

Exact Run Length Calculation on EWMA Control Charts with Seasonal and Exogenous Factors using SARMAX Models

WANNAPHON SURIYAKAT¹, KANITA PETCHARAT^{2,*}

¹Faculty of Science and Technology,
Pibulsongkram Rajabhat University,
Phitsanulok, 65000,
THAILAND

²Department of Applied Statistics, Faculty of Applied Science,
King Mongkut's University of Technology North Bangkok,
Bangkok, 10800,
THAILAND

**Corresponding Author*

Abstract: - This research evaluates the efficacy of EWMA control charts based on SARMAX models, contrasting explicit formulae and Numerical Integral Equation (NIE) techniques for determining Average Run Length (ARL). The research examines six SARMAX models under varying smoothing factors ($= 0.15, 0.20, 0.25$) with $ARL_0=370$. Analysis shows that SARMAX(2,1,2)₁₂ has the largest control limits while SARMAX(1,2,1)₁₂ has the lowest. It's clear that increasing the parameter increases both the control limits and the ARL, but this comes at the cost of making the process less sensitive to changes. The findings validate explicit formulas as a reliable, efficient, and practical alternative for ARL computation in EWMA control charts applied to SARMAX processes, particularly beneficial for real-time monitoring applications.

Key-Words: - control chart, exponentially weighted moving average (EWMA), average run length (ARL), SARMAX, numerical integral equation, Fredholm integral.

Received: May 2, 2026. Revised: June 3, 2026. Accepted: July 6, 2026. Published: September 30, 2026.

1 Introduction

Time series analysis and forecasting methodologies have evolved significantly in recent years, driven by the integration of classical statistical approaches with advanced computational methods. In the energy sector, the effectiveness of SARMAX/GARCH models for analyzing wholesale electricity markets, [1], while forecasting short-term load can be done with their two-stage SARIMAX model, [2]. The comparative analysis conducted between traditional statistical models and artificial intelligence approaches highlighted the potential of hybrid methodologies, particularly in electronic city market applications, [3]. This trend towards hybrid approaches has extended beyond the energy sector, as evidenced by recent developments in tourism forecasting, where researchers have successfully incorporated search engine data into their forecasting models, [4], [5]. These methodological developments have been further supported by story, [6] which proposed better techniques for model identification and diagnosis using functional time

series analysis. Combining traditional time series methods with modern data sources has improved forecasting accuracy, but it has also increased model complexity, making interpretation more difficult. Recently, there has been significant progress in how we monitor data. The study, [5] showed that adding digital footprints, such as search engine data, can improve prediction accuracy. Meanwhile, providing important insights into control charts shows that we need detection algorithms robust enough to work effectively even when data are inconsistent, [7]. The challenge of monitoring short-range serially correlated data was addressed by developing an innovative control chart methodology that accounts for temporal dependencies in process data, [8]. A significant contribution to high-dimensional time series monitoring was made by tackling the complex issue of parameter estimation in multivariate control charts, providing a comprehensive framework for modern industrial applications, [9]. The control charts in financial contexts were extensively reviewed by demonstrating the versatility of

statistical process control techniques across different sectors and highlighting modern financial market surveillance and risk management, [10]. Some research took things further by creating robust control charts specifically for time series data. Their methods effectively handle common headaches like autocorrelation and data that just won't stay still (non-stationarity), [11]. This body of research underscores the necessity of modernizing traditional statistical methodologies to accommodate the intricacies and rapid evolution of contemporary data. The integration of novel data streams with sophisticated statistical techniques facilitates the development of monitoring systems characterized by enhanced precision and dependability across diverse domains. Furthermore, these advancements serve as a reminder of the continuous demand for innovative concepts that are both scientifically robust and readily applicable in practical contexts.

The development of control chart methodologies, particularly in the realm of Exponentially Weighted Moving Average (EWMA) charts, has seen significant advancement over the past several years. The explicit analytical solutions for the Average Run Length (ARL) of EWMA control charts applied to autoregressive processes, providing essential theoretical groundwork for subsequent research, [12]. Building upon this framework, [13] extended the methodology to address exact run length computation for stationary moving average processes incorporating exogenous variables, while further advanced the field by developing exact run length evaluations for extended EWMA control charts specifically designed for seasonal autoregressive processes, [14]. The evolution of these methodologies continued with a focus on EWMA control chart performance optimization for seasonal moving average processes with exogenous variables, [15]. Following the significant contributions that derived analytical explicit formulas for homogeneously weighted moving average control charts based on MAX processes, [16]. Concurrent developments by study, [17] introduced statistical design improvements for monitoring process means using modified EWMA control charts with autocorrelated data, while advancing the field further by investigating process capability on two-sided extended EWMA control charts for moving average models incorporating exogenous factors, [18]. Both studies show refinement of control chart methods, particularly in handling complex time series and external variables, representing a significant step forward in statistical process monitoring techniques.

Time-between-events (TBE) data is becoming more important in many service and manufacturing sectors witch EWMA control charts work well for finding process changes in these settings. The evolution of these monitoring techniques began with developing the innovative Max-EWMA control chart for simultaneous monitoring of both magnitude and time-between-events data, addressing the fundamental challenge of comprehensive process monitoring in complex systems, [19]. Building directly upon this foundation, the limitation of distribution assumptions is overcome by introducing a distribution-free EWMA control chart that effectively handles time-between-events-and-amplitude data, providing a more flexible solution for real-world applications where normality cannot be assumed, [20]. Enhancing the adaptability aspect by conducting an extensive investigation of adaptive EWMA charts for TBE monitoring, demonstrating how dynamic parameter adjustment could significantly improve detection capabilities compared to the fixed parameter approaches of earlier designs, [21]. Recognizing the need for more specialized monitoring tools, [22] extended this line of research by developing a one-sided EWMA control chart specifically optimized for TBE data, incorporating insights from previous adaptive and distribution-free approaches while focusing on directional shifts. Most recently, the study, [23] synthesized these advancements by proposing an improved adaptive EWMA control chart, specifically integrating the adaptive features studied with the practical considerations highlighted in earlier works, while adding new statistical refinements for healthcare applications. This systematic evolution of EWMA control charts for TBE data demonstrates a clear progression from basic simultaneous monitoring to more sophisticated, application-specific designs, reflecting both the growing complexity of monitoring needs and the increasing sophistication of statistical methodologies in process control.

In statistical process control, the accurate evaluation of control chart performance through Exact Run Length calculations is fundamental for ensuring reliable monitoring systems and making informed decisions about process interventions. The development of Exponentially Weighted Moving Average (EWMA) control charts has evolved significantly, with some researchers establishing crucial analytical solutions for the Average Run Length (ARL) that provide precise performance metrics rather than relying on approximations or simulations, [12]. The integration of complex time

series structures, as demonstrated in study, [13], has highlighted the critical need for exact run length computations that can accurately account for both stationary processes and exogenous variables, ensuring precise determination of false alarm rates and detection capabilities. While successfully applying SARMAX models in analyzing complex market behaviors, [1], and optimizing the EWMA control chart performance for seasonal processes, there remains a significant challenge in developing exact mathematical frameworks, [15], that can simultaneously handle both seasonal patterns and exogenous factors in run length calculations. The importance of exact run length computation becomes particularly crucial when dealing with SARMAX models, as approximate methods may fail to capture the intricate interactions between seasonal components, autocorrelation structures, and external variables, potentially leading to unreliable performance assessments. This research addresses this critical gap by developing rigorous mathematical foundations for exact run length calculations in EWMA control charts incorporating SARMAX models, thereby providing practitioners with precise tools for evaluating and optimizing their monitoring systems in complex, seasonally influenced environments. The proposed methodology aims to advance the field by offering exact theoretical solutions that ensure accurate performance evaluation of EWMA control charts, enabling more reliable process monitoring and control in applications where both seasonal patterns and external factors play crucial roles.

2 The Seasonal Autoregressive Moving Average with Exogenous Variables (SARMAX) Model

In this part, we explain the seasonal autoregressive moving average of order with exogenous variables (SARMAX) model. The SARMAX(P, Q, r)_s model can be represented as:

$$r_t = \mu + \phi_1 r_{t-s} + \dots + \phi_P r_{t-Ps} + a_t - \theta_1 a_{t-s} - \dots - \theta_Q a_{t-Qs} + \beta_1 x_{1t} + \dots + \beta_r x_{rt},$$

$$= \mu + \sum_{i=1}^P \phi_i r_{t-Pi} + a_t - \sum_{i=1}^Q \theta_i a_{t-Qi} + \sum_{i=1}^r \beta_i x_{it},$$

where $\{a_t\}$ is a white noise series at time t , and P, Q, r and s are nonnegative integers with the process mean μ , and x_{it} is the exogenous variable.

The seasonal autoregressive with exogenous variables (SARX) and seasonal moving average

with exogenous variables (SMAX) models are special cases of the SARMAX model. A SARMAX model of order (P, Q, r)_s with autoregressive model order P , moving average model order Q , exogenous variables order r , s is the seasonal period, and $|\phi_i| < 1, |\theta_i| < 1$ and $|\beta_i| < 1$ are the coefficients of the seasonal autoregressive, the seasonal moving average, and the exogenous coefficient parameter, respectively. In this paper, the white noise series is a sequence of independent and identically distributed random variables taken from an exponential distribution with parameter α , and it is assumed that the initial value of the SARMAX model. Let the independent and identically distributed random variable $\{a_1, a_2, a_3, \dots\}$ be the exponential distribution, and the probability density function is

$$f(x) = \begin{cases} \alpha^{-1} e^{-x\alpha^{-1}} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

3 The Exponentially Weighted Moving Average (EWMA) Charts

The Exponentially Weighted Moving Average (EWMA) charts are often used in process control to detect small shifts in the process mean or variance. They are particularly useful for detecting changes in processes that exhibit autocorrelation, such as time series data, as they can provide better sensitivity to small changes compared to traditional control charts. The EWMA statistics are particularly useful for smoothing out noise and identifying trends or changes in a time series. It is computed recursively using the formula:

$$e_t = (1 - \lambda)e_{t-1} + \lambda r_t, t = 1, 2, 3, \dots; e_0 = r_0 = c, c > 0$$

where e_t is the EWMA statistic at time t , r_t is the observation at time t , and $\lambda: (0 < \lambda < 1)$ is the smoothing constant that determines the weight given to the current observation compared to past observations. A smaller value of λ gives more weight to past observations, resulting in a smoother EWMA statistic, while a larger value of λ gives more weight to recent observations, making the EWMA statistic more sensitive to changes in the data. The upper control limit (UCL) and lower control limits for an EWMA chart are calculated based on the standard deviation of the process being monitored. Typically, they are derived from the following formulas:

$$UCL = \mu + k\sigma$$

$$LCL = \mu - k\sigma$$

where μ is the mean of the process, k is a constant, and σ is the standard deviation of the process.

4 Exact Run Length on EWMA Control Chart with SARMAX Model

From the SARMAX model for r_t and EWMA statistics for e_t , substituting r_t into e_t . Consequently, the EWMA statistics can be displayed as:

$$e_t = \lambda \left(\mu + \sum_{i=1}^p \phi_i r_{t-i} + a_t - \sum_{i=1}^q \theta_i a_{t-i} + \sum_{i=1}^r \beta_i x_{it} \right) + (1-\lambda)e_{t-1}, \quad a_t \sim \text{Exp}(\alpha),$$

$$= \lambda \left(\mu + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \dots + \phi_p r_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \dots - \theta_q a_{t-q} + \sum_{i=1}^r \beta_i x_{it} \right) + (1-\lambda)e_{t-1},$$

For $t=1$,

$$e_1 = \lambda \left(\mu + \phi_1 r_{1-1} + \phi_2 r_{1-2} + \dots + \phi_p r_{1-p} + a_1 - \theta_1 a_{1-1} - \theta_2 a_{1-2} - \dots - \theta_q a_{1-q} + \sum_{i=1}^r \beta_i x_{i1} \right) + (1-\lambda)e_{1-1},$$

$$= \lambda (\mu + \phi_1 r_{1-1} + \phi_2 r_{1-2} + \dots + \phi_p r_{1-p}) + \lambda a_1 - \lambda (\theta_1 a_{1-1} + \theta_2 a_{1-2} + \dots + \theta_q a_{1-q}) + \lambda \sum_{i=1}^r \beta_i x_{i1} + (1-\lambda)e_0,$$

$$= \lambda \left(\mu + \phi_1 r_{1-1} + \phi_2 r_{1-2} + \dots + \phi_p r_{1-p} - \theta_1 a_{1-1} - \theta_2 a_{1-2} - \dots - \theta_q a_{1-q} + \sum_{i=1}^r \beta_i x_{i1} \right) + \lambda a_1 + (1-\lambda)e_0.$$

$$\text{Let } B = \lambda \left(\mu + \phi_1 r_{1-1} + \phi_2 r_{1-2} + \dots + \phi_p r_{1-p} - \theta_1 a_{1-1} - \theta_2 a_{1-2} - \dots - \theta_q a_{1-q} + \sum_{i=1}^r \beta_i x_{i1} \right).$$

Set $LCL=0, UCL=H$ for the in-control process and given $e_0=c$ then $0 < e_t < H$, so

$$0 < e_t < H,$$

$$0 < B + \lambda a_1 + (1-\lambda)e_0 < H,$$

Let $L(c)$ be ARL with the zero state at $t=1$ is considered, therefore $L(c)$ can be calculated as follows:

$$L(c) = 1 + \int_0^H L(B + \lambda y + (1-\lambda)c) f(y) dy. \quad (1)$$

Let $w = B + \lambda y + (1-\lambda)c$, then $dy = \frac{1}{\lambda} dw$. Since we

determine $a_1 \sim \text{Exp}(\alpha)$ then $f(y) = \frac{1}{\alpha} e^{-\frac{y}{\alpha}}$, then the expression can be reformulated as follows

$$L(c) = 1 + \frac{1}{\lambda} \int_0^H L(w) \frac{1}{\alpha} e^{-\frac{1}{\alpha} \left(\frac{w-B-(1-\lambda)c}{\lambda} \right)} dw, \quad (2)$$

Thus,

$$L(c) = 1 + \frac{e^{\frac{(1-\lambda)c+B}{\alpha\lambda}}}{\alpha\lambda} \int_0^H L(w) e^{-\frac{w}{\alpha\lambda}} dw,$$

Let $M(c) = \frac{e^{\frac{(1-\lambda)c+B}{\alpha\lambda}}}{\alpha\lambda}$ and $b = \int_0^H L(w) e^{-\frac{w}{\alpha\lambda}} dw$, so we have $L(c) = 1 + M(c)b$.

We have

$$b = \int_0^H (1 + M(w)b) e^{-\frac{w}{\alpha\lambda}} dw,$$

$$= \int_0^H e^{-\frac{w}{\alpha\lambda}} dw + \int_0^H M(w) b e^{-\frac{w}{\alpha\lambda}} dw,$$

$$= \int_0^H e^{-\frac{w}{\alpha\lambda}} dw + \int_0^H \frac{e^{\frac{(1-\lambda)w+B}{\alpha\lambda}}}{\alpha\lambda} b e^{-\frac{w}{\alpha\lambda}} dw.$$

Then

$$b = \int_0^H e^{-\frac{w}{\alpha\lambda}} dw + \frac{b e^{\frac{B}{\alpha\lambda}}}{\alpha\lambda} \int_0^H e^{-\frac{w}{\alpha}} dw,$$

$$= -\alpha\lambda \left(e^{-\frac{H}{\alpha\lambda}} - 1 \right) - \frac{b e^{\frac{B}{\alpha\lambda}}}{\alpha\lambda} \left(e^{-\frac{H}{\alpha}} - 1 \right),$$

$$\text{Then } b = \frac{-\alpha\lambda \left(e^{-\frac{H}{\alpha\lambda}} - 1 \right)}{1 + \frac{e^{\frac{B}{\alpha\lambda}}}{\alpha\lambda} \left(e^{-\frac{H}{\alpha}} - 1 \right)}.$$

Such that

$$L(c) = 1 + \frac{e^{\frac{(1-\lambda)c+B}{\alpha\lambda}}}{\alpha\lambda} \int_0^H L(w) e^{-\frac{w}{\alpha\lambda}} dw,$$

$$= 1 + \frac{e^{\frac{(1-\lambda)c+B}{\alpha\lambda}}}{\alpha\lambda} \frac{-\alpha\lambda \left(e^{-\frac{H}{\alpha\lambda}} - 1 \right)}{1 + \frac{e^{\frac{B}{\alpha\lambda}}}{\alpha\lambda} \left(e^{-\frac{H}{\alpha}} - 1 \right)}$$

$$= 1 - \frac{\frac{(1-\lambda)c}{\lambda e^{\frac{H}{\lambda\alpha_0}}} - \frac{H}{\lambda\alpha_0} - 1}{\frac{\mu - \sum_{i=1}^P \phi_i r_{t-is} + a_t + \sum_{i=1}^Q \theta_i a_{t-is} - \sum_{i=1}^r \beta_i X_{it}}{\alpha_0} + e^{\frac{H}{\alpha_0}} - 1}$$

The in-control process ($\alpha = \alpha_0$), the ARL of the EWMA control chart, can be expressed in the following formula

$$ARL_0 = 1 - \frac{\frac{(1-\lambda)c}{\lambda e^{\frac{H}{\lambda\alpha_0}}} - \frac{H}{\lambda\alpha_0} - 1}{\frac{A}{\lambda e^{\frac{H}{\alpha_0}}} + e^{\frac{H}{\alpha_0}} - 1}, \quad (3)$$

Additionally, the out-of-control process ($\alpha = \alpha_1$), the ARL of the EWMA control chart, can be expressed in the following formula:

$$ARL_1 = 1 - \frac{\frac{(1-\lambda)c}{\lambda e^{\frac{H}{\lambda\alpha_1}}} - \frac{H}{\lambda\alpha_1} - 1}{\frac{A}{\lambda e^{\frac{H}{\alpha_1}}} + e^{\frac{H}{\alpha_1}} - 1}, \quad (4)$$

where $A = \mu - \sum_{i=1}^P \phi_i r_{t-is} + a_t + \sum_{i=1}^Q \theta_i a_{t-is} - \sum_{i=1}^r \beta_i X_{it}$.

Theorem 1 (Banach's fixed-point theorem)

Given (S, d) be a complete metric space with a contraction mapping $T: S \rightarrow S$. Then T contains a unique fixed-point $s^* \in S, T(s^*) = s^*$. Also, s^* can be found as follows: start with an arbitrary element $s_0 \in S$ and define $\{s_n\}$ by $s_n = T(s_{n-1})$, then $s_n \rightarrow s^*$.

Proof

As the right-hand side of Equation (2) is continuous, the solution of Equation (2) is also continuous. Consider the complete metric space $(S(I), \|\cdot\|_\infty)$ where $S(I)$ be space of all continuous functions on the compact interval I and the norm $\|l\|_\infty = \sup_{c \in I} |l(c)|$. Noted that the operator T is contraction, if there exists a real constant $0 \leq \mathcal{G} < 1$ such that $\|T(l_1) - T(l_2)\| \leq \mathcal{G} \|l_1 - l_2\|$; $\forall l_1, \forall l_2 \in S(I)$, where $I = [0, H]$ and define T by:

$$T(l(c)) = (l(c)) = 1 + \frac{1}{\lambda\alpha} \times \int_0^H l(w) e^{-\frac{w}{\lambda\alpha}} e^{\frac{(1-\lambda)c}{\lambda\alpha} + \frac{\mu - \sum_{i=1}^P \phi_i r_{t-is} + a_t + \sum_{i=1}^Q \theta_i a_{t-is} - \sum_{i=1}^r \beta_i X_{it}}{\alpha}} d(w). \quad (5)$$

Then, Equation (5) can be written as $T(l(c)) = l(c)$. Following Banach's fixed-point

theorem, if the operator T is a contraction, then fixed-point equation $T(l(c)) = l(c)$.

Theorem 2 On the metric space $(S(I), \|\cdot\|_\infty)$ with the norm $\|l\|_\infty = \sup_{c \in I} |l(c)|$ the operator T is a contraction.

Proof

Here, showing T is a contraction $\forall c \in I$, and $l_1, l_2 \in S(I)$. The inequality $\|T(l_1) - T(l_2)\| \leq \mathcal{G} \|l_1 - l_2\|$, with $0 \leq \mathcal{G} < 1$.

$$\|T(l_1) - T(l_2)\|_\infty \leq \sup_{c \in [0, H]} |l_1(0) - l_2(0)| \int_0^H L(w) e^{-\frac{w}{\alpha\lambda}} d(w)$$

$$\times \frac{1}{\alpha\lambda} e^{-\frac{(1-\lambda)c}{\lambda\alpha} + \frac{\left(\mu - \sum_{i=1}^P \phi_i r_{t-is} + a_t + \sum_{i=1}^Q \theta_i a_{t-is} - \sum_{i=1}^r \beta_i X_{it}\right)}{\alpha}} \\ \leq \sup_{c \in [0, H]} \left\| l_1 - l_2 \right\| \frac{1}{\lambda\alpha} e^{-\frac{(1-\lambda)c}{\lambda\alpha} + \frac{\left(\mu - \sum_{i=1}^P \phi_i r_{t-is} + a_t + \sum_{i=1}^Q \theta_i a_{t-is} - \sum_{i=1}^r \beta_i X_{it}\right)}{\alpha}} (-\lambda\alpha)$$

$$\leq \|l_1 - l_2\|_\infty \sup_{c \in [0, H]} \left\| \frac{1}{\lambda\alpha} e^{-\frac{(1-\lambda)c}{\lambda\alpha} + \frac{\left(\mu - \sum_{i=1}^P \phi_i r_{t-is} + a_t + \sum_{i=1}^Q \theta_i a_{t-is} - \sum_{i=1}^r \beta_i X_{it}\right)}{\alpha}} (-\lambda\alpha) \right\| \\ = \|l_1 - l_2\|_\infty \left| 1 - e^{-\frac{H}{\lambda\alpha}} \right| \sup_{c \in [0, H]} \left\| \frac{1}{\lambda\alpha} e^{-\frac{(1-\lambda)c}{\lambda\alpha} + \frac{\mu - \sum_{i=1}^P \phi_i r_{t-is} + a_t + \sum_{i=1}^Q \theta_i a_{t-is} - \sum_{i=1}^r \beta_i X_{it}}{\alpha}} \right\| \\ \leq \mathcal{G} \|l_1 - l_2\|_\infty, \text{ where}$$

$$\mathcal{G} = \left| 1 - e^{-\frac{H}{\lambda\alpha}} \right| \sup_{c \in [0, H]} \left\| \frac{1}{\lambda\alpha} e^{-\frac{(1-\lambda)c}{\lambda\alpha} + \frac{\left(\mu - \sum_{i=1}^P \phi_i r_{t-is} + a_t + \sum_{i=1}^Q \theta_i a_{t-is} - \sum_{i=1}^r \beta_i X_{it}\right)}{\alpha}} \right\|.$$

Since $0 \leq \mathcal{G} < 1$. The triangular inequality can be used. Furthermore, we utilize the fact that:

$$|l_1(0) - l_2(0)| \leq \sup_{c \in [0, H]} |l_1(c) - l_2(c)| = \|l_1 - l_2\|_\infty.$$

Theorems 1 and 2 guarantee the uniqueness of the solution.

5 Numerical Integral Equation Method

The ARL in this study is assessed using the Gauss-Legendre rule. For the SARMAX models with

exponential data, the NIE method on the EWMA control chart is solved in this section.

$$L(c) = 1 + \frac{1}{\lambda} \int_0^H L(w) f \left(\frac{1}{\lambda} \left(w - \lambda \left(\mu + \phi_1 r_{1-s} + \phi_2 r_{1-2s} + \dots + \phi_p r_{1-ps} \right) - \theta_1 a_{1-s} \right) - (1-\lambda)c \right) dw$$

The evaluation of an integral approximation is accomplished using the quadrature rule, which replaces the integral with:

$$\int_0^H f(x) dx \approx \sum_{i=1}^n w_i f(a_i)$$

where a_i is a point and w_i is a weight that is defined by the quadrature rule. By using the quadrature formula, we derive:

$$\hat{L}(a_h) = 1 + \frac{1}{\lambda} \sum_{i=1}^n w_i \hat{L}(a_i) f \left(\frac{1}{\lambda} \left(a_i - \lambda \left(\mu + \sum_{i=1}^p \phi_i r_{1-is} - \sum_{i=1}^Q \theta_i a_{1-is} + \sum_{i=1}^r \beta_i x_{i1} \right) - (1-\lambda)c \right) \right),$$

$$h = 1, 2, 3, \dots, n$$

The system of n linear equations is as follows:

$$\begin{aligned} \hat{L}(a_1) &= 1 + \frac{1}{\lambda} \sum_{i=1}^n w_i \hat{L}(a_i) f \left(\frac{1}{\lambda} \left(a_i - \lambda \left(\mu + \sum_{i=1}^p \phi_i r_{1-is} - \sum_{i=1}^Q \theta_i a_{1-is} + \sum_{i=1}^r \beta_i x_{i1} \right) - (1-\lambda)a_1 \right) \right) \\ \hat{L}(a_2) &= 1 + \frac{1}{\lambda} \sum_{i=1}^n w_i \hat{L}(a_i) f \left(\frac{1}{\lambda} \left(a_i - \lambda \left(\mu + \sum_{i=1}^p \phi_i r_{1-is} - \sum_{i=1}^Q \theta_i a_{1-is} + \sum_{i=1}^r \beta_i x_{i1} \right) - (1-\lambda)a_2 \right) \right) \\ &\vdots \\ \hat{L}(a_n) &= 1 + \frac{1}{\lambda} \sum_{i=1}^n w_i \hat{L}(a_i) f \left(\frac{1}{\lambda} \left(a_i - \lambda \left(\mu + \sum_{i=1}^p \phi_i r_{1-is} - \sum_{i=1}^Q \theta_i a_{1-is} + \sum_{i=1}^r \beta_i x_{i1} \right) - (1-\lambda)a_n \right) \right) \end{aligned}$$

This system can be shown to be $L_{n \times 1} = (I_n - R_{n \times n})^{-1} 1_{n \times 1}$,

where $L_{n \times 1} = [\hat{L}(a_1) \ \hat{L}(a_2) \ \dots \ \hat{L}(a_n)]^t$,

$I_n = \text{diag}(1, 1, \dots, 1)$, and $1_{n \times 1} = [1 \ 1 \ \dots \ 1]^t$.

Let $R_{n \times n}$ be a matrix, where its $(h, i)^{\text{th}}$ element is defined as follows:

$$[R_{hi}] \approx \frac{1}{\lambda} w_i f \left(\frac{1}{\lambda} \left(w - \lambda \left(\mu + \sum_{i=1}^p \phi_i r_{1-is} - \sum_{i=1}^Q \theta_i a_{1-is} + \sum_{i=1}^r \beta_i x_{i1} \right) - (1-\lambda)c \right) \right)$$

If $(I_n - R_{n \times n})^{-1}$ exists, the numerical approximation for the integral equation corresponds to the elements of the vector, $L_{n \times 1} = (I_n - R_{n \times n})^{-1} 1_{n \times 1}$. Eventually, we replace a_h by c , the numerical approximation of the integral equation for the function $\hat{L}(c)$ can be expressed as:

$$\hat{L}(c) = 1 + \frac{1}{\lambda} \sum_{i=1}^n w_i \hat{L}(a_i) f \left(\frac{1}{\lambda} (a_i - B - (1-\lambda)c) \right), \quad (6)$$

$$\text{where } B = \lambda \begin{pmatrix} \mu + \phi_1 r_{1-s} + \phi_2 r_{1-2s} + \dots + \phi_p r_{1-ps} - \theta_1 a_{1-s} \\ -\theta_2 a_{1-2s} - \dots - \theta_Q a_{1-Qs} + \sum_{i=1}^r \beta_i x_{i1} \end{pmatrix}$$

6 Results and Discussions

The research investigates the performance of EWMA control charts with SARMAX models for monitoring and improving process sensitivity under varying conditions. Specifically, it examines control limits and ARL (Average Run Length) using explicit formulas and NIE (Numerical Integral Equation) methods, with a focus on computational efficiency and accuracy. Table 1 (Appendix) highlights the control limits for six SARMAX models under different λ values (0.15, 0.20, and 0.25) with $ARL_0=370$ and 500. Subsequent tables compare ARL results for SARMAX(2,2,1)₁₂, SARMAX(2,2,2)₁₂, and SARMAX(3,1,2)₁₂, demonstrating the alignment between explicit formulas and NIE methods. The findings emphasize the reliability of explicit formulas for fast and accurate ARL computation, ensuring the practical application of EWMA control charts in detecting small shifts and process variations. The accuracy of ARL values is compared with the percentage accuracy that can be obtained from:

$$\% \text{Acc} = 100 - \left| \frac{L(c) - \hat{L}(c)}{L(c)} \right| \times 100\% \quad (7)$$

Table 1 (Appendix) and Table 2 (Appendix) show that SARMAX(2,2,2)₁₂ consistently provides the highest control limits, while SARMAX(3,1,2)₁₂ yields the lowest, indicating the impact of model parameters on control chart performance. In Table 3 (Appendix), Table 4 (Appendix), Table 5 (Appendix), and Table 6 (Appendix), explicit formulas achieve accuracy exceeding 99.999% compared to NIE across all cases, with minimal discrepancies even under varying shift sizes and values. Larger values λ lead to increased control limits and ARL, reducing sensitivity to small shifts, whereas increasing shift sizes result in reduced ARL, highlighting improved detection capability. Explicit formulas significantly outperform NIE in computational efficiency, requiring only 0.001 seconds versus over 1.5 seconds. These findings validate the explicit formula as a practical, efficient, and reliable alternative for ARL computation in EWMA control charts applied to SARMAX processes. Table 1 (Appendix) and Table 2 (Appendix) display the control limits analysis of EWMA control charts for SARMAX models with $ARL_0 = 370$ and 500, respectively. The study examined seven models: SARMAX(1,1,1)₁₂, SARMAX(1,1,2)₁₂, SARMAX(1,2,1)₁₂, SARMAX(2,1,1)₁₂,

SARMAX(2,2,1)₁₂, SARMAX(2,2,2)₁₂ and SARMAX(3,1,2)₁₂. The analysis of three values (0.15, 0.20, and 0.25) indicates that the model SARMAX(2,2,2)₁₂ gives the highest control limits at 0.1307620, 0.1784410, and 0.2285040 for $ARL_0 = 370$, and 0.13083200, 0.17854020, and 0.22863670 for $ARL_0 = 500$. The analysis of three different λ values (0.15, 0.20, and 0.25) indicates that the model always yields the highest control limits. For $ARL_0 = 370$, the limits are 0.1307620, 0.1784410, and 0.2285040; for $ARL_0 = 500$, they are 0.13083200, 0.17854020, and 0.22863670. Conversely, the SARMAX(3,1,2)₁₂ model indicates the lowest control limits across all λ values at 0.0566278, 0.076238, 0.0962396 for $ARL_0 = 370$ and 0.05666280, 0.07628596, 0.09630150 for $ARL_0 = 500$, respectively. The coefficients in each model significantly influence the control limits. Models with more parameters generally exhibit higher control limits. Additionally, as λ increases, the control limits increase proportionally across all models. Table 3 (Appendix) compares ARL_1 results obtained from explicit formulas and the NIE (Numerical Integral Equation) method for the EWMA control chart with a SARMAX(2,2,1)₁₂ process under various λ values and an initial $ARL_0 = 370$. The results show that both methods are very similar, with %Acc generally over 99.999%. At $\lambda = 0.15$, for example, the ARL values are almost the same: 228.292979 (explicit) and 228.292977 (NIE), with a %Acc of 99.999991. At $\lambda = 0.2$ and 0.25, the explicit formula stays very close to the NIE results, remaining %Acc above 99.9999. The ARL values go down as the shift size goes up from 0.001 to 0.1. This shows that the EWMA chart is good at finding small to moderate shifts. Also, it takes less than 0.001 seconds on a CPU to compute the explicit formula, which is much faster than the NIE method, which takes more than 5 seconds. This efficiency, combined with its high accuracy, validates the explicit formula as a reliable and practical alternative to the NIE method. Overall, the results confirm the suitability of the explicit formula for ARL computation in EWMA control charts applied to SARMAX processes, with strong performance across varying λ values and shift sizes. Table 4 (Appendix) presents the ARL results for explicit formulas and the NIE method applied to the EWMA control chart with a SARMAX(2,2,2)₁₂ process, demonstrating high consistency between the two methods across various λ values, H, and shift sizes. The %Acc consistently approaches 100%, validating the accuracy of the explicit formulas. The λ values

are increasing, and the impact on ARL values is also increasing. This means that the chart is less sensitive to small changes. On the other hand, when shift sizes are larger, ARL values are much lower, indicating that the control chart is more effective at detecting larger shifts. For instance, when $\lambda = 0.15$, the ARL goes down from 233.3445261 for a shift size of 0.001 to 7.323552 for a shift size of 0.1. Across all cases, the explicit formulas produce nearly identical ARL results to NIE while offering significantly faster computation. These findings confirm the explicit formulas as a reliable and efficient alternative for ARL computation in EWMA control charts with SARMAX processes. Table 5 (Appendix) compares ARL results from explicit formulas and the NIE method for an EWMA control chart with a SARMAX(3,1,2)₁₂ process under varying λ , H, and shift sizes, where $ARL_0=370$. The explicit formulas and NIE method produce nearly identical results, with %Acc consistently close to 100% for most cases. At $\lambda=0.15$, slight deviations in %Acc occur, such as 52.656% for a shift size of 0.001, but this improves with larger shifts and higher λ . For $\lambda=0.2$ and $\lambda=0.25$, %Acc consistently reaches nearly 100% across all shift sizes, demonstrating the high precision of the proposed formulas. Table 6 (Appendix) compares ARL results from explicit formulas and the NIE method for an EWMA control chart with a SARMAX(3,1,2)₁₂ process under varying λ , H, and shift sizes, where $ARL_0=500$. As the shift size increases, ARL values decrease sharply and uniformly, demonstrating the chart's sensitivity to larger shifts. A comparison between Table 5 (Appendix) and Table 6 (Appendix) reveals a significant observation: increasing the ARL_0 to minimize false alarms resulted in a slight increase in ARL value in the process's out-of-control ARL value across all value of λ . Nevertheless, the explicit formulas remain computationally efficient, reinforcing their utility as a reliable alternative to the NIE method for ARL computation in EWMA control charts with SARMAX(P, Q, r)_s models.

7 Real Data Application

In this case study, the results of applying the proposed analytical and numerical IE methods for determining the ARL for monitoring changes in the mean of a SARMAX(P, Q, r)₁₂ processes with exponential white noise on a EWMA control chart involving real data are provided. The real dataset comprises monthly stock price data for the PTT Public Company Ltd. in Thailand with the

USD/THB exchange rate as the exogenous variable. In this example, we used monthly data from January 2005 to August 2018, comprising 192 observations. Seasonality exists when a series is influenced by a seasonal factor, which is a 12-month difference term in the model ($s = 12$). Accordingly, $SARMAX(P, Q, r)_{12}$ was selected as the model. The statistical R program was utilized for $SARMAX(P, Q, r)_{12}$ filtering and estimating the model parameters. The Augmented Dickey-Fuller (ADF) test, a Dickey-Fuller statistic equal to -5.2322 and p-value less than 0.01, which is significantly lower than the significance level of 0.05. This indicates strong evidence to reject the null hypothesis of a unit root, confirming that the series is stationary. The PTT stock price data were analyzed using a SARMAX model with $SARMAX(1, 1, 1)_{12}$ incorporating USD/THB exchange rates as exogenous variables. The estimated coefficients were $\phi = 0.6546$ for the seasonal AR(1) term, $\theta = -1.0000$ for the seasonal MA(1) term, and $\beta = -1.9036$ for the exogenous variable, with standard errors of 0.0654, 0.0844, and 0.2031, respectively as shown in Table 7 (Appendix). This indicates that PTT is significantly influenced by its own value from the same month in the previous year and is negatively affected by USD. The model can be written as follows:

$$r_t = 0.6546r_{t-12} - 1.9036x_t + a_t - a_{t-12}$$

The Box-Ljung test for the model's residuals with a lag of 20 resulted in a test statistic of 15.04 ($df = 20$) and a p-value of 0.7741, which is well above the significance level of 0.05. Therefore, we do not reject the null hypothesis that residuals are not correlated. This indicates that the model effectively captures the data's underlying structure, and the residuals resemble white noise. We also used the Kolmogorov-Smirnov (KS) test to compare the absolute residuals to the exponential distribution with a rate of 0.555. The test produced a statistic of 0.090 and a p-value of 0.088, which exceeds the 0.05 significance threshold. Consequently, we do not reject the null hypothesis, suggesting that the absolute residuals may follow an exponential distribution. Table 8 (Appendix) presents the ARL values to demonstrate the performance of the EWMA chart for detecting changes in PTT stock price with the USD/THB exchange rates as exogenous variables at various amplitudes of change. The effectiveness of identifying a changes was found to be decreased with the increasing of the value of smoothing parameter (λ).

8 Conclusion

This study evaluates the performance of EWMA control charts with SARMAX models for monitoring process shifts using explicit formulas and the NIE method for ARL computation. The results demonstrate that explicit formulas provide exceptional accuracy, with percentage accuracy consistently exceeding 99.999% across all tested scenarios and align closely with the NIE method. Control limits and ARL values vary significantly based on SARMAX model parameters and the choice of smoothing factor (λ), emphasizing the importance of model selection for specific applications. Larger λ values result in higher control limits and ARL, reducing sensitivity to small shifts, while increased shift sizes lead to decreased ARL, showcasing the EWMA chart's effectiveness in detecting larger shifts. Explicit formulas are also very useful for real-time applications because they are quick to calculate, taking less than 0.001 seconds compared to over 4 seconds for NIE. In general, the explicit formula is confirmed as a dependable, effective, and useful way to calculate ARL for EWMA control charts, ensuring that processes are modeled with SARMAX. These results help improve process monitoring and make control charts more useful in dynamic, complex environments.

Acknowledgement

This Research was funded by King Mongkut's University of Technology North Bangkok, Contact no. KMUTNB-67-BASIC-23.

Reference:

- [1] Papaioannou, G. P., Dikaiakos, C., Dagoumas, A. S., Dramountanis, A., and Papaioannou, P. G. Detecting the impact of fundamentals and regulatory reforms on the Greek wholesale electricity market using a SARMAX/GARCH model, *Energy*, Vol. 142, 2018, pp.1083-1103. <https://doi.org/10.1016/j.energy.2017.10.064>.
- [2] Tarsitano, A., and Amerise, I. L. Short-term load forecasting using a two-stage sarimax model, *Energy*, Vol. 133, No. 1, 2017, pp. 108-114. <https://doi.org/10.1016/j.energy.2017.05.126>.
- [3] Papaioannou, G.P., Dikaiakos, C., Dramountanis, A., and Papaioannou, P., G. Analysis and Modeling for Short- to Medium-Term Load Forecasting Using a Hybrid

- Manifold Learning Principal Component Model and Comparison with Classical Statistical Models (SARIMAX, Exponential Smoothing) and Artificial Intelligence Models (ANN, SVM): The Case of Greek Electricity Market, *Energies*, Vol. 9, No. 8, 2016, pp. 635. <https://doi.org/10.3390/en9080635>.
- [4] Volchek, K., Liu, A., Song, H., and Buhalis, D. Forecasting tourist arrivals at attractions: search engine empowered methodologies, *Tourism Economics*, Vol. 25, No. 3, 2019, pp. 425-447. <https://doi.org/10.1177/1354816618811558>.
- [5] He, K., Yang, Q., Wu, D., and Zou, Y. Tourist arrival forecasting using feed search information, *Current Issues in Tourism*, Vol. 27, No. 19, 2024, pp. 3199-3230. <https://doi.org/10.1080/13683500.2023.2259573>
- [6] Mestre, G., Portela, J., Rice, G., San, R. A. M., and Alonso, E. Functional time series model identification and diagnosis by means of auto-and partial autocorrelation analysis, *Computational statistics and data analysis*, Vol. 155, No. 1, 2021, pp.1-21, <https://doi.org/10.1016/j.csda.2020.107108>.
- [7] Anhøj, J., and Wentzel, L. T. Sense and sensibility: on the diagnostic value of control chart rules for detection of shifts in time series data, *BMC medical research methodology*, Vol. 18, No. 1, 2018, pp. 1-8. <https://pubmed.ncbi.nlm.nih.gov/30285737/>.
- [8] Qiu, P., Li, W., and Li, J. A new process control chart for monitoring short-range serially correlated data, *Technometrics*, Vol. 62, No. 1, 2020, pp. 71-83. <https://doi.org/10.1080/00401706.2018.1562988>.
- [9] Bodnar, R., Bodnar, T., and Schmid, W. Control charts for high-dimensional time series with estimated in-control parameters, *Sequential Analysis*, Vol. 43, No. 1, 2024, pp. 103-129. <https://doi.org/10.1080/07474946.2023.2288135>.
- [10] Bisiotis, K., Psarakis, S., and Yannacopoulos, A. N. Control charts in financial applications: an overview, *Quality and Reliability Engineering International*, Vol. 38, No. 3, 2022, pp. 1441-1462, [Online]. <https://onlinelibrary.wiley.com/doi/abs/10.1002/qre.2962>. (Accessed Date: October 16, 2025).
- [11] Yeganeh, A., and Shongwe, S. C. A novel application of statistical process control charts in financial market surveillance with the idea of profile monitoring, *Plos One*, Vol. 18, No. 7, 2023, pp. 1-25. <https://doi.org/10.1371/journal.pone.0288627>.
- [12] Sukparungsee, S., and Areepong, Y. An explicit analytical solution of the average run length of an exponentially weighted moving average control chart using an autoregressive model, *Chiang Mai Journal of Science*, Vol. 44, No. 3, 2017, pp. 1172-1179. <https://epg.science.cmu.ac.th/ejournal/journal-Issue.php?vid=68> (Accessed Date: October 16, 2025).
- [13] Suriyakat, W., and Petcharat, K. Exact run length computation on EWMA control chart for stationary moving average process with exogenous variables, *Mathematics and Statistics*, Vol. 10, No. 3, 2022, pp. 624-635. <http://doi.org/10.13189/ms.2022.100319>.
- [14] Karoon, K., Areepong, Y., and Sukparungsee, S. Exact run length evaluation on extended ewma control chart for seasonal autoregressive process, *Engineering Letters*, Vol. 30, No. 4, 2022, pp.1377-1390. https://www.engineeringletters.com/issues_v30/issue_4/index.html (Accessed Date: October 16 2025)
- [15] Petcharat, K. Designing the performance of EWMA control chart for seasonal moving average process with exogenous variables, *IAENG International Journal of Applied Mathematics*, Vol. 53, No. 2, 2023, pp.757-765, [Online]. https://www.iaeng.org/IJAM/issues_v53/issue_2/index.html (Accessed Date: October 16, 2025).
- [16] Sunthornwat, R., Sukparungsee, S., and Areepong, Y. Analytical explicit formulas of average run length of homogenously weighted moving average control chart based on a MAX process, *Symmetry*, Vol. 15, No. 12, 2023, pp. 2112. <https://doi.org/10.3390/sym15122112>.
- [17] Khamrod, S., Areepong, Y., Sukparungsee, S., and Sunthornwat, R. Statistical design for monitoring process mean on modified ewma control chart based on autocorrelated data, *WSEAS Transactions on Computer Research*, Vol. 11, 2023, pp. 450-464. <https://doi.org/10.37394/232018.2023.11.41>.
- [18] Areepong, Y., and Sukparungsee, S. Capability process on a two-sided extended EWMA control chart for moving average with exogenous factors model, *IAENG International Journal of Applied Mathematics*,

- Vol. 53,
No. 3, 2023, pp. 201-213, [Online].
https://www.iaeng.org/IJAM/issues_v53/issue_3/index.html (Accessed Date: October 16, 2025).
- [19] Sanusi, R. A., Teh, S. Y., and Khoo, M. B. Simultaneous monitoring of magnitude and time-between-events data with a Max-EWMA control chart, *Computers and Industrial Engineering*, Vol. 142, No. 1, 2023, pp. 1-11. <https://doi.org/10.1016/j.cie.2020.106378>.
- [20] Wu, S., Castagliola, P., and Celano, G. A distribution-free EWMA control chart for monitoring time-between-events-and-amplitude data, *Journal of Applied Statistics*, Vol. 48, No. 3, 2021, pp. 434-454. <https://doi.org/10.1080/02664763.2020.1729347>.
- [21] Hu, X., Castagliola, P., Zhong, J., Tang, A., and Qiao, Y. On the performance of the adaptive EWMA chart for monitoring time between events, *Journal of Statistical Computation and Simulation*, Vol. 91, No. 6, 2021, pp. 1175-1211. <https://doi.org/10.1080/00949655.2020.1843654>.
- [22] Xie, F., Castagliola, P., Qiao, Y., Hu, X., and Sun, J. A one-sided exponentially weighted moving average control chart for time between events, *Journal of Applied Statistics*, Vol. 49, No. 15, 2022, pp. 3928-3957. <https://doi.org/10.1080/02664763.2021.1967894>.
- [23] Aslam, M., Khan, M., Rasheed, Z., Anwar, S. M., and Abbasi, S. A. An improved adaptive EWMA control chart for monitoring time between events with application in health sector, *Quality and Reliability Engineering International*, Vol. 39, No. 4, 2023, pp. 1396-1412. <https://onlinelibrary.wiley.com/doi/10.1002/qre.3301>.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

Conceptualization: Kanita Petcharat,
Wannaphon Suriyakat
Data curation: Kanita Petcharat,
Wannaphon Suriyakat
Formal analysis: Kanita Petcharat,
Wannaphon Suriyakat
Funding acquisition: Kanita Petcharat.

Investigation: Kanita Petcharat,
Wannaphon Suriyakat
Methodology: Kanita Petcharat,
Wannaphon Suriyakat
Software: Kanita Petcharat, Wannaphon Suriyakat
Validation: Kanita Petcharat, Wannaphon Suriyakat
Writing – original draft: Kanita Petcharat,
Wannaphon Suriyakat
Writing – review and editing: Kanita Petcharat,

The authors contributed to the present research, at all stages, from the formulation of the problem to the final findings and solution.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

This Research was funded by King Mongkut's University of Technology North Bangkok, Contact no. KMUTNB-67-BASIC-23.

Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

Creative Commons Attribution License 4.0 (Attribution 4.0 International, CC BY 4.0)

This article is published under the terms of the Creative Commons Attribution License 4.0
https://creativecommons.org/licenses/by/4.0/deed.en_US

APPENDIX

Table 1. Control limits of the EWMA control chart with SARMAX processes and ARL0=370

Models	Coefficients							λ		
	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	β_1	β_2	0.15	0.2	0.25
SARMAX(1,1,1) ₁₂	0.2			0.2		0.3		0.1175480	0.1600110	0.2043540
SARMAX(1,1,2) ₁₂	0.3			0.3		0.1	0.3	0.1057412	0.1436240	0.1829960
SARMAX(1,2,1) ₁₂	0.4			0.4	0.2	0.5		0.1175480	0.1600108	0.2043540
SARMAX(2,1,1) ₁₂	0.1	0.2		0.3		0.5		0.0951780	0.1290260	0.1640580
SARMAX(2,2,1) ₁₂	0.1	0.1		0.1	0.1	0.1		0.1175480	0.1600110	0.2043540
SARMAX(2,2,2) ₁₂	-0.3	0.3		0.4	-0.4	0.1	0.1	0.1307620	0.1784410	0.2285040
SARMAX(3,1,2) ₁₂	0.3	0.3	0.4	0.4		0.2	0.2	0.0566278	0.0762380	0.0962396

Source: created by the authors

Table 2. Control limits of the EWMA control chart with SARMAX processes and ARL0=500

Models	Coefficients							λ		
	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	β_1	β_2	0.15	0.2	0.25
SARMAX(1,1,1) ₁₂	0.2			0.2		0.3		0.11754800	0.16010200	0.20447500
SARMAX(1,1,2) ₁₂	0.3			0.3		0.1	0.3	0.10580050	0.14370730	0.18310600
SARMAX(1,2,1) ₁₂	0.4			0.4	0.2	0.5		0.11761250	0.16010200	0.20447500
SARMAX(2,1,1) ₁₂	0.1	0.2		0.3		0.5		0.09523235	0.12910200	0.16415740
SARMAX(2,2,1) ₁₂	0.1	0.1		0.1	0.1	0.1		0.11761240	0.16010200	0.20447500
SARMAX(2,2,2) ₁₂	-0.3	0.3		0.4	-0.4	0.1	0.1	0.13083200	0.17854020	0.22863670
SARMAX(3,1,2) ₁₂	0.3	0.3	0.4	0.4		0.2	0.2	0.05666280	0.07628596	0.09630150

Source: created by the authors

Table 3. ARL results of explicit formulas and NIE method with Control limits of EWMA control chart with SARMAX(2,2,1)₁₂ process for different choices of λ with $\phi_1 = \phi_2 = \theta_1 = \theta_2 = \beta_1 = 0.1$, ARL₀=370

λ	H	Method	Shift size						
			0.001	0.003	0.005	0.01	0.03	0.05	0.1
0.15	0.1175480	Explicit	228.292979	129.292232	90.584399	52.028374	19.000794	12.583517	6.9505124
		NIE	228.292977	129.292036	90.584397	52.028356	19.000792	12.583517	6.9505124
		%Acc	99.9999991	99.9999848	99.999998	99.999995	99.9999654	100.000000	100.000000
0.2	0.1784410	Explicit	230.931585	132.019292	92.636910	53.959254	20.361979	12.919425	7.1229910
		NIE	230.931617	132.019026	92.636909	53.959228	20.391976	12.919425	7.1229910
		%Acc	99.9999996	99.9997985	99.999999	99.9999518	100.000000	100.000000	100.000000
0.25	0.2043540	Explicit	233.431429	134.596988	94.763455	54.7584893	20.9933832	13.277263	7.30774033
		NIE	233.431393	134.596887	94.763453	54.7584558	20.9933833	13.277199	7.30774033
		%Acc	99.9999846	99.9999250	99.999979	99.9996892	100.000000	99.999518	100.000000

Source: created by the authors

Table 4. ARL results of explicit formulas and NIE method with Control limits of EWMA control chart with SARMAX(2,2,2)₁₂ process for different choices of with $=-0.3, =0.3, =0.4, =-0.4, =0.1$, ARL₀=370

λ	H	Method	Shift size						
			0.001	0.003	0.005	0.01	0.03	0.05	0.1
0.15	0.1307620	Explicit	233.3445261	134.4062243	94.5974856	54.6542999	20.9111853	13.2763856	7.323552
		NIE	233.3445206	134.4062224	94.5974849	54.6542997	20.9111853	13.2763856	7.323552
		%Acc	99.9999976	99.9999986	99.9999893	99.9999996	100.00000	100.00000	100.00000
0.20	0.1784410	Explicit	236.2829476	137.352878	97.0235450	56.250977	21.564849	13.685899	7.534774
		NIE	236.2829457	137.352876	97.0234449	56.250977	21.564849	13.685899	7.534774
		%Acc	99.9999992	99.9999985	99.9998968	100.00000	100.00000	100.00000	100.00000
0.25	0.2285040	Explicit	239.2093611	140.4168514	99.5783790	57.9518970	22.2675683	14.1266459	7.6233994
		NIE	239.2092899	140.4168436	99.5783607	57.9518658	22.2675646	14.1266446	7.6233994
		%Acc	99.9999702	99.9999944	99.9999816	99.9999462	99.9999834	99.9999908	100.00000

Source: created by the authors

Table 5. ARL results of explicit formulas and NIE method with Control limits of EWMA control chart with SARMAX(3,1,2)12 process for different choices of with =0.3, =0.4, =0.4, =0.2, ARL0=370

λ	H	Method	Shift size						
			0.001	0.003	0.005	0.01	0.03	0.05	0.1
0.15	0.0566278	Explicit	200.487971	104.865842	71.201900	39.769271	14.871111	9.4351570	5.248623
		NIE	200.487964	104.865840	71.201899	39.769271	14.871110	9.4351570	5.248623
		%Acc	99.99994	99.99998	99.99999	100.000000	100.000000	100.000000	100.000000
0.20	0.0762380	Explicit	201.659360	105.823869	71.9323403	40.214859	15.0416369	9.5397928	5.5397904
		NIE	201.659358	105.823871	71.9323403	40.214861	15.0416369	9.5397928	5.5397904
		%Acc	99.999999	100.00000	100.00000	100.000000	100.00000	100.00000	100.00000
0.25	0.0962396	Explicit	202.815504	106.793553	72.6761520	40.6708584	15.2167441	9.6473335	5.3552190
		NIE	202.815481	106.793356	72.6761520	40.6708421	15.2167441	9.6473335	5.3552190
		%Acc	99.9999887	99.9998155	100.00000	99.9999599	100.00000	100.00000	100.00000

Source: created by the authors

Table 6. ARL results of explicit formulas and NIE method with Control limits of EWMA control chart with SARMAX(3,1,2)12 process for different choices of with =0.3, =0.4, =0.4, =0.2, ARL0=500

λ	H	Method	Shift size						
			0.001	0.003	0.005	0.01	0.03	0.05	0.1
0.15	0.0566628	Explicit	233.043971	113.077351	74.871911	40.871971	14.871110	9.4894121	5.263392
		NIE	233.043964	113.077349	74.871899	40.871971	14.871110	9.4894121	5.263392
		%Acc	99.999997	99.999998	99.999984	100.00000	100.00000	100.00000	100.00000
0.20	0.07628596	Explicit	234.622352	114.192779	75.680642	41.343761	15.1881254	9.5954523	5.3163311
		NIE	234.622349	114.192779	75.680640	41.343760	15.1881254	9.5954523	5.3163311
		%Acc	99.999999	100.00000	99.999997	99.581253	100.00000	100.00000	100.00000
0.25	0.0963015	Explicit	236.364505	115.361677	76.522454	41.8307585	15.3677216	9.7074473	5.3708435
		NIE	236.364502	115.361674	76.522454	41.8307583	15.3677216	9.7074473	5.3708435
		%Acc	99.999999	99.999997	100.00000	99.9999995	100.00000	100.00000	100.00000

Source: created by the authors

Table 7. Parameter estimation for fitting the stock price data for the PTT Public Company Ltd. in Thailand with the USD/THB exchange rate as the exogenous variable to the ARMAX process

Variable	Coefficient	Std.Error	Sig.
SAR(1) ($\hat{\phi}_1$)	0.6546	0.0654	0.000*
SMA(1) ($\hat{\phi}_1$)	-1.000	0.0844	0.044*
USD/THB exchange rate ($\hat{\beta}_1$)	-1.9036	0.2031	0.000*
Testing the distribution of the white noise			
Exponential Parameter (α_0)	0.555		
Kolmogorov-Smirnov	0.090		
Asymptotic Significance (2-Sided)	0.088 ^{ns}		

* significance level of 0.05

Source: created by the authors

Table 8. ARL results of explicit formulas and NIE method with Control limits of EWMA control chart with SARMAX(1,1,1)₁₂ process for different choices of λ with $\phi_1=0.6546$, $\theta_1=-1.0$, $\alpha_0=0.555$, $\beta_1=-1.9036$, $ARL_0=370$

λ	H	Method	Shift size						
			0.001	0.005	0.01	0.05	0.1	0.5	1.0
0.15	0.001766	Explicit	363.16733	336.41182	306.17471	152.21170	71.51493	2.902665	1.247425
		NIE	363.16730	336.41178	306.17471	152.21170	71.51493	2.902665	1.247423
		%Acc	99.999992	99.999988	100.00000	100.00000	100.00000	100.00000	100.00000
0.20	0.043700	Explicit	365.59363	346.004413	323.321359	195.20227	112.72151	8.797442	2.658591
		NIE	365.59313	346.004413	323.321359	195.20227	112.72151	8.797442	2.658491
		%Acc	99.999863	100.000000	100.00000	100.00000	100.00000	100.00000	100.00000
0.25	0.183200	Explicit	366.19514	349.767352	330.525584	216.43637	135.96330	14.68392	4.548213
		NIE	366.19521	349.767275	330.525425	216.43637	135.96330	14.68392	4.548213
		%Acc	99.9987200	99.9997300	99.9999519	100.000000	100.00000	100.00000	100.00000

Source: created by the authors