

Using a jump-diffusion process to estimate return-period flows

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Abstract: To estimate return-period flows in hydrology, a model based on a jump-diffusion process is used. In addition to the four model parameters, the dispersion parameter is also estimated for periods of rapid increase or decrease in flow. Simulations of the stochastic process are then performed. Using these simulations, the probabilities of exceeding significant flow values are estimated. Return-period flow estimates are obtained by interpolation or extrapolation, and are compared with the corresponding values calculated by hydrologists for the Delaware River in the United States.

Key-Words: Brownian motion, Poisson process, estimation, climate change.

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1 Introduction

Let $X(t)$ denote the flow of a river at time t , where t is in days. In hydrology, a return period for a river is the average time required for a given flow (or water level) to recur. By definition, the flow R_n that corresponds to a value that occurs at least once in n years is such that the probability of exceedance of R_n (at least once) within a year, which is usually expressed in percentages, is equal to $1/n$. Hence, we can write that

$$P \left[\max_{0 \leq t \leq 365} X(t) > R_n \right] = \frac{1}{n}. \quad (1)$$

Return-period flows are important for the design of dams and bridges, as well as for risk analysis. Consequently, many papers have been written on their computation; see, for example, [1] and, [2], as well as the review paper, [3].

The method used by hydrologists to estimate return-period flows is based on long-term historical data. However, due to climate change, it is likely that data collected several decades ago are no longer reliable; see, [4], [5] and, [6]. Moreover, an assumption that is generally made when estimating return-period flows is that they do not vary over time and do not depend on past events. According to, [7], these assumptions are now questionable.

In this paper, we will propose a new technique to estimate the return-period flows by first modelling $X(t)$ as a jump-diffusion process. This type of stochastic process is often used in financial mathematics. The author has shown in, [8] that they can also serve as models for variations in river flows over time.

With the method that we propose, it is sufficient to use data from a few (recent) years to estimate return-period flows. Moreover, it is easy to update the estimated return-period flows when new data become available. It is also possible to observe the potential effects of climate change by modifying one or more parameters of the model.

Of course, a longer period, such as 20 years of data, can be used when implementing the proposed technique. However, it is not necessary to use data collected perhaps more than 100 years ago, the accuracy of which may sometimes be questionable. Furthermore, the proposed method should be seen as a complement to those used by hydrologists, not as a replacement for them.

In the next section, the mathematical background will be presented. In Section 3, the problem of estimating the model parameters will be considered and the technique to obtain point estimates of the return-period flows will be explained. Next, in Section 4, an application to the Delaware River, located in the United States, will be presented. We will compare our estimates with the corresponding ones calculated by hydrologists, as well as with the estimates obtained by the author in, [9] by making use of filtered Poisson processes.

2 Jump-diffusion processes

A jump-diffusion process $\{X(t), t \geq 0\}$ can be defined as follows:

$$X(t) = X(0) + \int_0^t m[X(s)]ds \quad (2) \\ + \int_0^t \{v[X(s)]\}^{1/2} dB(s) + \sum_{i=1}^{N(t)} Y_i,$$

where $m(\cdot) \in \mathbb{R}$, $v(\cdot) > 0$, $\{B(t), t \geq 0\}$ is a standard Brownian motion, $\{N(t), t \geq 0\}$ is a Poisson process with rate λ , and $Y_1, Y_2, \dots$ are independent random variables that have the same distribution as the random variable Y . Furthermore, the processes $\{B(t), t \geq 0\}$ and $\{N(t), t \geq 0\}$ are assumed to be independent. The functions $m(\cdot)$ and $v(\cdot)$ must be such that the continuous part of the stochastic process is a (pure) diffusion process, which satisfies the stochastic differential equation

$$dX(t) = m[X(t)]dt + \{v[X(t)]\}^{1/2} dB(t). \quad (3)$$

In this paper, we assume that $m(x) \equiv \mu \in \mathbb{R}$ and $v(x) \equiv \sigma^2$. Then, continuous part of $\{X(t), t \geq 0\}$ is a Wiener process (or Brownian motion) with drift μ and dispersion parameter σ (> 0). We will see in the Section 4 that this assumption is realistic in the application presented. We also assume that $Y_1, Y_2, \dots$ are independent random variables that are all exponentially distributed with parameter α , and that are independent of the Poisson process. The discrete part of $\{X(t), t \geq 0\}$, that is $\sum_{i=1}^{N(t)} Y_i$, is a compound Poisson process.

The parameter λ of the Poisson process is the rate at which hydrological events occur, and the average size of the flow increase caused by an event of the Poisson process is equal to $1/\alpha$.

There are four parameters in the model: μ , σ , λ and α . In, [8], the author explained how it is possible to estimate these parameters when we have real observations of the flow $X(t)$. Once the parameters have been estimated, we could try to compute the probability (see Eq. (1)) $P[\max_{0 \leq t \leq 365} X(t) > \xi]$ in order to determine the value of R_n . However, we assumed that the functions $m(x)$ and $v(x)$ do not vary over time. In reality, these functions depend on time. In particular, the parameter σ , known as the *volatility parameter* in financial mathematics, is certainly much larger during the period following the occurrence of an event of the Poisson process. Indeed, since the units of σ and those of $X(t)$ are the same, the larger $X(t)$ is, the larger σ should also be. Therefore, in order to obtain better point estimates of the return-period flows, we will consider the periods when the value of $X(t)$ increased or decreased very

rapidly, and we will calculate the point estimate of σ for these periods.

Remark. As mentioned above, the author used filtered Poisson processes in, [9] to produce point estimates of return-period flows. A filtered Poisson process is a continuous-time stochastic process $\{Z(t), t \geq 0\}$ defined by (see, [10])

$$Z(t) = Z(0) + \sum_{n=1}^{N(t)} w(t, \tau_n, Y_n), \quad (4)$$

where $\tau_1, \tau_2, \dots$ are the arrival times of the events of the Poisson process $\{N(t), t \geq 0\}$, and $w(t, \tau_n, Y_n)$ is the *response function*. It is therefore a generalization of a compound Poisson process.

In an application to hydrology, an appropriate response function w is

$$w(t, \tau_n, Y_n) = Y_n e^{-(t-\tau_n)/c}, \quad (5)$$

where c is a parameter (that must be estimated) that depends on the rate at which the effect of an event on the river flow decreases.

A filtered Poisson process can be generalized by assuming that the process $\{N(t), t \geq 0\}$ is a renewal process. The stochastic process $\{Z(t), t \geq 0\}$ would then be a filtered renewal process.

Although the author made use of filtered Poisson or renewal processes with success in various hydrological applications, it can be argued that jump-diffusion processes are more realistic for modelling river flows, as filtered processes behave deterministically between events, which is not the case in practice.

3 Estimation of the model parameters and of the return-period flows

Here is a summary of the method presented in, [8] to estimate the model parameters: let

$$X_0(t) := X(t) - X(0). \quad (6)$$

We find that

$$E[X_0(1)] = \mu + \frac{\lambda}{\alpha}, \quad (7)$$

$$E[X_0^2(1)] = \mu^2 + \sigma^2 + \frac{\lambda(\lambda + 2)}{\alpha^2} + \frac{2\mu\lambda}{\alpha}, \quad (8)$$

$$E[X_0^3(1)] = \mu^3 + 3\mu\sigma^2 + \frac{\lambda(\lambda^2 + 6\lambda + 6)}{\alpha^3} + \frac{3\mu\lambda(\lambda + 2)}{\alpha^2} + \frac{3\lambda(\mu^2 + \sigma^2)}{\alpha}, \quad (9)$$

$$E[X_0^4(1)] = \mu^4 + 6\mu^2\sigma^2 + 3\sigma^4 + \frac{4\mu\lambda(\lambda^2 + 6\lambda + 6)}{\alpha^3} + \frac{6\lambda(\mu^2 + \sigma^2)(\lambda + 2)}{\alpha^2} + \frac{4\mu\lambda(\mu^2 + 3\sigma^2)}{\alpha} + \frac{\lambda(\lambda^3 + 12\lambda^2 + 36\lambda + 24)}{\alpha^4}. \quad (10)$$

Now, suppose that we have a set of data points $x_1, \dots, x_n$. The point estimate of $\mu_k := E[X_0^k(1)]$, for $k = 1, 2, 3, 4$, is given by

$$\hat{\mu}_k := \sum_{i=1}^{n-1} \frac{y_i^k}{n-1}, \quad (11)$$

where $y_i := x_{i+1} - x_i$ for $i = 1, \dots, n-1$.

We can make use of the method of moments: we set $\hat{\mu}_k = \mu_k$ for each value of k . Then, in theory, we can obtain point estimates of the four model parameters by solving (numerically) the system of four non-linear equations.

Another possibility is the following:

1. Equation (7) and the equation $\hat{\mu}_1 = \mu_1$ imply that

$$\mu = \hat{\mu}_1 - \frac{\lambda}{\alpha}. \quad (12)$$

2. Substitute the above expression for μ into the equation $\hat{\mu}_2 = \mu_2$ and solve for σ in terms of λ and α .
3. Express the parameter λ in terms of α by using the equation $\hat{\mu}_3 = \mu_3$.
4. Obtain a polynomial equation (of degree 9) in α from the equation $\hat{\mu}_4 = \mu_4$. If there is a single positive root of this polynomial equation that leads to admissible values for the other three parameters, then they can be calculated by proceeding in reverse order.

Once the four parameters have been estimated, we can simulate the jump-diffusion process $\{X(t), t \geq 0\}$ to determine the probability of exceeding various thresholds. However, as mentioned above, in reality the model parameters vary over time, especially the parameter σ . Therefore, we will consider the days on which the increase or decrease in flow was very large. Based on these observations, we will calculate a new point estimate of the parameter σ , which we denote by σ_1 . In practice, this is necessary to obtain good point estimates of flows corresponding to given return periods, especially those occurring every 100 years, for example, because these very high flows are most likely to occur during rapid increases in flow, when the model parameters change. Even jump-diffusion processes, which are powerful tools, cannot model these phenomena well if their parameters are assumed to be constants.

Next, since we assumed that the hydrological events occur according to a Poisson process with rate λ , we can state that the probability of an event on a given day is given by

$$p := 1 - e^{-\lambda}. \quad (13)$$

Moreover, the probability of two (or more) events is negligible.

Suppose that $\bar{x}$ is the average flow rate of the river of interest over a given time period. Let

- W be a Bernoulli random variable with parameter $\hat{p} := 1 - e^{-\hat{\lambda}}$,
- Z be a Gaussian random variable with parameters $\hat{\mu}$ and $\hat{\sigma}^2$,
- Z_1 be a Gaussian random variable with parameters $\hat{\mu}$ and $\hat{\sigma}_1^2$,
- U be an exponential random variable with parameter $\hat{\alpha}$.

We define

$$X = (1 - W) \times Z + W \times (Z_1 + U). \quad (14)$$

Suppose that we want to estimate the probability that the river flow will exceed the threshold R over a period of one year. We can simulate observations of the random variables W , Z , Z_1 and U for each day and, starting from $\bar{x}$, calculate the flow rate for consecutive days by adding a simulated value of X defined in Eq. (14) each day. We can stop the procedure as soon as a value greater than R has been reached. By repeating this procedure a large number of times, the probability of exceeding R can be easily estimated.

Remark. The above procedure uses the fact that a Wiener process has independent and stationary increments.

In the next section, we will present an application of jump-diffusion processes to estimate the return-period flows for the Delaware River, located in the United States. The point estimates obtained by applying our method will be compared with the estimates calculated by hydrologists.

4 An application to the Delaware River

The observed daily (average) flows in cfs (cubic feet per second) of the Delaware River, which is located in the mid-Atlantic region of the United States, are available on the website <https://nwis.waterdata.usgs.gov/nwis>. The average flow value for the period 2020-2024 at the Montague, N.J., station is equal to 6777.

In Figure 1, we can see the various peaks that occurred during this period. A pure diffusion process cannot reproduce this type of behaviour.

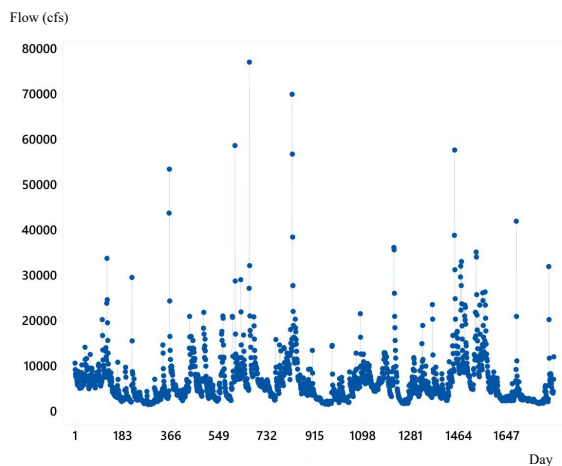


Figure 1: Daily flows of the Delaware River in cfs during the period 2020-2024.

Figure 2 presents the Q-Q plot for the difference $X(t+1) - X(t)$. The normality assumption is clearly rejected. The large gaps in the tails, on both sides, are typically the result of jumps. Moreover, there is an asymmetry in the tails, with the right tail being longer than the left one. This is consistent with rapid increases in flow and is what a model with positive exponential jumps is supposed to produce.

Next, Figures 3, 4, 5, 6 and 7 display the differences $X(t+k) - X(t)$ between the current flow and the flow k days later, for $k = 1, \dots, 5$, when the flow was smaller than or equal to 3000 cfs. When the river flow is low, it should be possible to model it using a diffusion process without jumps, as we can conclude that there are no significant jumps during this period. The curve superimposed on the

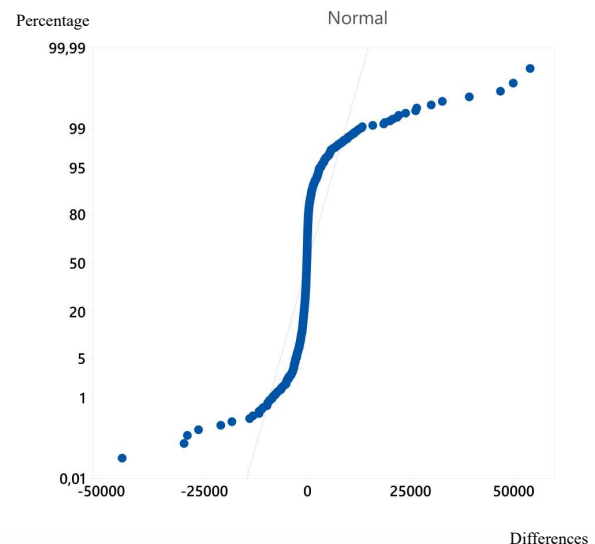


Figure 2: Q-Q plot for the difference $X(t+1) - X(t)$ of the daily flows of the Delaware River in cfs during the period 2020-2024.

histograms in these figures is the probability density function of a Gaussian random variable that possesses the same mean and variance as the differences $X(t+k) - X(t)$, for each value of k .

Although the normality assumption must once again be rejected, we see that the differences increasingly resemble a Gaussian distribution as k increases. Furthermore, it must be taken into account that the samples are large. As is well known in statistics, by taking enough observations, any model will be rejected. Therefore, it is reasonable to accept that fluctuations in daily flow can be modelled as a Gaussian process. This is a necessary compromise in order to obtain a model that is mathematically tractable. Finally, there is no simple formula for the probability density function of a jump-diffusion process. It can be expressed as an infinite sum of convolutions. Consequently, it is difficult to test the fit of the model to the data for all non-negative values of the river flow.

To verify that the proposed model applies to other rivers, we performed the same analysis for the Mississippi River, also located in the United States. Figure 8 shows the river flow during the period 2020-2024, Figure 9 presents the Q-Q plot for the differences $X(t+1) - X(t)$, and Figures 10 and 11 display the histograms of $X(t+1) - X(t)$ and $X(t+5) - X(t)$, respectively, when $X(t) \leq 10,000$ cfs.

The results are quite similar to those for the Delaware River, but there are a few differences. First, according to a goodness-of fit statistical test, the

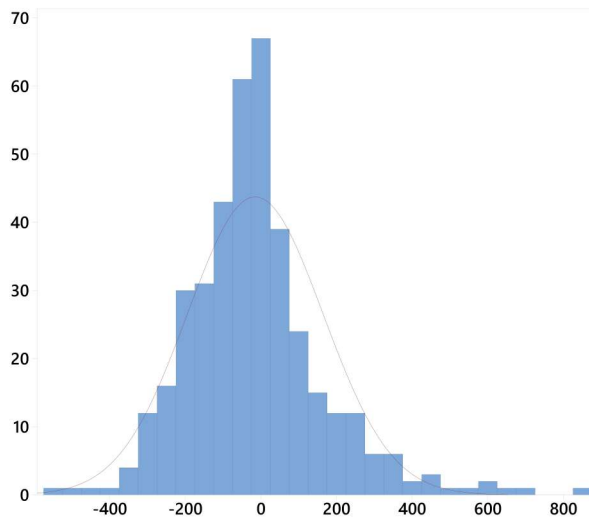


Figure 3: Histogram of the difference $X(t+1) - X(t)$ for the Delaware River when $X(t) \leq 3000$ cfs, during the period 2020-2024.

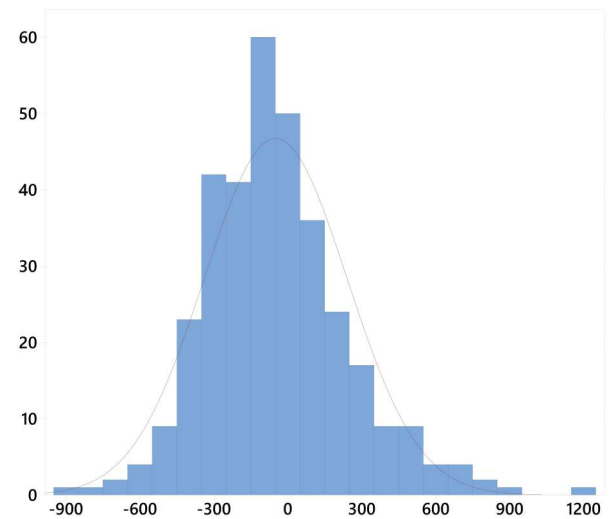


Figure 5: Histogram of the difference $X(t+3) - X(t)$ for the Delaware River when $X(t) \leq 3000$ cfs, during the period 2020-2024.

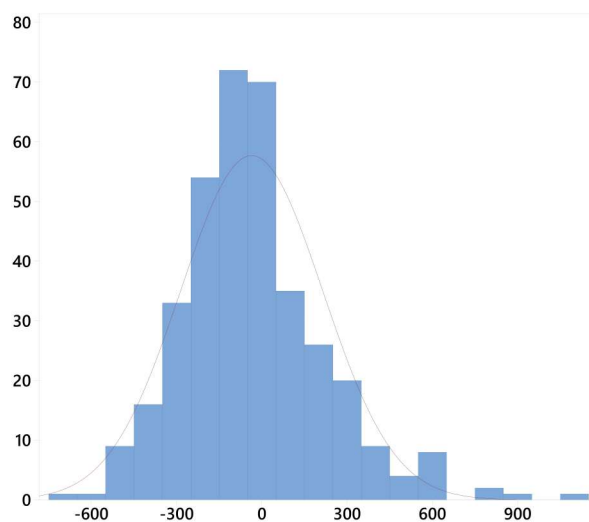


Figure 4: Histogram of the difference $X(t+2) - X(t)$ for the Delaware River when $X(t) \leq 3000$ cfs, during the period 2020-2024.

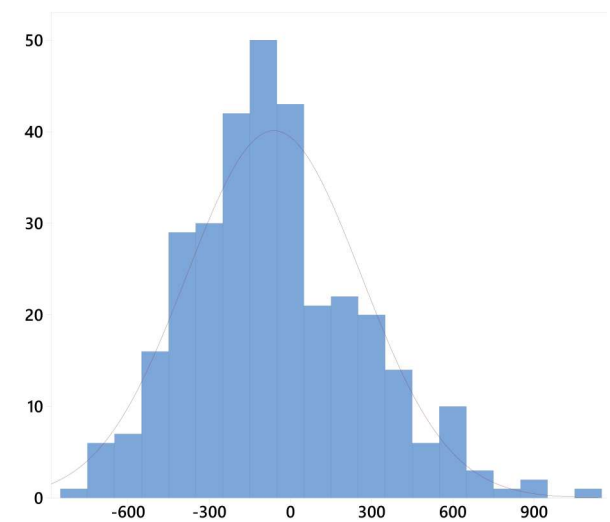


Figure 6: Histogram of the difference $X(t+4) - X(t)$ for the Delaware River when $X(t) \leq 3000$ cfs, during the period 2020-2024.

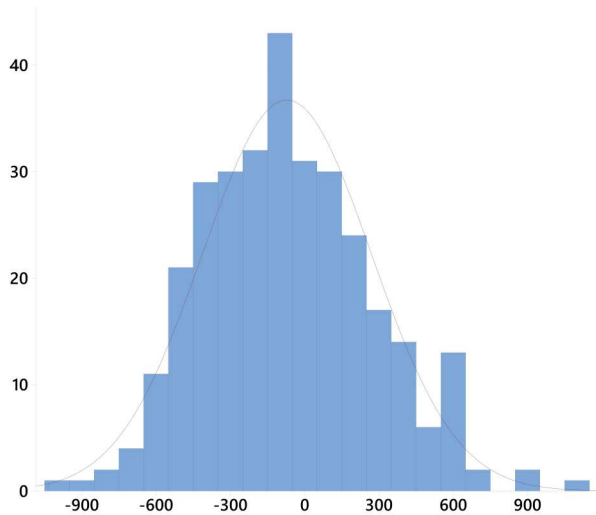


Figure 7: Histogram of the difference $X(t+5) - X(t)$ for the Delaware River when $X(t) \leq 3000$ cfs, during the period 2020-2024.

variables $X(t+k) - X(t)$ are slightly more Gaussian than in the case of the Delaware River. Moreover, the Q-Q plot is different. For this type of graph, an asymmetric double exponential distribution for the jumps could be an appropriate model. This distribution can in fact be justified using geographical and hydrological arguments.

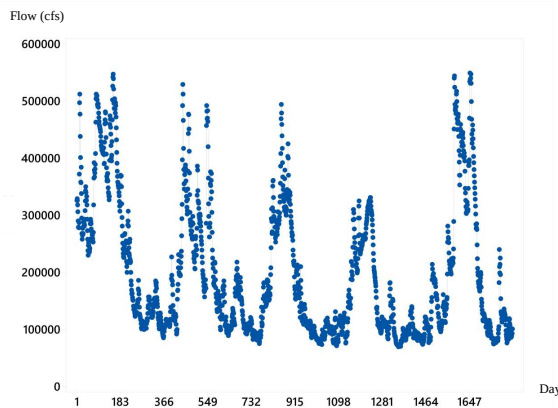


Figure 8: Daily flows of the Mississippi River in cfs during the period 2020-2024.

Next, let $Z_k := X(t+k) - X(t)$. Table 1 gives the mean $\bar{z}_k$ and the variance $s_{z_k}^2$ of the observed values of Z_k , for $k = 1, 2, 3, 4, 5$, when $X(t) \leq 3000$ cfs. In the case of a Wiener process, $E[Z_k] = \mu k$, and $\text{VAR}[Z_k] = \sigma^2 k$. Looking at Figures 12 and 13, we can conclude that both $\bar{z}_k$ and $s_{z_k}^2$ are approximately linear functions of k , so that a Wiener process is indeed an appropriate model for fluctuations in flow when it is low, so that jumps can be neglected.

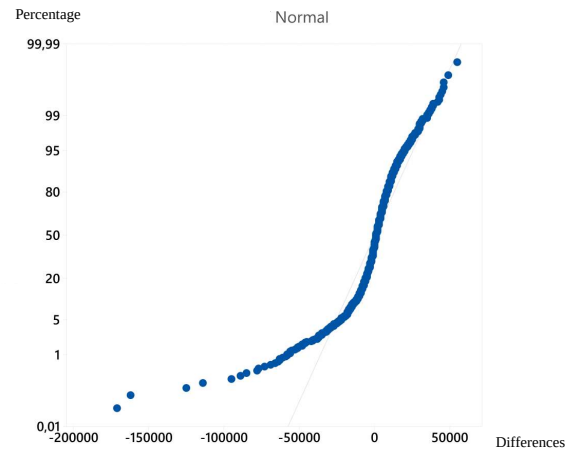


Figure 9: Q-Q plot for the difference $X(t+1) - X(t)$ of the daily flows of the Mississippi River in cfs during the period 2020-2024.

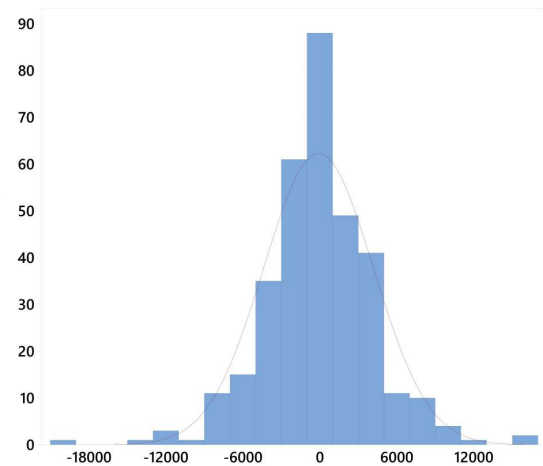


Figure 10: Histogram of the difference $X(t+1) - X(t)$ for the Mississippi River when $X(t) \leq 10,000$ cfs, during the period 2020-2024.

Table 1: Mean $\bar{z}_k$ (in cfs) and variance $s_{z_k}^2$ (in cfs²) of the observed differences $z_k := x(t+k) - x(t)$ in the case of the Delaware River, for $k = 1, 2, 3, 4, 5$, when $X(t) \leq 3000$.

Variable	Observations	$\bar{z}_k$	$s_{z_k}^2$
z_1	394	-15.3	32284
z_2	362	-36.0	62661
z_3	340	-49.2	84327
z_4	324	-60.8	103849
z_5	314	-71.5	116385

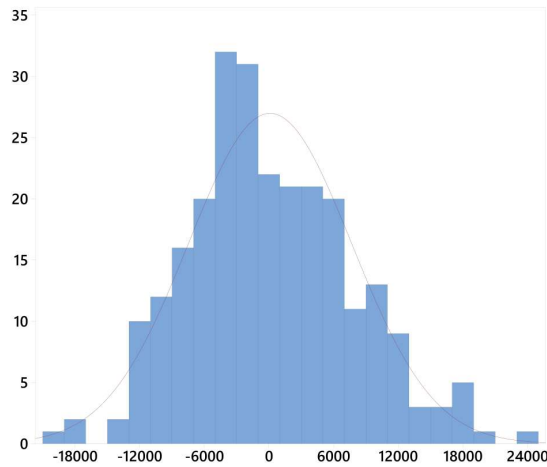


Figure 11: Histogram of the difference $X(t+5) - X(t)$ for the Mississippi River when $X(t) \leq 10,000$ cfs, during the period 2020-2024.

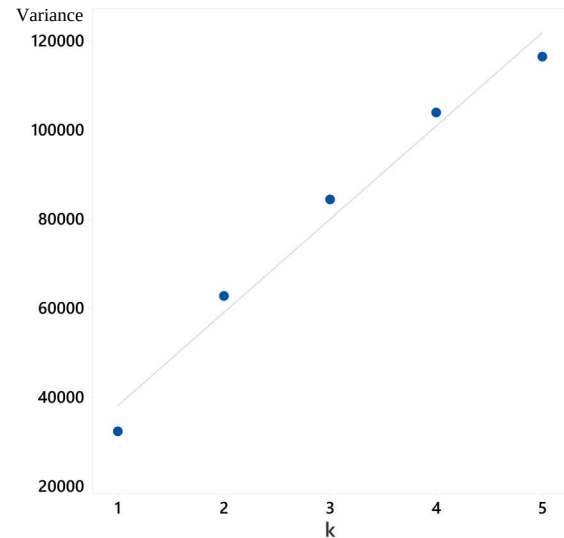


Figure 13: Regression line for the variance $s^2_{z_k}$ (in cfs^2) of the observed differences $z_k = x(t+k) - x(t)$ in the case of the Delaware River, as a function of k , when $X(t) \leq 3000$.

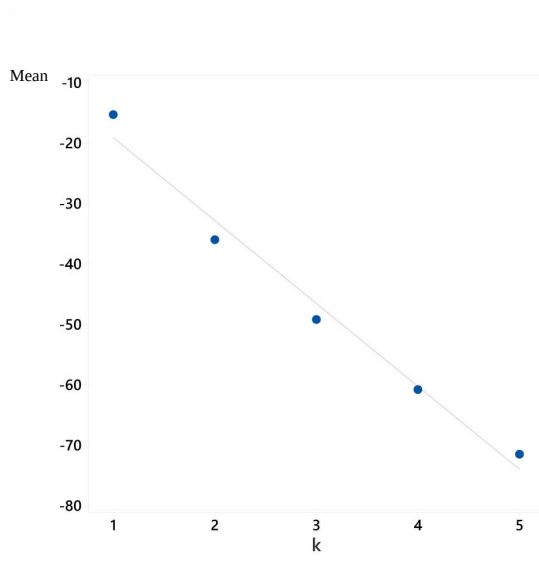


Figure 12: Regression line for the mean $\bar{z}_k$ (in cfs) of the observed differences $z_k = x(t+k) - x(t)$ in the case of the Delaware River, as a function of k , when $X(t) \leq 3000$.

For other rivers, rather than a Wiener process, a geometric Brownian motion, or perhaps an Ornstein-Uhlenbeck process, might be more appropriate for the continuous part of the jump-diffusion process. However, it must be possible to estimate the parameters of the proposed model.

Now, using the observed values of the Delaware River flow for the period 2020-2024, we find the point estimates of μ_k , for $k = 1, 2, 3, 4$, presented in Table 2.

Table 2: Point estimates $\hat{\mu}_k$ (in cfs^k), $k = 1, 2, 3, 4$, for the Delaware River

k	1	2
$\hat{\mu}_k$	0.8	115,705,554
k	3	4
$\hat{\mu}_k$	2.42155×10^{12}	1.52738×10^{16}

From Eq. (12), we deduce that

$$\mu = 0.8 - \frac{\lambda}{\alpha}. \quad (15)$$

Then, proceeding as explained in Section 3, we can write that

$$\sigma = \sqrt{15705553.36 - 2 \frac{\lambda}{\alpha^2}}, \quad (16)$$

$$\lambda = 40365448890 \alpha^3 \quad (17)$$

and

$$\begin{aligned} 0 \approx & 1 \times 10^{33} \alpha^9 + 6071532166 \times 10^7 \alpha^8 \quad (18) \\ & + 4000054703 \times 10^{14} \alpha^7 - 1 \times 10^{23} \alpha^6 \\ & - 3377527334 \times 10^5 \alpha^5 + 11197306260 \alpha^4 \\ & + 1002085419 \times 10^4 \alpha^3 \\ & - 1001049106 \alpha^2 - 1453307099 \times 10^7 \alpha \\ & + 968770773500. \end{aligned}$$

Making use of the mathematical software program *Maple*, we find that the above polynomial equation has two positive roots:

$$\alpha_1 \approx 0.00006665974275, \alpha_2 \approx 0.007849510255. \quad (19)$$

With α_2 , we find that

$$\hat{\sigma}^2 \approx -6.1799 \times 10^8, \quad (20)$$

which is not admissible. With $\hat{\alpha} = \alpha_1$, we calculate

$$\begin{aligned} \hat{\lambda} &\approx 0.011956, \quad \hat{\sigma} \approx 3213.106361, \quad (21) \\ \hat{\mu} &\approx -178.564732. \end{aligned}$$

Remark. The fact that all four point estimates of the model parameters are admissible is another indication that the model is plausible. Indeed, there was no guarantee that these point estimates would be valid in the application considered.

Next, we will consider the days on which the flow increased or decreased significantly, in order to estimate the parameter σ_1 . This step is somewhat subjective, as it depends on the choice of threshold. If we consider only days when the difference between the observed flow rate and that of the previous day was at least 5000 cfs (in absolute value), we find that $\hat{\sigma}_1 \approx 5.05 \times \hat{\sigma}$. With a difference of 10,000 cfs, $\hat{\sigma}_1 \approx 7.28 \times \hat{\sigma}$, and $\hat{\sigma}_1 \approx 8.86 \times \hat{\sigma}$ for a difference of 15,000 cfs. We decided to set $\hat{\sigma}_1 = 7 \times \hat{\sigma}$ (since $(5.05 + 7.28 + 8.86)/3 \approx 7$).

The next step is to generate observations of the jump-diffusion process with the parameters that we have estimated. To do this, we used the software program *Maple*. We repeated 1,000 times the simulation of the process to determine the probability of exceeding each of the following thresholds R over a one-year period: 100,000, 150,000, 200,000 and 250,000 cfs. The results are shown in Table 3.

By interpolating or extrapolating in Table 3, we can now provide estimates of return-period flows R_n for important values of n . These estimates are displayed in Table 4.

Remark. To produce the estimates of R_n , we could also have used linear or quadratic regression with the

Table 3: Estimated probability $\hat{p}_R$ of exceeding the threshold R (in cfs) during a one-year period for the Delaware River

R	100,000	150,000	200,000	250,000
$\hat{p}_R$	0.253	0.092	0.031	0.01

Table 4: Estimated return-period flows R_n (in cfs) for important values of n for the Delaware River

n	5	10	25	50
R_n	116,460	147,516	192,623	226,190
n	100	500		
R_n	250,000	303,333		

four pairs in Table 2, rather than interpolating between adjacent values of $\hat{p}_R$.

Now in, [11] estimates of return-period flows R_n for the Delaware River at the Montague, N.J., station were calculated by hydrologists using the computer program PeakFQ. Their estimates are presented in Table 5.

Table 5: Estimated return-period flows R_n (in cfs) for the Delaware River calculated by hydrologists

n	5	10	25	50
R_n	101,000	127,000	164,000	194,000
n	100	500		
R_n	226,000	308,000		

Comparing the values in Tables 4 and 5, we observe that for $n = 5, 10, 25$ and 50, the estimates in Table 3 are approximately 15% larger than the corresponding ones in Table 5. This may in fact be a consequence of climate change, since, unlike hydrologists, we used data from the most recent years available. When $n = 100$, the difference is approximately 10%, and for $n = 500$, the two estimates are very close. It should be noted that the estimates of R_{100} and (in particular) R_{500} are most likely less reliable. For such long periods, no method can produce truly accurate estimates, so only the order of magnitude of the estimates should be relied upon.

The estimates presented in Table 3 will differ if we perform other simulations. Our objective was to show that the proposed method produces estimates of return-period flows R_n that are consistent with those

calculated by hydrologists.

One advantage of our method is that it is easy to recalculate estimates of R_n when new data become available. Another advantage is that it is also easy to see the potential effect of varying the model parameters. Suppose, for example, that the parameter λ , which represents the rate at which events of the Poisson process occur, increases by 10% due to climate change. We can repeat the procedure and produce new estimates that take this increase in λ into account. The same applies to the parameters σ and α .

Finally, we will also compare our estimates of R_n with those calculated in, [9] using a filtered Poisson process. These are provided in Table 6.

Table 6: Estimated return-period flows R_n (in cfs) for the Delaware River calculated in, [9]

n	5	10	25	50
R_n	101,000	127,000	161,000	186,350
n	100	500		
R_n	211,450	269,500		

The estimates of R_5 and R_{10} in Table 6 are the same as the corresponding ones in Table 5. This is because these quantities were used to estimate the two parameters of the model proposed by the author in, [9]. Then, contrary to what is observed in Table 4, the point estimates begin to underestimate the quantities calculated by hydrologists. Due to climate change, it would be surprising if return-period flows decreased compared to values obtained using data collected a long time ago.

Remark. In, [9], return-period flows were also estimated for a river with a much lower average flow value than the Delaware River. The results were very similar to the return-period flows estimated by hydrologists. This should also probably be the case if the method presented in this paper is applied to estimate return-period flows for a river with a moderate average flow.

5 Conclusion

In this paper, we have presented a method that allows us to estimate the return-period flows of rivers or streams if we have observations of the flow of these watercourses. The method is based on modelling the flow as a jump-diffusion process.

The method does not require the use of flow observations that were collected a long time ago and which, due to climate change, may now be unreliable for calculating return-period flows.

In addition, our technique requires only a relatively small amount of real data to be implemented, and it allows us to see possible changes in return-period flow values if one or more model parameters are modified.

As mentioned in the Introduction, one can of course use as many years of data as one wishes. Furthermore, the potential effects of climate change are implicitly taken into account with this method because it is based mainly on relatively recent data.

In Section 4, we presented an application of the proposed method to the problem of estimating return-period flows for the Delaware River, which is a major waterway in the United States. We saw that the return-period flow estimates obtained using our technique are consistent with the estimates calculated by hydrologists.

In a preliminary version of this paper that was presented at an international conference, we used a (pure) diffusion process to model the river flow. We were then able to obtain a mathematical formula for estimating return-period flows. However, when using a jump-diffusion process, obtaining such a mathematical formula is a difficult problem in mathematics. That is why we proceeded with simulations. It should be emphasized, however, that a jump-diffusion process is generally more realistic than a pure diffusion process for modelling variations in river flows.

Finally, we explained how to estimate the model parameters when the continuous part of the jump-diffusion process is a Wiener process, and the discrete part is a compound Poisson process with exponentially distributed jumps. If we replace the Wiener process with a geometric Brownian motion, for example, the problem of estimating all the model parameters can become even more difficult.

References:

- [1] M. E. Jennings, W. O. Thomas, Jr and H. C. Riggs, *Nationwide Summary of US Geological Survey Regional Regression Equations for Estimating Magnitude and Frequency of Floods for Ungaged Sites*, 1993, Report 94-4002, USGS Publications Warehouse, 1994. <https://doi.org/10.3133/wri944002>
- [2] C. Onyutha and P. Willems, Empirical statistical characterization and regionalization of amplitude-duration-frequency curves for extreme peak flows in the Lake Victoria Basin, East Africa, *Hydrological Sciences Journal*, Vol. 60, No. 6, pp. 997–1012, 2015. <https://doi.org/10.1080/02626667.2014.898846>

- [3] A. Ahmed, G. Yildirim, K. Haddad and A. Rahman, Regional flood frequency analysis: A bibliometric overview, *Water*, Vol. 15, No. 9, 1658, 2023. <https://doi.org/10.3390/w15091658>
- [4] H. Vangelis, I. Zotou, I. M. Kourtis, V. Bellos and V. A. Tsihrintzis, Relationship of rainfall and flood return periods through hydrologic and hydraulic modeling, *Water*, Vol. 14, No. 22, Article 3618, 2022. <https://doi.org/10.3390/w14223618>
- [5] E. Koliokosta, Return periods in assessing climate change risks: uses and misuses, *Environmental Sciences Proceedings*, Vol. 26, No. 1, Article 75, 2023. <https://doi.org/10.3390/environsciproc2023026075>
- [6] F. Willkofer, R. R. Wood and R. Ludwig, Assessing the impact of climate change on high return-period levels of peak flows in Bavaria applying the CRCM5 large ensemble, *Hydrology and Earth System Sciences*, Vol. 28, No. 13, pp. 2969–2989, 2024. <https://doi.org/10.5194/hess-28-2969-2024>
- [7] F. Serinaldi, Dismissing return periods!, *Stochastic Environmental Research and Risk Assessment*, Vol. 29, pp. 1179–1189, 2015. <https://doi.org/10.1007/s00477-014-0916-1>
- [8] M. Lefebvre, Modelling river flows as jump-diffusion processes, *Atti della Accademia Peloritana dei Pericolanti - Classe di Scienze Fisiche, Matematiche e Naturali*, Vol. 102, No. 1, A4, 2024. <https://doi.org/10.1478/AAPP.1021A4>
- [9] M. Lefebvre, Estimation of the return periods in hydrology, *Advances in Environmental and Engineering Research*, Vol. 3, No. 4, Article 049, 2022. <https://doi.org/10.21926/aer.2204049>
- [10] E. Parzen, *Stochastic Processes*, Holden-Day, San Francisco, CA, USA, 1962. <https://doi.org/10.1137/1.9781611971125>
- [11] R. D. Schopp and G. D. Firda, *Flood Magnitude and Frequency of the Delaware River in New Jersey, New York, and Pennsylvania*, U.S. Geological Survey Open-File Report 2008-1203, Reston, VA, USA, 2008.

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