2750
		0.5	1.4688	1.5973	1.5270	1.5358	1.5423	1.5387	1.3357	1.4732	1.4012
100	0.5	0.1	0.2183	0.2203	0.2191	0.2185	0.2184	0.2184	0.2128	0.2148	0.2137
		0.3	0.2341	0.2428	0.2383	0.2399	0.2399	0.2398	0.2128	0.2348	0.2303
		0.5	0.2536	0.2764	0.2646	0.2721	0.2726	0.2722	0.2416	0.265	0.2531
	1	0.1	0.4367	0.4408	0.4385	0.4367	0.4365	0.4364	0.4271	0.4312	0.4288
		0.3	0.4679	0.4852	0.4762	0.4804	0.4803	0.4802	0.4526	0.4706	0.461
		0.5	0.5056	0.5509	0.5276	0.5434	0.5444	0.5437	0.4836	0.5307	0.5066
	1.5	0.1	0.6563	0.6624	0.6590	0.6553	0.6549	0.6549	0.6399	0.6462	0.6425
		0.3	0.7020	0.7280	0.7145	0.7198	0.7197	0.7196	0.6806	0.7074	0.6935
		0.5	0.7622	0.8302	0.7950	0.8169	0.8182	0.8172	0.7241	0.7942	0.7592
	2	0.1	0.8744	0.8823	0.8778	0.8732	0.8727	0.8727	0.8516	0.8600	0.8552
		0.3	0.9371	0.9718	0.9536	0.9607	0.9605	0.9602	0.9043	0.9401	0.9213
		0.5	1.0171	1.1084	1.0611	1.0864	1.0883	1.0870	0.9671	1.0618	1.0138

n	σ	δ	ALs								
			GCI			NA			PBCI		
			VST	W	H	VST	W	H	VST	W	H
150	0.5	0.1	0.1779	0.1797	0.1787	0.1785	0.1784	0.1784	0.1747	0.1765	0.1755
		0.3	0.1905	0.1976	0.1940	0.1960	0.1959	0.1959	0.1860	0.1933	0.1895
		0.5	0.2061	0.2248	0.2153	0.2220	0.2222	0.2220	0.1995	0.2185	0.2091
	1	0.1	0.3558	0.3592	0.3574	0.3572	0.3570	0.3570	0.3495	0.3528	0.3509
		0.3	0.3805	0.3948	0.3874	0.3921	0.3920	0.3920	0.3724	0.3867	0.3790
		0.5	0.4106	0.4477	0.4290	0.4432	0.4437	0.4434	0.3997	0.4379	0.4188
	1.5	0.1	0.5338	0.5389	0.5361	0.5352	0.5349	0.5349	0.5255	0.5308	0.5279
		0.3	0.5710	0.5925	0.5815	0.5877	0.5876	0.5874	0.5588	0.5808	0.5695
		0.5	0.6164	0.6724	0.6441	0.6657	0.6664	0.6659	0.5985	0.6560	0.6266
	2	0.1	0.7118	0.7185	0.7149	0.7146	0.7142	0.7143	0.6992	0.7062	0.7022
		0.3	0.7613	0.7902	0.7753	0.7841	0.7839	0.7838	0.7436	0.7731	0.7578
		0.5	0.8231	0.8976	0.8598	0.8867	0.8876	0.8870	0.7990	0.8750	0.8366
200	0.5	0.1	0.1539	0.1554	0.1546	0.1546	0.1546	0.1546	0.1524	0.1538	0.1531
		0.3	0.1646	0.1708	0.1676	0.1697	0.1696	0.1696	0.1618	0.1682	0.1649
		0.5	0.1777	0.1938	0.1857	0.1920	0.1921	0.1920	0.1739	0.1902	0.1820
	1	0.1	0.3075	0.3105	0.3089	0.3093	0.3093	0.3093	0.3037	0.3068	0.3049
		0.3	0.3286	0.3410	0.3347	0.3394	0.3393	0.3393	0.3241	0.3367	0.3304
		0.5	0.3549	0.3872	0.3709	0.3847	0.3850	0.3848	0.3463	0.3792	0.3629

	1.5	0.1	0.4616	0.4661	0.4637	0.4644	0.4643	0.4643	0.4562	0.4608	0.4583
		0.3	0.4938	0.5126	0.5030	0.5094	0.5093	0.5092	0.4846	0.5038	0.4941
		0.5	0.5326	0.5810	0.5568	0.5770	0.5774	0.5770	0.5204	0.5704	0.5453
	2	0.1	0.6148	0.6209	0.6177	0.6185	0.6183	0.6183	0.6097	0.6155	0.6124
		0.3	0.6571	0.6821	0.6694	0.6790	0.6788	0.6787	0.6490	0.6739	0.6614
		0.5	0.7099	0.7747	0.7419	0.7679	0.7686	0.7681	0.6949	0.7601	0.7277

Source: Created by the authors

Table 3. The 95% estimated CIs for the percentile of car accident mortality in central Thailand from 11-17 April 2021

Methods	**Interval**		**Length**
	Lower	**Upper**	
GCI-VST	1.4201	2.8157	1.3955
GCI-W	1.3393	2.8413	1.5021
GCI-H	1.3872	2.8165	1.4293
NA-VST	1.2164	2.5659	**1.3495**
NA-W	1.2721	2.6314	1.3592
NA-H	1.2825	2.6442	1.3617
PBCI-VST	1.2686	2.6515	1.3828
PBCI-W	1.2686	2.6515	1.3828
PBCI-H	1.2185	2.6624	1.4438

Source: Created by the authors