

On Supermodularity and Applications of the CPT-like Functionals

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Abstract: In this paper, expanding the concept of supermodularity of the Choquet integral, we present results from the recent authors paper related to the supermodularity of CPT-like functionals. Applications of CPT-like monotone integrals are illustrated with examples through the application of CPT-like functionals in decision-making for the representation of a preference relation and as actuarial integral-based premium principles.

Key-Words: capacity, supermodularity, Choquet integral, symmetric operations, CPT-like functional, premium principles

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1 Introduction

The basic properties of the Choquet integral are monotonicity and comonotone additivity, [1], [2], [3], [4]. In general, the Choquet integral is not an additive monotone integral based on a capacity. If the capacity is supermodular (resp. submodular), then the Choquet integral defined with respect to it is supermodular (resp. submodular), consequently superadditive (resp. subadditive). Recently, CPT-like functionals have been introduced and studied in [5], inspired by the concept of CPT functionals, originally introduced in [6]. As possible applications in decision-making and actuarial science of these concepts, an unified approach to the integral-based premium principles based on the monotone integral with respect to a capacity was presented. In an analogous way to the general CPT premium principle, the CPT-like integral-based premium principle has been introduced and characterized in [5]. Such integral-based premium principle generalizes some well-known premium principles in actuarial science such as the mean premium principle and the general CPT premium principle.

One of the most popular tools in multi-criteria decision-making related to the Choquet integral, are ordered weighted averages (OWA) introduced and studied in [7], [8], [9]. Bipolar ordered weighted averages (BOWA) were presented in [10]. In bipolar

context it appears that more effective than OWA are BOWA, recently introduced in [11]. They are related to the concept of bi-capacities, proposed in [12]. Further consideration on bipolarity in preference modeling were considered in [13], [14], [15]. Numerical characteristics of OWA were studied in [16], while the entropy of bi-capacities was introduced in [17].

The aim of this paper is to present some results of the ϕ -supermodularity of CPT-type functionals and illustrate their application in decision-making as monotone integral-based functionals. The paper is organized as follows. In the following section an overview of the basic concepts and definitions of theory of fuzzy integration are recalled, as well as the supermodularity theorem of the Choquet integral. In the third section, the main results related to supermodularity of capacities and the ϕ -supermodularity of CPT-like functionals are presented. Finally, their application is illustrated with examples and some concluding remarks are given.

2 Supermodularity of capacities and the Choquet integral

Let $(X, \mathcal{A})$ be a measurable space, with X non-empty and σ -algebra $\mathcal{A}$. The following definitions and statements are based on [1], [3], [18].

Definition 2.1. A set function $m : \mathcal{A} \rightarrow \mathbb{R}$, $m(\emptyset) = 0$, is called

- (i) *capacity* if m is non-negative and for all $A, B \in \mathcal{A}$, $A \subset B$ it holds $m(A) \leq m(B)$ (monotonicity),
- (ii) *t-monotone* if for some integer $t \geq 2$, and $A_1, \dots, A_t \in \mathcal{A}$, $A = \mathcal{P}(X)$, X finite, it holds $m(A_1 \cup \dots \cup A_t) \geq$

$$\geq \sum_{I \subset \{1, \dots, t\}, I \neq \emptyset} (-1)^{|I|+1} m\left(\bigcap_{i \in I} A_i\right), \quad (1)$$

- (iii) *supermodular* if for all $A, B \in \mathcal{A}$ it holds

$$m(A \cup B) + m(A \cap B) \geq m(A) + m(B). \quad (2)$$

If m is t -monotone for every $t \geq 2$, $t \in \mathbb{N}$, then m is called a *totally monotone* set function. If m is totally monotone, then it is also supermodular, but not vice-versa. In the theory of non-additive measures, 2-monotonicity of a set function is called supermodularity. Note that the non-negativity of a set function m together with the total monotonicity provides its monotonicity. A capacity m is *normalized* if $m(X) = 1$. If the inequality sign in (2) is reversed, we get the definition of a *submodular* set function m . A set function m is said to be *super/subadditive*, if the inequality (2)/(inverse inequality) is satisfied for all disjoint pairs of $\mathcal{A}$ -measurable sets.

Definition 2.2. Let $(X, \mathcal{A}, m)$ be a space of capacity. The Choquet integral, with respect to m , $\mathcal{A}$ -measurable function $f : X \rightarrow [0, \infty]$ is defined by:

$$Ch_m(f) = \int_0^\infty m(\{x | f(x) \geq t\}) dt. \quad (3)$$

Let us recall, that the integral defined above is monotone non-decreasing with respect to f , and also with respect to m , it is positively homogenous and comonotone additive. If $X = \{x_1, x_2, \dots, x_n\}$, then

$$Ch_m(f) = \sum_{i=1}^n (f(x_{\alpha(i)}) - f(x_{\alpha(i-1)})) m(A_{\alpha(i)}), \quad (4)$$

where $\alpha = (\alpha(1), \dots, \alpha(n))$ is a permutation such that $f(x_{\alpha(i)}) \leq f(x_{\alpha(j)})$, for all i, j , $i \leq j$, $f(x_{\alpha(0)}) = 0$, $A_{\alpha(n+1)} = \emptyset$ and $A_{\alpha(i)} = \{x_{\alpha(i)}, \dots, x_{\alpha(n)}\}$.

Denote with $\mathcal{F}(X)^+$ set of all $\mathcal{A}$ -measurable functions, $f : X \rightarrow [0, \infty]$. We have the next theorem on the supermodularity of Choquet integral (see, [3], [4]).

Theorem 2.3. A functional $Ch_m : \mathcal{F}(X)^+ \rightarrow [0, \infty]$, given by (3) for all $f \in \mathcal{F}(X)^+$ is supermodular on $\mathcal{F}(X)^+$, i.e. $\forall f, h \in \mathcal{F}(X)^+$ it holds:

$$Ch_m(f \vee h) + Ch_m(f \wedge h) \geq Ch_m(f) + Ch_m(h), \quad (5)$$

if m is a supermodular capacity.

Remark 2.4. Assuming that a capacity m is submodular, the Choquet integral with respect to m satisfies the opposite inequality, i.e. the Choquet integral on $\mathcal{F}(X)^+$ is a submodular functional.

3 ϕ -supermodularity of CPT-like functionals

In this section, we present our results from [18] related to the ϕ -supermodularity of CPT-like monotone integrals.

First, we recall that by composing a convex, non-decreasing function with a supermodular capacities, we also get a supermodular capacity. The following statement was proved in [18] (compare with Lemma 1 and Example 2 in [4]).

Proposition 3.1. If $g : [0, 1] \rightarrow [0, 1]$ is a convex non-decreasing function such that $g(0) = 0$ and $g(1) = 1$, and $m : \mathcal{A} \rightarrow [0, 1]$ is a supermodular normalized capacity, then $g \circ m$ is a supermodular normalized capacity.

Remark 3.2. Analogously, by composing a concave, non-decreasing function and submodular capacity, a submodular capacity is obtained.

Denote with $\mathcal{F}(X)$ set of all $\mathcal{A}$ -measurable functions $f : X \rightarrow \mathbb{R}$. Recall, for each $f \in \mathcal{F}(X)$, it holds $f = f^+ - f^-$, where $f^+ = f \vee 0$ and $f^- = (-f) \vee 0$, $f^+, f^- \in \mathcal{F}(X)^+$.

For an odd, increasing, continuous function $\phi : [-a, a] \rightarrow [-\infty, \infty]$, $a \in \mathbb{R}^+$, $\phi(0) = 0$, $\phi(a) = \infty$, the symmetric addition $\oplus : [-a, a]^2 \rightarrow [-a, a]$, generated by ϕ , with $\infty - \infty = \infty$, is given by:

$$x \oplus y = \phi^{-1}(\phi(x) + \phi(y)). \quad (6)$$

We say that $CPT_{m_1, m_2}^\phi : \mathcal{F}(X) \rightarrow \mathbb{R}$, defined for all $f \in \mathcal{F}(X)$ (with convention $\infty - \infty = \infty$) by:

$$CPT_{m_1, m_2}^\phi(f) = \phi^{-1}(Ch_{m_1}(\phi(f^+)) - Ch_{m_2}(\phi(f^-))) \quad (7)$$

where $\phi : \mathbb{R} \rightarrow \mathbb{R}$ is odd, continuous and increasing and m_1 and m_2 are capacities, is a *CPT-like functional*.

In the next theorem from [18], the ϕ -supermodularity of a CPT-like functional is considered.

Theorem 3.3. Let $CPT_{m_1, m_2}^\phi : \mathcal{F}(X) \rightarrow \bar{\mathbb{R}}$ be a CPT-like functional given by (1), where $\phi : \bar{\mathbb{R}} \rightarrow \bar{\mathbb{R}}$ is odd, continuous, and increasing function, and capacities $m_i, i = 1, 2$ given by $m_i = g_i \circ P$, where $g_i : [0, 1] \rightarrow [0, 1], g_i(0) = 0, g_i(1) = 1, i = 1, 2$ and $P : \mathcal{A} \rightarrow [0, 1]$ is a probability function. Then, for all functions f and $h, f, h \in \mathcal{F}(X)$, such that $CPT_{m_1, m_2}^\phi(f), CPT_{m_1, m_2}^\phi(h) \in \mathbb{R}$ it holds:

$$\begin{aligned} CPT_{m_1, m_2}^\phi(f \vee h) \oplus CPT_{m_1, m_2}^\phi(f \wedge h) &\geq \\ &\geq CPT_{m_1, m_2}^\phi(f) \oplus CPT_{m_1, m_2}^\phi(h), \end{aligned} \quad (8)$$

if g_1 is convex, and g_2 is concave function.

Proof. Let $f, h \in \mathcal{F}(X)$, be such that $CPT_{m_1, m_2}^\phi(f), CPT_{m_1, m_2}^\phi(h) \in \mathbb{R}$. Define $f \vee h$ and $f \wedge h$, where $\vee$ is the maximum operation, and $\wedge$ the minimum operation, with:

$$(f \vee h)(x) = f(x) \vee h(x), \quad x \in X; \quad (9)$$

$$(f \wedge h)(x) = f(x) \wedge h(x), \quad x \in X. \quad (10)$$

Using the assumed properties of ϕ , we obtain that $\phi(f) = \phi \circ f$ and $\phi(h) = \phi \circ h$, defined for all $x \in X$ with $\phi(f)(x) = \phi(f(x)), \phi(h)(x) = \phi(h(x))$, respectively, are $\mathcal{A}$ -measurable functions. Since ϕ is odd, continuous and increasing function, we obtain $\phi(f \vee h) = \phi(f) \vee \phi(h)$, hence:

$$\phi(f \vee h)^+ = \phi(f)^+ \vee \phi(h)^+; \quad (11)$$

$$\phi(f \vee h)^- = \phi(f)^- \wedge \phi(h)^-. \quad (12)$$

Analogously, we obtain $\phi(f \wedge h) = \phi(f) \wedge \phi(h)$, and that fact implies $\phi(f \wedge h)^+ = \phi(f)^+ \wedge \phi(h)^+$, as well as $\phi(f \wedge h)^- = \phi(f)^- \vee \phi(h)^-$. Therefore, it holds:

$$Ch_{m_1}(\phi(f \vee h)^+) = Ch_{m_1}(\phi(f^+) \vee \phi(h^+)); \quad (13)$$

$$Ch_{m_1}(\phi(f \wedge h)^+) = Ch_{m_1}(\phi(f^+) \wedge \phi(h^+)); \quad (14)$$

$$Ch_{m_2}(\phi(f \vee h)^-) = Ch_{m_2}(\phi(f^-) \wedge \phi(h^-)); \quad (15)$$

$$Ch_{m_2}(\phi(f \wedge h)^-) = Ch_{m_2}(\phi(f^-) \vee \phi(h^-)). \quad (16)$$

Now, by adding the first two equations, subtracting the next two, applying Proposition 3.1 and its analogue for concave functions, Theorem 2.3, Remark 2.4, the definition of symmetric addition generated by ϕ , and using the fact that ϕ^{-1} is also an increasing function, we get the statement. $\square$

Notice that the reverse inequality holds under the assumption that g_1 is concave and g_2 is a convex function.

The above result is illustrated in the next example from [18].

Example 3.4. Let $X = \{x_1, x_2, x_3, x_4\}$, $\text{card}(X) = 4$, $P(A) = \frac{\text{card}(A)}{4}$, for all $A \in \mathcal{P}(X)$. Let $g_1(x) = x^3, g_2(x) = 1 - (1 - x)^3, x \in [0, 1]$. Consider $m_i = g_i \circ P, i = 1, 2$, such that

$\text{card}(A), A \subset X$	$m_1(A)$	$m_2(A)$
$\text{card}(A) = 1$	$\frac{1}{64}$	$\frac{37}{64}$
$\text{card}(A) = 2$	$\frac{1}{8}$	$\frac{7}{8}$
$\text{card}(A) = 3$	$\frac{27}{64}$	$\frac{63}{64}$
$\text{card}(A) = 4$	1	1

By Proposition 3.1 and Remark 3.2, since g_1 is a convex function and g_2 is concave, while the probability function P is modular (both supermodular and submodular), it follows that m_1 is a supermodular capacity, while m_2 is a submodular capacity. Notice that the supermodularity of the capacity m_1 ensures its superadditivity, while the submodularity of m_2 ensures its subadditivity.

Let $f, h \in \mathcal{F}(X)$ be defined by: $f(x_1) = 2, f(x_2) = 5, f(x_3) = f(x_4) = 0, h(x_1) = 4, h(x_2) = 3, h(x_3) = -2, h(x_4) = 0$. Let $\phi(x) = \text{sgn}(x) \cdot x^4, x \in \bar{\mathbb{R}}$. By (4) and (1), we get:

F	$\phi^{-1}(Ch_{m_1}(\phi(F^+)))$	$\phi^{-1}(Ch_{m_2}(\phi(F^-)))$
f	1.84214	0
h	1.89367	1.74396
$f \vee h$	2.47899	0
$f \wedge h$	1.31778	1.74396

For $f, h, f \vee h, f \wedge h$, by (1), we obtain $CPT_{m_1, m_2}^\phi(f) = 1.84214, CPT_{m_1, m_2}^\phi(h) = 1.37833, CPT_{m_1, m_2}^\phi(f \vee h) = 2.47899$, and $CPT_{m_1, m_2}^\phi(f \wedge h) = -1.58016$.

Since we have $CPT_{m_1, m_2}^\phi(f \vee h) \oplus CPT_{m_1, m_2}^\phi(f \wedge h) \approx 2.37, CPT_{m_1, m_2}^\phi(f) \oplus CPT_{m_1, m_2}^\phi(h) \approx 1.97$, the inequality presented in Theorem 3.3 holds.

3.1 Application of the CPT-like functional

Inspired by [5], [19], [20], we consider applications of the CPT-like integral-based functional in decision theory for a preference relation representation and as actuarial premium principles.

We will present a problem such that the monotone integral, the Lebesgue-integral based functional, the asymmetric Choquet integral-based functional, the general CPT-integral based functional are not appropriate for modeling decision-making when the utility of decision-maker is the identity function.

Example 3.5. Let us consider ranking of four risks in four different banks B_1, B_2, B_3, B_4 which present

investments based on the foreign currency savings. We will assess what bank offers most favorable the interest rates on foreign currency savings under the same term in currencies c_1, c_2 and c_3 . Let the interest rates per annum be given as a percent by

	c_1	c_2	c_3
B_1	0.1	0.7	0.9
B_2	0.2	0.7	0.8
B_3	0.15	0.4	0.7
B_4	0.25	0.4	0.6

The depositors make mostly deposits in currencies c_2 and c_3 and then in currency c_1 . Since in the banks B_1 and B_2 the currencies c_2 and c_3 are favorable, more regard is given to currency c_1 . Thus we will write $B_2 \succ B_1$. Furthermore, due to the banks B_3 and B_4 offer unfavorable the interest rates on savings in the currency c_2 , more regard is given to currency c_3 , thus $B_3 \succ B_4$.

Denote

$$I(f) = Ch_m(f), \quad (17)$$

$$\Pi_{CPT_\phi}(f, m_1, m_2) = CPT_{m_1, m_2}^\phi(f). \quad (18)$$

If we use (4) to calculate monotone integral related to B_1 and B_2 , we get

$$I(B_1) =$$

$$= 0.1 m(\{c_1, c_2, c_3\}) + 0.6 m(\{c_2, c_3\}) + 0.2 m(\{c_3\}),$$

$$I(B_2) =$$

$$= 0.2 m(\{c_1, c_2, c_3\}) + 0.5 m(\{c_2, c_3\}) + 0.1 m(\{c_3\}).$$

$B_2 \succ B_1$ implies $I(B_1, m) < I(B_2, m)$, i.e.,

$$m(\{c_1, c_2, c_3\}) > m(\{c_2, c_3\}) + m(\{c_3\}). \quad (19)$$

On the other side, since we require $B_3 \succ B_4$ and $I(B_3) =$

$$= 0.1 m(\{c_1, c_2, c_3\}) + 0.2 m(\{c_2, c_3\}) + 0.4 m(\{c_3\}),$$

$$I(B_4) =$$

$$= 0.2 m(\{c_1, c_2, c_3\}) + 0.1 m(\{c_2, c_3\}) + 0.3 m(\{c_3\}),$$

we have

$$m(\{c_1, c_2, c_3\}) < m(\{c_2, c_3\}) + m(\{c_3\}). \quad (20)$$

The contradiction based on (19) and (20) yields that the monotone integral is not suitable for representing this interaction between criteria.

The currencies c_1, c_2 and c_3 have been considered on a 0 – 1 scale, now we will observe the same currencies but on a scale –0.5 – 0.5, i.e., it holds

	c_1	c_2	c_3
B_1	–0.4	0.2	0.4
B_2	–0.3	0.2	0.3
B_3	–0.35	–0.1	0.2
B_4	–0.25	–0.1	0.1

For $\phi(x) = x, x \in \mathbb{R}$, where $X = \{c_1, c_2, c_3\}$, we have $\Pi_{CPT_\phi}(B_1, m_1, m_2) =$

$$= 0.2 m_1(\{c_2, c_3\}) + 0.2 m_1(\{c_3\}) - 0.4 m_2(\{c_1\}),$$

$$\Pi_{CPT_\phi}(B_2, m_1, m_2) =$$

$$= 0.2 m_1(\{c_2, c_3\}) + 0.1 m_1(\{c_3\}) - 0.3 m_2(\{c_1\}),$$

$$\Pi_{CPT_\phi}(B_3, m_1, m_2) =$$

$$= 0.2 m_1(\{c_3\}) - 0.1 m_2(\{c_2, c_1\}) - 0.25 m_2(\{c_1\}),$$

$$\Pi_{CPT_\phi}(B_4, m_1, m_2) =$$

$$= 0.1 m_1(\{c_3\}) - 0.1 m_2(\{c_2, c_1\}) - 0.15 m_2(\{c_1\}).$$

Due to $B_2 \succ B_1$ and $B_3 \succ B_4$, i.e., $\Pi_{CPT_\phi}(B_1, m_1, m_2) < \Pi_{CPT_\phi}(B_2, m_1, m_2)$ and $\Pi_{CPT_\phi}(B_4, m_1, m_2) < \Pi_{CPT_\phi}(B_3, m_1, m_2)$, the following inequalities are valid, respectively,

$$m_1(\{c_3\}) < m_2(\{c_1\}), \quad (21)$$

$$m_2(\{c_1\}) < m_1(\{c_3\}). \quad (22)$$

Hence, in this case we also get contradiction and we can conclude the general CPT integral-based functional is not appropriate for representation of this problem. Notice that neither the Lebesgue-integral based functional nor the asymmetric Choquet integral-based functional are appropriate for representing required interaction among criteria.

Now let $\phi(x) = \sqrt[3]{x}, x \in \mathbb{R}$. It holds

$$\Pi_{CPT_\phi}(B_1, m_1, m_2) = \left(\sqrt[3]{0.2} m_1(\{c_2, c_3\}) + (\sqrt[3]{0.4} - \sqrt[3]{0.2}) m_1(\{c_3\}) - \sqrt[3]{0.4} m_2(\{c_1\}) \right)^3,$$

$$\Pi_{CPT_\phi}(B_2, m_1, m_2) = \left(\sqrt[3]{0.2} m_1(\{c_2, c_3\}) + (\sqrt[3]{0.3} - \sqrt[3]{0.2}) m_1(\{c_3\}) - \sqrt[3]{0.3} m_2(\{c_1\}) \right)^3,$$

$$\begin{aligned} \Pi_{CPT_\phi}(B_3, m_1, m_2) &= \\ &= \left(\sqrt[3]{0.2} m_1(\{c_3\}) - \sqrt[3]{0.1} m_2(\{c_2, c_1\}) - (\sqrt[3]{0.35} - \sqrt[3]{0.1}) m_2(\{c_1\}) \right)^3, \end{aligned}$$

$$\begin{aligned} \Pi_{CPT_\phi}(B_4, m_1, m_2) &= \\ &= \left(\sqrt[3]{0.1} m_1(\{c_3\}) - \sqrt[3]{0.1} m_2(\{c_2, c_1\}) - (\sqrt[3]{0.25} - \sqrt[3]{0.1}) m_2(\{c_1\}) \right)^3. \end{aligned}$$

Due to $B_2 \succ B_1$ and $B_3 \succ B_4$ the following inequalities are valid, respectively,

$$m_1(\{c_3\}) < m_2(\{c_1\}), \quad (23)$$

$$(\sqrt[3]{0.35} - \sqrt[3]{0.25}) m_2(\{c_1\}) < (\sqrt[3]{0.2} - \sqrt[3]{0.1}) m_1(\{c_3\}) \quad (24)$$

i.e.,

$$m_1(\{c_3\}) < m_2(\{c_1\}) < \frac{\sqrt[3]{0.2} - \sqrt[3]{0.1}}{\sqrt[3]{0.35} - \sqrt[3]{0.25}} m_1(\{c_3\}) \quad (25)$$

In this case, there is no contradiction. For example, if $m_1(\{c_3\}) = 0.5$ and $m_2(\{c_1\}) = 0.6$ then the inequalities (25) are satisfied. Hence, the CPT-like integral-based functional related to function ϕ defined by $\phi(x) = \sqrt[3]{x}$, $x \in \mathbb{R}$ can be applied in modeling observed problem.

In insurance, the integral-based premium principle is a rule, i.e. a real-valued functional Π , for assigning a premium $\Pi(f)$ to an insurance risk f . For more details about integral-based premium principles, appropriate for calculation insurance prices, we refer to [5], [6], [21].

Using the definition of CPT-like functional, in the next example some integral-based premium principles are considered (for details, see [5]).

Example 3.6. Let $f \in \mathcal{F}(X)$ be a discrete insurance risk.

Net premium principle:

Consider $\phi(x) = x$, i.e. $\phi = id$, and $m_1 = m_2 = P$, where P is a probability measure. We have

$$\Pi_N(f) = CPT_{P,P}^{id}(f) \quad (26)$$

i.e. CPT-like functional is the mathematical expectation $E(f)$ of f .

Mean premium principle:

Consider odd, continuous and strictly increasing ϕ , and $m_1 = m_2 = P$, where P is a probability measure. Then

$$\Pi_M(f) = CPT_{P,P}^{\phi}(f) \quad (27)$$

i.e. it coincides with the mean premium principle for a insurance risk f .

Distortion (Wang's) premium principle:

For $\phi = id$, a normalized capacity m and its dual capacity $\bar{m}$, $\bar{m}(A) = 1 - m(X \setminus A)$, where $m = g \circ P$ for a non-decreasing distortion function $g : [0, 1] \rightarrow [0, 1]$, $g(0) = 0$, $g(1) = 1$, we define

$$\Pi_{CPT_g}(f) = CPT_{m, \bar{m}}^{id}(f) \quad (28)$$

CPT-like premium principle:

In general, for capacities m_1 and m_2 , and odd, continuous and strictly increasing ϕ , we define

$$\Pi_{CPT_{\phi}}(f) = CPT_{m_1, m_2}^{\phi}(f) \quad (29)$$

4 Conclusion

We have presented the concept of CPT-like functionals, originally introduced and studied in [5], through an application in decision-making as numerical models for the representation of the preference relation. The ϕ -supermodularity of CPT-like functionals has been presented according to [18]. The CPT-like functionals can be applied in actuarial sciences as a common frame for some insurance premium principles. A generalization of this concept was considered in literature, namely, based on the bipolar Choquet g -integrals, the g -BIOWA operators was introduced. In the future work, numerical characteristics of the g -BIOWA operators will be studied.

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Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work, the author(s) utilized artificial intelligence (AI) technologies solely for the purpose of linguistic refinement and grammatical correction. Following the application of these tools, the author(s) comprehensively reviewed, verified, and edited the resulting text. The author(s) maintain full accountability and responsibility for the accuracy, integrity, and final content of the publication.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

Mirjana Štrboja carried out: Conceptualization, Formal analysis, Methodology, and Writing - Original Draft.

Biljana Mihailović carried out: Validation, Formal analysis, Writing - Original Draft.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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