

A Refined Decomposition Method for Addressing Multistage Transportation Problems in a Neutrosophic Environment

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Abstract: - This study addresses multistage transportation problems within a neutrosophic framework using single-valued trapezoidal neutrosophic numbers (SVTrNNs). A score function (SF) is employed to convert the model into a crisp form, followed by a decomposition-based solution approach that preserves the structure of each stage. The proposed method integrates classical transportation techniques with large-scale optimization strategies, making it well suited for complex, real-world logistics systems. A mathematical illustration is included to exemplify the effectiveness of the approach.

Key-Words: - SVTrNNs, score function, multistage transportation problem, decomposition method, initial basic feasible solution, optimal solution.

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1 Introduction

A transportation problem (TP) is recognized as a foundational optimization model with extensive applications in applied mathematics and operations research. It has extensive applications not only in transportation systems but also in production planning, scheduling, and assignment problems. In a typical transportation model, parameters such as unit costs, supply, and demand play a key role, but in practical settings, these parameters are typically uncertain and cannot be specified exactly. The standard formulation of TP was first introduced by [1]. Various heuristic and solution approaches are proposed for different objectives such as cost minimization and time optimization are developed by [2], [3], [4], [5], [6], [7]. Additional studies have introduced alternative computational strategies, including graph-based techniques and allocation rules for both balanced and unbalanced problems, [8], [9], [10]. A comprehensive review of TPs and their solution methods is presented by [11].

In practical environments, uncertainty in cost, supply, and demand has led to the development of

fuzzy transportation models. A fuzzy framework using the extension principle is applied by [12]. Subsequently, many researchers have contributed various approaches based on different fuzzy representations such as trapezoidal, triangular, and Fermatean fuzzy numbers, [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24]. These approaches mainly convert fuzzy models into crisp equivalents using ranking, transformation, or optimization techniques.

In real-world applications, TP often occurs in multiple stages, where decisions at one stage influence subsequent stages. Thus, multistage TPs (MSTPs) differ significantly from classical models due to inter-stage dependencies and increased complexity. Depending on the nature of transportation, the number of stages, and interconnections between stages, different MSTP formulations can arise.

One of the most important methods for handling complex optimization problems is the decomposition method, which transforms a problem into smaller, manageable subproblems that can be solved independently to obtain the overall

solution, [25],[26], [27], [28].The remaining of the paper is illustrated in Figure 1.

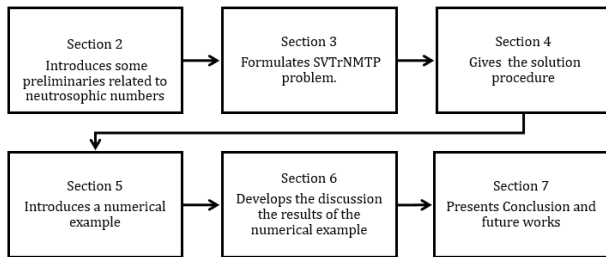


Fig. 1: Layout of Remaining Paper

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2 Preliminaries

In this section, we present some basic definitions of SVTNNs along with their arithmetic operations.

Definition 1. ([29]). Let H be the universe of discourse with a generic element ζ . A fuzzy set (FS) $\tilde{A}$ in H is defined as: $\tilde{A} = \{x: \mu_{\tilde{A}}(x), x \in H\}$, $\mu_{\tilde{A}}: H(\zeta) \rightarrow [0,1]$, where $\mu_{\tilde{A}}(x)$ represents the membership function satisfying $0 \leq \mu_{\tilde{A}}(x) \leq 1$.

Definition 2. ([29]). Let R denote the set of real numbers and $\mu_{\tilde{A}}: R \rightarrow [0,1]$ be a FS. Then $\tilde{A}$ is called a fuzzy number if it satisfies the following conditions:

- There exists $x^0 \in R$ such that $\mu_{\tilde{A}}(x^0) = 1$ (normality),
- $\mu_{\tilde{A}}(\gamma x + (1 - \gamma)y) \geq \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}$, for all $x, y \in R$ and $\gamma \in [0,1]$ (convexity),
- $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 3. [30]. Let U and V be two single-valued neutrosophic sets (SVNSs). Then $U \subseteq V$ if and only if $\mu_U(x) \leq \mu_V(x)$, $\lambda_U(x) \geq \lambda_V(x)$, $\kappa_U(x) \geq \kappa_V(x)$, $\forall x \in H$, where μ , λ , and κ represent truth, indeterminacy, and falsity membership functions, respectively.

Definition 4. [31]. A trapezoidal fuzzy number is represented as $\tilde{A}^{\text{TrFN}} = (p_1, p_2, p_3, p_4)$, with membership function defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < p_1, \\ \frac{x - p_1}{p_2 - p_1}, & p_1 \leq x \leq p_2, \\ 1, & p_2 \leq x \leq p_3, \\ \frac{p_4 - x}{p_4 - p_3}, & p_3 \leq x \leq p_4, \\ 0, & x > p_4. \end{cases}$$

Definition 5. ([32]) Let H be a non-empty set. A neutrosophic set $\tilde{A}^{\text{NS}}$ in H is defined as $\tilde{A}^{\text{NS}} = \{ \langle x; T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x) \rangle : x \in H \}$, where $T_{\tilde{A}}(x)$, $I_{\tilde{A}}(x)$, and $F_{\tilde{A}}(x)$ denote the truth, indeterminacy, and falsity membership degrees, respectively, with values in the non-standard interval $]0^-, 1^+[$. These functions satisfy $0^- \leq T_{\tilde{A}}(x) + I_{\tilde{A}}(x) + F_{\tilde{A}}(x) \leq 3^+$.

Definition 6. ([33]). (Arithmetic operations on SVTrN) Let $\tilde{p}^{\text{SVTrN}} = \langle (p_1, p_2, p_3, p_4); \mu_p, \lambda_p, \kappa_p \rangle$ and $\tilde{q}^{\text{SVTrN}} = \langle (q_1, q_2, q_3, q_4); \mu_q, \lambda_q, \kappa_q \rangle$ be two SVTrN numbers and $k \neq 0$. Then

$$\tilde{p} \oplus \tilde{q} = \langle (p_1 + q_1, p_2 + q_2, p_3 + q_3, p_4 + q_4); \mu_p \wedge \mu_q, \lambda_p \vee \lambda_q, \kappa_p \vee \kappa_q \rangle$$

$$\tilde{p} \ominus \tilde{q} = \langle (p_1 - q_4, p_2 - q_3, p_3 - q_2, p_4 - q_1); \mu_p \wedge \mu_q, \lambda_p \vee \lambda_q, \kappa_p \vee \kappa_q \rangle$$

$$\tilde{p} \otimes \tilde{q} = \langle (p_i q_i); \mu_p \wedge \mu_q, \lambda_p \vee \lambda_q, \kappa_p \vee \kappa_q \rangle$$

$$\tilde{p} \oslash \tilde{q} = \langle (p_i / q_{5-i}); \mu_p \wedge \mu_q, \lambda_p \vee \lambda_q, \kappa_p \vee \kappa_q \rangle, q_i \neq 0$$

$$k\tilde{p} = \begin{cases} \langle (kp_1, kp_2, kp_3, kp_4); \mu_p, \lambda_p, \kappa_p \rangle, & k > 0 \\ \langle (kp_4, kp_3, kp_2, kp_1); \mu_p, \lambda_p, \kappa_p \rangle, & k < 0 \end{cases}$$

$$\tilde{p}^{-1} = \langle (1/p_4, 1/p_3, 1/p_2, 1/p_1); \mu_p, \lambda_p, \kappa_p \rangle, \tilde{p} \neq 0$$

Definition 7. Let $\tilde{p}^{\text{SVTrN}} = \langle (p_1, p_2, p_3, p_4); \mu_p, \lambda_p, \kappa_p \rangle$ be a SVTrN. Then its score and accuracy functions are defined as:

$$SC(\tilde{p}) = \frac{1}{16} [(p_1 + p_2 + p_3 + p_4) + \mu_p + (1 - \lambda_p) + (1 - \kappa_p)],$$

$$AC(\tilde{p}) = \frac{1}{16} [(p_1 + p_2 + p_3 + p_4) + \mu_p + (1 - \lambda_p) + (1 + \kappa_p)].$$

Definition 8. Let

$$\tilde{p}^{\text{SVTrN}} = \langle (p_1, p_2, p_3, p_4); \mu_p, \lambda_p, \kappa_p \rangle,$$

$$\tilde{q}^{\text{SVTrN}} = \langle (q_1, q_2, q_3, q_4); \mu_q, \lambda_q, \kappa_q \rangle$$

be two SVTrN numbers. Then their comparison is defined as follows:

- If $SC(\tilde{p}) < SC(\tilde{q})$, then $\tilde{p} < \tilde{q}$.
- If $SC(\tilde{p}) = SC(\tilde{q})$, then:
 - If $AC(\tilde{p}) < AC(\tilde{q})$, then $\tilde{p} < \tilde{q}$
 - If $AC(\tilde{p}) > AC(\tilde{q})$, then $\tilde{p} > \tilde{q}$.
 - If $AC(\tilde{p}) = AC(\tilde{q})$, then $\tilde{p} = \tilde{q}$.

3 Problem Statement

This section presents different multistage TP (MSTP) formulations that model several practical decision-making situations. These formulations allow decision-makers to directly determine optimal transportation strategies under different structural settings.

3.1 Multistage Classical TP (Type I)

This case considers a multistage transportation system without any intermediate-stage restrictions. Let the supply at the first stage be a_i , the demand at the final stage be b_j , and the unit transportation cost be c_{ij} . The model is formulated as:

$$\begin{aligned} \text{(MTPP1): Min } Z &= \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \\ \text{Subject to } \sum_{j=1}^n x_{ij} &= d_i, i = 1, \dots, m, \\ \sum_{i=1}^m x_{ij} &= e_j, j = \overline{1, n}, x_{ij} \geq 0; \forall i, j. \end{aligned}$$

This formulation reduces to a classical single-stage TP.

3.2 Multistage Classical TP (Type II)

This case assumes that transportation at each stage is independent of other stages. Let $k = 1, 2, \dots, N$ denote the stages where $a_{i_k}^k$ represents supply at the stage k , $b_{j_k}^k$ represents demand, and $c_{i_k j_k}^k$ denotes transportation cost. The model is given by:

$$\begin{aligned} \text{(MTPP2): Min } Z^k &= \sum_{i_k=1}^{m_k} \sum_{j_k=1}^{n_k} c_{i_k j_k}^k x_{i_k j_k}^k, \\ \text{Subject to } \sum_{j_k=1}^{n_k} x_{i_k j_k}^k &= d_{i_k}^k, i_k = 1, 2, \dots, m_k, \\ \sum_{i_k=1}^{m_k} x_{i_k j_k}^k &= e_{j_k}^k, j_k = 1, 2, \dots, n_k, \\ x_{i_k j_k}^k &\geq 0; \forall i_k, j_k, k = \overline{1, N}. \end{aligned}$$

The minimum transportation cost is:

$$\min Z = \sum_{k=1}^N Z^k.$$

3.3 Multistage classical TP of the Third Kind

This case describes multistage TPs with additional constraints at intermediate stages, without altering the basic transportation structure at each stage. The model is formulated as:

(MTPP3): Min Z = The objective function of MTPP2
Subject to

$$\begin{aligned} \sum_{j_1=1}^{n_1} x_{i_1 j_1}^1 &= d_{i_1}^1, i_1 = 1, 2, \dots, m_1, \\ \sum_{i_1=1}^{m_1} x_{i_1 j_1}^1 &= e_{j_1}^1, j_1 = 1, 2, \dots, n_1, \\ &\dots \\ \sum_{j_k=1}^{n_k} x_{i_k j_k}^k &= d_{i_k}^k, i_k = 1, 2, \dots, m_k, \\ \sum_{i_k=1}^{m_k} x_{i_k j_k}^k &= e_{j_k}^k, j_k = 1, 2, \dots, n_k, \\ &\dots \end{aligned}$$

$$\begin{aligned} \sum_{j_N=1}^{n_N} x_{i_N j_N}^N &= d_{i_N}^N, i_k = 1, 2, \dots, m_N, \\ \sum_{i_N=1}^{m_N} x_{i_N j_N}^N &= e_{j_N}^N, j_N = 1, 2, \dots, n_N, \\ F_{s_k}(x_{i_{k-1} j_{k-1}}^{k-1}, x_{i_k j_k}^k, x_{i_{k+1} j_{k+1}}^{k+1}) &= 0, \\ x_{i_k j_k}^k &\geq 0; \forall i_k, j_k, k = \overline{1, N} \end{aligned}$$

where $F_{s_k}, k = 1, \dots, N$ denotes linear constraint functions at the stage N , and s_k is the number of functions at k . This problem can be efficiently solved using decomposition techniques that exploit the structure of transportation models.

3.4 Multistage Classical TP of the Fourth Kind

This case assumes that the difference between input and output flow at each intermediate stage is known, which modifies the structure of each stage. The model is:

(MTPP4): Min Z = The objective function of MTPP2

Subject to:

$$\begin{aligned} \sum_{j_1=1}^{n_1} x_{i_1 j_1}^1 &= d_{i_1}^1, i_1 = 1, 2, \dots, m_1 \\ \sum_{i_1=1}^{m_1} x_{i_1 j_1}^1 - \sum_{j_2=1}^{n_2} x_{i_2 j_2}^2 &= e_{j_1}^1, \\ j_1 &= 1, 2, \dots, n_1; i_2 = 1, 2, \dots, m_2, \dots \\ \sum_{i_k=1}^{m_k} x_{i_{k-1} j_{k-1}}^{k-1} - \sum_{j_k=1}^{n_k} x_{i_k j_k}^k &= e_{j_{k-1}}^{k-1}, \\ j_{k-1} &= 1, 2, \dots, n_{k-1}; i_k = 1, 2, \dots, m_k; \\ \sum_{i_{N-1}=1}^{m_{N-1}} x_{i_{N-1} j_{N-1}}^{N-1} - \sum_{j_N=1}^{n_N} x_{i_N j_N}^N &= e_{j_{N-1}}^{N-1}, \\ j_{N-1} &= 1, 2, \dots, n_{N-1}; i_N = 1, 2, \dots, N; \\ \sum_{i_N=1}^{m_N} x_{i_N j_N}^N &= e_{j_N}^N, j_N = 1, 2, \dots, n_N; \\ x_{i_k j_k}^k &\geq 0; \forall i_k, j_k, k = \overline{1, N}. \end{aligned}$$

This formulation can be solved using the simplex method.

In this paper, we consider MTPP3 under uncertainty, where all parameters are expressed as SVTrN:

$$\tilde{Z} = \text{Min } Z^k = \sum_{i_k=1}^{m_k} \sum_{j_k=1}^{n_k} \tilde{c}_{i_k j_k}^k x_{i_k j_k}^k$$

Subject to

$$\begin{aligned} \sum_{j_1=1}^{n_1} x_{i_1 j_1}^1 &= \tilde{d}_{i_1}^1, i_1 = 1, 2, \dots, m_1, \\ \sum_{i_1=1}^{m_1} x_{i_1 j_1}^1 &= \tilde{e}_{j_1}^1, j_1 = 1, 2, \dots, n_1, \end{aligned}$$

$$\begin{aligned} \sum_{j_k=1}^{n_k} x_{i_k j_k}^k &= \tilde{d}_{i_k}^k, i_k = 1, 2, \dots, m_k, \\ \sum_{i_k=1}^{m_k} x_{i_k j_k}^k &= \tilde{e}_{j_k}^k, j_k = 1, 2, \dots, n_k, \end{aligned}$$

(1)

$$\begin{aligned} \sum_{j_N=1}^{n_N} x_{i_N j_N}^N &= \tilde{d}_{i_N}^N, i_k = 1, 2, \dots, m_N, \\ \sum_{i_N=1}^{m_N} x_{i_N j_N}^N &= \tilde{e}_{j_N}^N, j_N = 1, 2, \dots, n_N, \\ F_{s_k}(x_{i_{k-1} j_{k-1}}^{k-1}, x_{i_k j_k}^k, x_{i_{k+1} j_{k+1}}^{k+1}) &= 0, \end{aligned}$$

$$x_{ijk}^k \geq 0; \forall i_k, j_k, k = \overline{1, N}.$$

Using the ranking function of SVTrN, the problem (1) is transformed into

$$\text{Min } d^k = \sum_{i_k=1}^{m_k} \sum_{j_k=1}^{n_k} Q_{i_k j_k}^k x_{i_k j_k}^k,$$

Subject to:

$$\begin{aligned} \sum_{j_1=1}^{n_1} x_{i_1 j_1}^1 &= \zeta_{i_1}^1, i_1 = 1, 2, \dots, m_1, \\ \sum_{i_1=1}^{m_1} x_{i_1 j_1}^1 &= \psi_{j_1}^1, j_1 = 1, 2, \dots, n_1, \\ \sum_{j_k=1}^{n_k} x_{i_k j_k}^k &= \zeta_{i_k}^k, i_k = 1, 2, \dots, m_k, \\ \sum_{i_k=1}^{m_k} x_{i_k j_k}^k &= \psi_{j_k}^k, j_k = 1, 2, \dots, n_k, \end{aligned}$$

$$\sum_{j_N=1}^{n_N} x_{i_N j_N}^N = \zeta_{i_N}^N, i_N = 1, 2, \dots, m_N, \quad (2)$$

$$\sum_{i_N=1}^{m_N} x_{i_N j_N}^N = \psi_{j_N}^N, j_N = 1, 2, \dots, n_N,$$

$$F_{S_k}(x_{i_{k-1} j_{k-1}}^{k-1}, x_{i_k j_k}^k, x_{i_{k+1} j_{k+1}}^{k+1}) = 0,$$

$$x_{ijk}^k \geq 0; \forall i_k, j_k, k = \overline{1, N}$$

4 Solution Scheme

The multistage TP (particularly Type III) can be efficiently solved using the decomposition technique of LP.

Let D^k constraint coefficient matrix of the k -th subproblem, A^k linking constraint matrix for the stage k , $\tilde{b}$ SVTrN vector of tie-in constraints, $\tilde{b}^k$ SVTrN vector of supplies and demands at the stage k , R_0 matrix formed by the first m_0 columns of B^{-1} , where m_0 is the size of $\tilde{b}$, $\tilde{c}^k$ SVTrNN cost vector of stage k , c_B vector of basic variable costs. Assume an initial basic feasible solution $(v_1, v_2, \dots, v_{o+1})$ with the basis matrix B of order $m_0 + N$.

Algorithm

Step 1. Solve each subproblem

$$w^k = (c^k - c_B R_0 A^k) x^k$$

Subject to

$$D^k x^k = b^k, x^k \geq 0, k = 1, 2, \dots, N. \quad (3)$$

Let $w^{(R_k)}$ be the optimal value at x^k (using LINGO 20.0). Proceed to Step 2.

Step 2. Compute the reduced costs

$$\sigma^k = w^{(R_k)} - c_B \tau^k,$$

where τ^k is the $(m_0 + k)$ th column of B^{-1} . Then

$$\sigma = \min_k \sigma^k$$

- If $\sigma^k \geq 0$ for all k , the current solution is optimal $\rightarrow$ Stop.
- Otherwise, proceed to Step 3.

Step 3. Introduce the entering variable ω^k corresponding to the most σ . Determine the leaving variable using feasibility conditions, update B^{-1} via the revised simplex method and repeat from Step 1. The general process is presented in Figure 2.

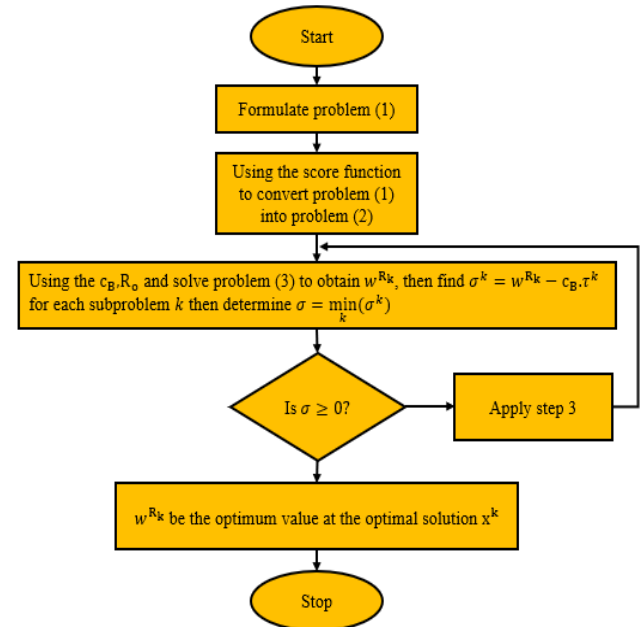


Fig. 2: The proposed solution procedure's framework

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5 Numerical Example

Consider the following two-stage TP with data illustrated in Table 1, Table 2, Table 3 and Table 4.

Table 1. Neutrosophic Data for 1st Stage TP

Warehouses	customers	supply	
	D_1	D_2	
S_1	$x_{11}^1(\tilde{c}_{11}^{1SVTrN})$	$x_{12}^1(\tilde{c}_{12}^{1SVTrN})$	$\tilde{d}_1^{1SVTrN}$
S_2	$x_{21}^1(\tilde{c}_{21}^{1SVTrN})$	$x_{22}^1(\tilde{c}_{22}^{1SVTrN})$	$\tilde{d}_2^{1SVTrN}$
S_3	$x_{31}^1(\tilde{c}_{31}^{1SVTrN})$	$x_{32}^1(\tilde{c}_{32}^{1SVTrN})$	$\tilde{d}_3^{1SVTrN}$
Demand	$\tilde{e}_1^{1SVTrN}$	$\tilde{e}_2^{1SVTrN}$	

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Table 2. Neutrosophic Data for 2nd Stage TP

Warehouses	Customers		Supply	
k_1	k_1	k_1		
D_1	$x_{11}^2(\tilde{c}_{11}^{2SVTrN})$	$x_{12}^2(\tilde{c}_{12}^{2SVTrN})$	$x_{13}^2(\tilde{c}_{13}^{2SVTrN})$	$\tilde{d}_1^{2SVTrN}$
D_2	$x_{21}^2(\tilde{c}_{21}^{2SVTrN})$	$x_{22}^2(\tilde{c}_{22}^{2SVTrN})$	$x_{23}^2(\tilde{c}_{23}^{2SVTrN})$	$\tilde{d}_2^{2SVTrN}$
Demand	$\tilde{e}_1^{2SVTrN}$	$\tilde{e}_2^{2SVTrN}$	$\tilde{e}_3^{2SVTrN}$	

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Where,

$$\tilde{c}_{11}^{1SVTrN} = \langle (3, 5, 6, 8); 0.6, 0.5, 0.4 \rangle,$$

$$\begin{aligned}\tilde{c}_{12}^{1SVTrN} &= \langle (4, 8, 11, 15); 0.6, 0.3, 0.2 \rangle, \\ \tilde{c}_{21}^{1SVTrN} &= \langle (0, 1, 3, 6); 0.7, 0.5, 0.3 \rangle, \\ \tilde{c}_{22}^{1SVTrN} &= \langle (9, 11, 14, 16); 0.5, 0.4, 0.7 \rangle, \\ \tilde{c}_{31}^{1SVTrN} &= \langle (15, 17, 19, 22); 0.4, 0.8, 0.4 \rangle, \\ \tilde{c}_{32}^{1SVTrN} &= \langle (5, 8, 10, 14); 0.3, 0.6, 0.6 \rangle, \\ \tilde{c}_{11}^{2SVTrN} &= \langle (1, 3, 4, 6); 0.6, 0.3, 0.5 \rangle, \\ \tilde{c}_{12}^{2SVTrN} &= \langle (5, 8, 10, 14); 0.3, 0.6, 0.6 \rangle, \\ \tilde{c}_{13}^{2SVTrN} &= \langle (15, 17, 19, 22); 0.4, 0.8, 0.4 \rangle, \\ \tilde{c}_{21}^{2SVTrN} &= \langle (4, 8, 11, 15); 0.6, 0.3, 0.2 \rangle, \\ \tilde{c}_{22}^{2SVTrN} &= \langle (3, 5, 6, 8); 0.6, 0.5, 0.4 \rangle, \\ \tilde{c}_{23}^{2SVTrN} &= \langle (9, 11, 14, 16); 0.5, 0.4, 0.7 \rangle, \\ \tilde{a}_1^{1SVTrN} &= \langle (15, 17, 19, 22); 0.4, 0.8, 0.4 \rangle, \\ \tilde{a}_2^{1SVTrN} &= \langle (9, 11, 14, 16); 0.5, 0.4, 0.7 \rangle, \\ \tilde{a}_3^{1SVTrN} &= \langle (3, 5, 6, 8); 0.6, 0.5, 0.4 \rangle, \\ \tilde{a}_1^{2SVTrN} &= \langle (12, 15, 20, 25); 0.8, 0.2, 0.6 \rangle, \\ \tilde{a}_2^{2SVTrN} &= \langle (5, 9, 14, 19); 0.3, 0.7, 0.6 \rangle, \\ \tilde{b}_1^{1SVTrN} &= \langle (12, 15, 20, 25); 0.8, 0.2, 0.6 \rangle, \\ \tilde{b}_2^{1SVTrN} &= \langle (5, 9, 14, 19); 0.3, 0.7, 0.6 \rangle, \\ \tilde{b}_1^{2SVTrN} &= \langle (4, 8, 11, 15); 0.6, 0.3, 0.2 \rangle, \\ \tilde{b}_2^{2SVTrN} &= \langle (5, 7, 8, 10); 0.5, 0.4, 0.7 \rangle, \\ \tilde{b}_3^{2SVTrN} &= \langle (9, 11, 14, 16); 0.5, 0.4, 0.7 \rangle.\end{aligned}$$

Table 3. Crisp Data for 1st Stage TP

Warehouses	Customers	Supply
D ₁	D ₂	
S ₁ x ₁₁ ¹ (2)	x ₁₂ ¹ (5)	6
S ₂ x ₂₁ ¹ (1)	x ₂₂ ¹ (4)	4
S ₃ x ₃₁ ¹ (6)	x ₃₂ ¹ (3)	2
Demand 9	3	12

Source: created by the authors

Table 4. Crisp Data for 2nd Stage TP

Warehouses	Customers	
k ₁	k ₁	k ₁
D ₁ x ₁₁ ² (2)	x ₁₂ ² (3)	x ₁₃ ² (6)
D ₂ x ₂₁ ² (5)	x ₂₂ ² (2)	x ₂₃ ² (4)
Demand 5	3	4
12		

Source: created by the authors

The mathematical model for the problem is

$$\begin{aligned}Z = & 2x_{11}^1 + 5x_{12}^1 + x_{21}^1 + 4x_{22}^1 + 6x_{31}^1 + 3x_{32}^1 \\ & + 2x_{11}^2 + 5x_{21}^2 + 3x_{12}^2 + 2x_{22}^2 \\ & + 6x_{13}^2 + 4x_{23}^2\end{aligned}$$

Subject to:

$$\begin{aligned}x_{11}^1 + x_{12}^1 &= 6, x_{21}^1 + x_{22}^1 = 4, \\ x_{31}^1 + x_{32}^1 &= 2, x_{11}^1 + x_{21}^1 + x_{31}^1 = 9, \\ x_{12}^1 + x_{22}^1 + x_{32}^1 &= 2, x_{11}^2 + x_{21}^2 = 5,\end{aligned}$$

$$\begin{aligned}x_{12}^2 + x_{22}^2 &= 3, x_{13}^2 + x_{23}^2 = 4, \\ x_{11}^2 + x_{12}^2 + x_{13}^2 &= 9, \\ x_{21}^2 + x_{22}^2 + x_{23}^2 &= 3, \\ x_{ij}^1 &\geq 0 \text{ and } x_{ij}^2 \geq 0.\end{aligned}$$

Where,

$$\begin{aligned}x^2 &= x_{11}^2 \quad x_{21}^2 \quad x_{12}^2 \quad x_{22}^2 \quad x_{13}^2 \quad x_{23}^2 \\ x^1 &= x_{11}^1 \quad x_{12}^1 \quad x_{21}^1 \quad x_{22}^1 \quad x_{31}^1 \quad x_{32}^1 \\ c^1: & 2 \quad 5 \quad 1 \quad 4 \quad 6 \quad 3, \\ c^2: & 2 \quad 5 \quad 3 \quad 2 \quad 6 \quad 4\end{aligned}$$

A ¹	1 0 0 0 0 0	A ²	1 0 0 0 1 0
D ¹	1 1 0 0 0 0 0 0 1 1 0 0 1 0 0 0 1 1 1 0 1 0 1 0 0 1 0 1 0 1		
		D ²	1 1 0 0 0 0 0 0 1 1 0 0 1 0 0 0 1 1 1 0 1 0 1 0 0 1 0 1 0 1

b	0
b ¹	6
	4
	2
	9
	3
b ²	5
	3
	4
	9
	3

The starting basic solution is given by $x_b = [v_1 \ v_2 \ v_3]$, $b = [0 \ 1 \ 1]$, and the corresponding basic matrix is

$$\begin{aligned}B &= [1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1] = B^{-1}, \\ c_B &= [1 \ 1 \ 1], R_0 = [1 \ 0 \ 0], \tau^1 = [1 \ 0 \ 0], \\ \tau^2 &= [0 \ 0 \ 1], c^1 = [0 \ 0 \ 0 \ 0 \ 0], c^2 = [0 \ 0 \ 0 \ 0 \ 0]\end{aligned}$$

First iteration

Step 1: For $k = 1, 2$

$$w^{x_1} = -6 \quad x_1^{x_1} = (6, 0, 3, 1, 0, 2)^T,$$

$$w^{x_2} = 1, \quad x_1^{x_2} = (5, 0, 3, 0, 1, 3)^T$$

Step 2: For $k = 1, 2$

$$\sigma^1 = -7, \quad \sigma^2 = 0. \text{ Then}$$

$$\sigma = (\sigma^1, \sigma^2) = -7 \text{ corresponding to } k = 1.$$

Step 3: The variable ω_1^1 corresponding to $x_1^{x_1}$ must enter the solution and the leaving variable v_1 .

Now, to determine the new basic solution:

$$x_b = [\omega_1^1 \ v_2 \ v_3], \quad B_{\text{new}}^{-1} = \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{6} & 1 & 0 & 0 & 1 \\ 6 & & & & & & & \end{bmatrix},$$

$$B^{-1}b = [0 \ 1 \ 1], \quad c_B = [0 \ 1 \ 1], \quad R_0 = \begin{bmatrix} 1 & -1 & 0 \\ 6 & & 0 \end{bmatrix}, \quad \tau^1 = [0 \ 1 \ 0], \quad \tau^2 = [0 \ 0 \ 1].$$

Second iteration

Step 1: For $k = 1, 2$

$$w^{x_1} = \frac{1}{2}, \quad x_2^{x_1} = (3, 3, 4, 0, 2, 0)^T, \quad w^{x_2} = -\frac{2}{3}, \quad x_2^{x_2} = (5, 0, 0, 3, 4, 0)^T$$

Step 2: For $k = 1, 2$

$$\sigma^1 = -\frac{1}{2}, \quad \sigma^2 = -\frac{5}{3}. \text{ Then}$$

$$\sigma = (\sigma^1, \sigma^2) = -\frac{5}{3}, \text{ corresponding to } k = 2.$$

Step 3: The variable ω_2^2 corresponding to $x_2^{x_2}$ must enter the solution and the leaving variable v_3 .

Now, to determine the new basic solution:

$$x_b = [\omega_1^1 \ v_2 \ \omega_2^2]$$

$$B_{\text{new}}^{-1} = \begin{bmatrix} 1 & 2 & -1 \\ 6 & 0 & 3 & -\frac{1}{6} & 1 & -\frac{2}{3} & 0 & 0 & 1 \end{bmatrix}$$

$$B^{-1}b = \begin{bmatrix} 2 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$c_B = [0 \ 1 \ 1]$$

$$R_0 = \begin{bmatrix} 1 & -1 \\ 6 & 6 & 0 \end{bmatrix}$$

$$\tau^1 = [0 \ 1 \ 0]$$

$$\tau^2 = \begin{bmatrix} 2 & -2 \\ 3 & 3 & 1 \end{bmatrix}.$$

Third iteration

Step 1: For $k = 1, 2$

$$w^{x_1} = \frac{1}{2}, \quad x_3^{x_1} = (3, 3, 4, 0, 2, 0)^T, \quad w^{x_2} = -\frac{2}{3}, \quad x_3^{x_2} = (5, 0, 0, 3, 4, 0)^T$$

Step 2: For $k = 1, 2$

$$\sigma^1 = -\frac{1}{2}, \quad \sigma^2 = 0. \text{ Then}$$

$$\sigma = (\sigma^1, \sigma^2) = -\frac{1}{2}, \text{ corresponding to } k = 1.$$

Step 3: The variable ω_3^1 corresponding to $x_3^{x_1}$ must enter the solution and the leaving variable v_2 .

Now, to determine the new basic solution:

$$x_b = [\omega_1^1 \ \omega_3^1 \ \omega_2^2],$$

$$B_{\text{new}}^{-1} = \begin{bmatrix} 1 & -1 & 4 & -1 \\ 3 & 3 & 3 & -\frac{1}{3} & 2 & -\frac{4}{3} & 0 & 0 & 1 \end{bmatrix},$$

$$B^{-1}b = \begin{bmatrix} 1 & 2 \\ 3 & 3 & 1 \end{bmatrix},$$

$$c_B = [0 \ 0 \ 0],$$

$$R_0 = \begin{bmatrix} 1 & -1 \\ 3 & -\frac{1}{3} & 0 \end{bmatrix},$$

$$\tau^1 = [-1 \ 2 \ 0],$$

$$\tau^2 = \begin{bmatrix} 4 & -4 \\ 3 & 3 & 1 \end{bmatrix}.$$

Fourth iteration

Step 1: For $k = 1, 2$

$$w^{x_1} = 0, \quad w^{x_2} = 0.$$

Step 2: For $k = 1, 2$

$$\sigma^1 = 0, \quad \sigma^2 = 0. \text{ Then}$$

$\sigma = (\sigma^1, \sigma^2) = 0$. Hence, the current solution for phase one is finished.

Phase two procedure:

$$c_B = [1 \ 1 \ 1], \quad c^1 = [0 \ 0 \ 0 \ 0 \ 0],$$

$$c^2 = [0 \ 0 \ 0 \ 0 \ 0].$$

Step 1: For $k = 1, 2$

$$w^{x_1} = 45, \quad x_4^{x_1} = (5, 1, 4, 0, 0, 2)^T, \quad w^{x_2} = 24,$$

$$x_4^{x_2} = (5, 0, 0, 3, 4, 0)^T$$

Step 2: For $k = 1, 2$

$$\sigma^1 = -4, \quad \sigma^2 = 0. \text{ Then}$$

$$\sigma = (\sigma^1, \sigma^2) = -4, \text{ corresponding to } k = 1.$$

Step 3: The variable ω_4^1 corresponding to $x_4^{x_1}$ must enter the solution and the leaving variable ω_1^1 .

Now, to determine the new basic solution:

$$x_b = [\omega_4^1 \ \omega_3^1 \ \omega_2^2]$$

$$B_{\text{new}}^{-1} = \begin{bmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & \frac{5}{2} \\ 2 & \frac{3}{2} & \frac{5}{2} & -2 & 0 & 0 & 1 \end{bmatrix}$$

$$B^{-1}b = \begin{bmatrix} 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

$$c_B = [25 \ 37 \ 40]$$

$$R_0 = \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ 2 & -\frac{1}{2} & 0 \end{bmatrix}$$

$$\tau^1 = \begin{bmatrix} -\frac{3}{2} & \frac{5}{2} & 0 \end{bmatrix}, \quad \tau^2 = [2 \ -2 \ 1].$$

Fifths iteration

Step 1: For $k = 1, 2$

$$w^{x_1} = 55, \quad x_5^{x_1} = (5, 1, 4, 0, 0, 2)^T,$$

$$w^{x_2} = 16, \quad x_5^{x_2} = (5, 0, 0, 3, 4, 0)^T$$

Step 2: For $k = 1, 2$

$$\sigma^1 = 0, \quad \sigma^2 = 0. \text{ Then}$$

$$\sigma = (\sigma^1, \sigma^2) = 0.$$

Therefore, optimality is achieved at the current iteration, and the algorithm stops. The resulting optimal solution is presented as:

$$x^1 = x_4^{x_1} \omega_4^1 + x_3^{x_1}$$

$$\omega_3^1 = (4, 2, 4, 0, 1, 1)^T$$

$$x^2 = x_2^{x_2} \omega_2^2 = (5, 0, 0, 3, 4, 0)^T, \text{ and}$$

$$\tilde{Z}^{\text{SVTrN}} = c^1 x^1 + c^2 x^2$$

$$= c_{i_1 j_1}^1 x_{i_1 j_1}^1 + c_{i_1 j_1}^2 x_{i_1 j_1}^2$$

$$= \langle (114, 163, 201, 264); 0.3, 0.8, 0.6 \rangle.$$

6 Discussion of the Results

The optimal neutrosophic transportation cost is obtained as:

$$\tilde{Z}^{SVTrN} = \langle (114, 163, 201, 264); 0.3, 0.8, 0.6 \rangle.$$

This result indicates that the total minimum transportation cost ranges from 114 to 264, with the most plausible values lying within the interval [163, 201]. The associated truth-membership degree is 0.3, reflecting the level of confidence in this estimate, while the corresponding indeterminacy and falsity degrees are 0.8 and 0.6, respectively.

$$\omega_{\tilde{p}}(x) = \begin{cases} 0.3 \frac{x-114}{49}, & 114 \leq x \leq 163, \\ 0.3, & 163 \leq x \leq 201, \\ 0.3 \frac{264-x}{63}, & 201 \leq x \leq 264, \\ 0, & \text{otherwise} \end{cases}$$

$$\phi_{\tilde{p}}(x) = \begin{cases} \frac{163-x+0.8(x-114)}{49}, & 114 \leq x \leq 163, \\ 0.8, & 163 \leq x \leq 201, \\ \frac{x-201+0.8(264-x)}{63}, & 201 \leq x \leq 264, \\ 0, & \text{otherwise} \end{cases}$$

$$\rho_{\tilde{p}}(x) = \begin{cases} \frac{163-x+0.8(x-114)}{49}, & 114 \leq x \leq 163, \\ 0.8, & 163 \leq x \leq 201, \\ \frac{x-201+0.8(264-x)}{63}, & 201 \leq x \leq 264, \\ 0, & \text{otherwise} \end{cases}$$

Thus, the decision-maker can estimate the total neutrosophic transportation cost within the interval [364, 908], along with its associated truth, indeterminacy, and falsity degrees. Based on this information, appropriate transportation planning and budget allocation decisions can be made.

7 Conclusions

This work developed new formulations of multistage transportation problems within a neutrosophic environment, where all parameters are signified by SVTrN. A SF was employed to convert the neutrosophic model into an equivalent crisp form, which facilitated a decomposition-based solution strategy. The proposed approach preserves the structural properties of transportation problems at each stage and efficiently applies classical techniques for large-scale linear programming. Moreover, the stability set of the first kind consistent to the obtained solution was derived. Future studies may extend this framework to other uncertainty models such as interval-valued FSs, intuitionistic FSs, Pythagorean FSs, spherical FSs, and picture FSs, with further

emphasis on real-world applications and advanced decision-making environments.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Hamiden Khalifa conceptualized the study, supervised the overall research process, provided critical revisions to improve the manuscript, contributed to data collection, experimental design, validation of results, and assisted in data curation, literature review, and proofreading of the final version.
- Sultan Alodhaibi participated in model development, formal analysis, and manuscript drafting.
- Hissah Ibrahim Almuzini contributed to methodology formulation, result interpretation, and visualization.

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