The Effect of Loading Rates on Mechanical Behaviour of Inelastic Solids

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Abstract: - In this paper, a computational study is presented to analyze the structural behaviour of solid materials undergoing inelastic strains when they are subject to various loading rates. An implicit integration algorithm is applied for the mechanical simulation of solids that exhibit plastic deformations. A numerical procedure is discussed that is useful to be applied to different types of constitutive models by suitable specialization of the proper flow function. Numerical algorithms are implemented, and computational examples are illustrated by denoting the effectiveness of the adopted numerical procedure.

Key-Words: - loading rates, inelastic solids, constitutive relations, computational solid mechanics, plastic behaviour, viscoplastic behaviour.

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1 Introduction

Computational mechanics approaches in the numerical simulation of solids with inelastic deformations have recently been the object of many research works, and different contributions have been formulated from radial return schemes to complex integration procedures. A consolidated approach to numerical integration schemes applies an operator split method consisting of an elastic prediction part and a subsequent plastic correction algorithm, [1]. Numerical integration procedures suitable for rate-independent and rate-dependent elastoplasticity problems have been presented, among others, in [1] and [2]. A computational formulation based on an algorithmic compliance tensor for rate-dependent solids exhibiting inelastic behaviour was proposed in [3]. A numerical approach with computational algorithms suitable for general yield criteria was presented in [4].

Alternative numerical integration schemes with computational algorithms for elastoplasticity with nonlinear kinematic hardening are illustrated e.g. in [5] and [6]. In this research study, the mechanical behaviour of solid materials frequently described as rate-sensitive materials is analyzed, [7]. Such mechanical behaviour of materials is typically shown by solids in which viscous deformations are displayed beyond elastic terms.

In this analysis, numerical integration schemes are considered for a computational simulation of evolutionary problems in inelastic solids. The adopted integration scheme holds for different types of constitutive relations, and the numerical algorithmic procedure is suitably implemented with the formulation of a consistent tangent operator, which can be suitably specialized to arbitrary yield criteria, [4].

Numerical results and computational applications are discussed for a benchmark problem with the objective of illustrating the effectiveness and usefulness of the discussed computational analysis.

2 Flow Laws in Inelastic Materials

Let us consider a continuum B which occupies the domain $\Omega \subset R^n$, in which we consider $1 \leq n \leq 3$, with $T \subset R^+$ indicating a suitable interval of time, V denoting a space of definition for displacement vectors, D a space of definition for strain tensors, and S a space of definition for stress tensors. A displacement is defined by $\mathbf{u}$: Ω x T $\rightarrow$V, a stress tensor is defined by $\boldsymbol{\sigma}$: $\Omega \times T \rightarrow S$, and a compatible strain tensor is assumed as defined by $\boldsymbol{\varepsilon} = \nabla^s \mathbf{u}$: Ω x T$\rightarrow$D, in which ∇^s represents the symmetric of the gradient.

In the present analysis, we adopt the assumption of infinitesimal strains and quasi-static loading programs, see among others [8] and [9]. Elastic material behaviour is assumed by defining an elastic energy W: D$\rightarrow$R together with an elastic

complementary energy W*: S→R. For linear elastic behaviour, the elastic energy is denoted by $W(\boldsymbol{\varepsilon}^{e}) = \frac{1}{2}\langle \mathbf{C}\boldsymbol{\varepsilon}^{e}, \boldsymbol{\varepsilon}^{e}\rangle$ and the elastic complementary energy is denoted by $W^{*}(\boldsymbol{\sigma}) = \frac{1}{2}\langle \boldsymbol{\sigma}, \mathbf{C}^{-1}\boldsymbol{\sigma}\rangle$, where $\boldsymbol{\varepsilon}^{e}$ indicates an elastic deformation, $\mathbf{C}$ represents the elastic stiffness, and $\langle\cdot,\cdot\rangle$ represents a non-degenerate bilinear form. Accordingly, the inelastic deformation is assumed as $\boldsymbol{\varepsilon}^{vp} = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{e}$, by combining plastic and viscoplastic deformations. By assuming a convex yield function $g(\boldsymbol{\sigma})$, the elastic domain is accordingly defined as $E = \{\boldsymbol{\sigma} \in S: g(\boldsymbol{\sigma}) \leq 0\}$. Herein, an internal variable treatment is suitably adopted for representing hardening variables, see for instance, [10].

Constitutive relationships between state variables and the corresponding associated variables may also be suitably illustrated within a convex analysis approach; in this regard see for instance the mathematical framework presented by [4]. A proper variational framework for plasticity and viscoplasticity problems in solid mechanics was also proposed in [4]. The corresponding computational algorithmic schemes and numerical applications were discussed in [11].

In rate-dependent inelastic formulations the constitutive equations are defined by optimality requirements of the regularized functional defining the rate-dependent plastic dissipation. The rate-dependent problem in plasticity is considered a regularization with penalization of the plasticity problem. Various specializations of the rate-dependent plastic constitutive formulations are defined by suitably specializing the penalty function, see for instance [4] and [11].

The rate-dependent plasticity evolutionary law is formulated by the Perzyna model, [7]

$$\dot{\boldsymbol{\varepsilon}}^{vp} = (1/\eta) < \Psi\,(g(\boldsymbol{\sigma}, \boldsymbol{\xi})) > \partial_{\boldsymbol{\sigma}}\, g(\boldsymbol{\sigma}, \boldsymbol{\xi}), \quad (1)$$

in which the MacAuley bracket $\langle\cdot\rangle$ is assumed to be $\langle z\rangle = (z+|z|)/2$ and $\eta>0$ represents a viscosity parameter. The flow function is herein defined by the expression associated with linear viscous terms and is assumed as $\Psi(g(\boldsymbol{\sigma}, \boldsymbol{\xi}))=g(\boldsymbol{\sigma}, \boldsymbol{\xi})$. Formulations of the flow function with nonlinear viscous terms are provided, for instance in [8].

Rate-dependent plastic models are properly reduced to the inviscid behaviour when $\eta\to 0$. The von Mises yield limit criterion is considered herein by assuming linear hardening and expressed as $g(\boldsymbol{\sigma}, \boldsymbol{\xi}_{kin}, \xi_{iso}) = \|\mathrm{dev}\boldsymbol{\sigma} - \boldsymbol{\xi}_{kin}\| - \upsilon(\xi_{iso}) = \|\boldsymbol{\rho}\| - (\sqrt{(2/3)})(\sigma_{yo} + \xi_{iso}) \leq 0$, in which $\mathrm{dev}\boldsymbol{\sigma}$ represents the deviator of the stress $\boldsymbol{\sigma}$, the relative stress is represented by $\boldsymbol{\rho} = \mathrm{dev}\boldsymbol{\sigma} - \boldsymbol{\xi}_{kin}$, the back stress is represented by $\boldsymbol{\xi}_{kin}$, the intersection between the deviatoric plane and the limit yield criterion is given by the radius $\upsilon(\xi_{iso}) = (\sqrt{(2/3)})(\sigma_{yo} + \xi_{iso})$, in which the uniaxial yield stress for a virgin material is represented by σ_{yo}. The static internal variable for linear isotropic hardening is given by $\xi_{iso} = H_{iso}\alpha_{iso}$, where H_{iso} is the isotropic hardening modulus, and α_{iso} is the associated kinematic internal variable herein expressed by $e^{vp} = \int(\sqrt{(2/3)})\|\dot{\boldsymbol{\varepsilon}}^{vp}\|\, dt$, and representing the equivalent viscoplastic strain.

A nonlocal modelling of plasticity and viscoplasticity associated to an optimization condition of nonlocal maximum dissipation property has been proposed in [12].

A proper computational approach with the numerical integration scheme of evolutive processes of plastic and viscoplastic solids was presented e.g. in [1], [2], [3], [4]. Extensions of formulations to account for various constitutive models have been proposed among others in [4], [11].

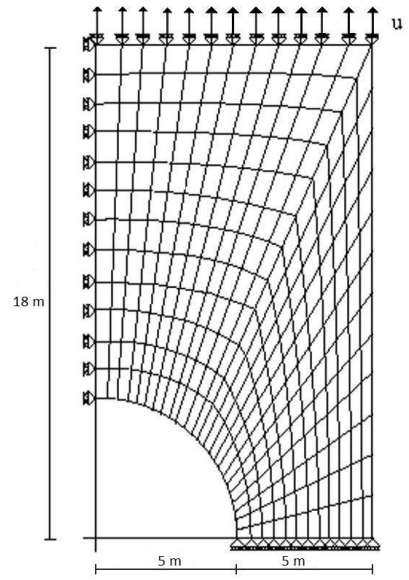

Fig. 1: The solid representing a perforated strip discretized by a finite element mesh
Source: created by the author

The discussed inelastic constitutive relations can be properly adopted in many applicative engineering problems, such as the hysteretic mechanical behaviour in cyclic loading processes for nonlinear structural analysis, at this regard see for instance, [1], [2], [4], [5], [6]. Non-smooth problems in plasticity and viscoplasticity can be analyzed by referring to [4], [13] and [14].

A comprehensive theoretical framework and a computational mechanics analysis in the simulation of problems in elastoplasticity and viscoplasticity is provided in [15], [16], [17], [18], [19], [20], [21],

[22], [23], [24], [25], [26], [27], [28], [29], [30], [31].

For the development of extremum properties and variational principles in elastoplasticity and viscoplasticity and the formulation of variationally consistent computational algorithms, among others see [32], [33], [34], [35], [36], [37], [38], [39], [40], [41], [42], [43], [44].

For an extensive presentation of continuum and computational mechanics methods for inelastic solids we refer for instance to [1], [2], [45], [46], [47], [48], [49], [50], [51], [52].

The modelling discussed in the present study can be usefully applied in the computational analysis for simulating the effects of loading rates in the mechanical behaviour of solid materials, see for instance its application to prospective extensions in areas of research analyzed in [53], [54], [55], [56], [57], [58], [59], [60], [61], [62].

3 Numerical Results

In the present investigation we analyze the plane strain problem of a perforated strip. Due to symmetry conditions only quarter of the strip is considered in the analysis. In Figure 1 the geometrical details of the strip and the applied loadings are shown. The adopted finite element mesh in modelling the solid is represented by 288 elements with 325 nodes. The mechanical parameters of the solid are defined by the elastic modulus $E=2.1\cdot10^5$ MPa, the yield limit $\sigma_y=240$ MPa, the Poisson ratio $\nu=0.3$, the kinematic hardening modulus $H_{kin}=2.1\cdot10^3$ MPa, and the isotropic hardening modulus $H_{iso}=6.0\cdot10^2$ MPa. Herein, a computational analysis is investigated by assuming a prescribed displacement imposed at the upper edge of the solid strip. The displacement is prescribed by applying incremental steps Δu until a total prescribed displacement $u_{max}=10$ cm.

In the performed numerical simulations, the rate-dependent constitutive relations of the Perzyna model (1) have been adopted by assuming linear viscous terms. The rate-dependent mechanical analysis of the adopted constitutive relations is described in Figure 2, in which the stress vs. displacement plot is represented. Stress plotted in Figure 2 is the vertical stress computed in element No. 1 at the horizontal abscissa at 5.0 m, that is, at the inner circle of the perforated strip. Displacement plotted in Figure 2 is considered as the displacement prescribed at the upper edge of the solid strip.

Different curves are plotted in Figure 2 corresponding to the different rates of application of the loading. The different curves are associated with different values assumed by the dimensionless loading rate parameter $\tau = (t_R/L_c)(\Delta u/\Delta t)$, which depends on the different loading rate $\Delta u/\Delta t$ with which Δu can be imposed. The intrinsic material properties are accounted for by specifying the value of the relaxation time $t_R = \eta/2G$, and by specifying a characteristic length $L_c=L/c$ which characterizes the geometry of the problem, in which the length L of the strip is herein set equal to 18 m, c is a suitable constant herein assumed as 2900, and where G is the shear modulus.

The rate-independent analysis corresponding to the inviscid material behaviour of the elastoplastic solid is properly exhibited as $\tau \to 0$.

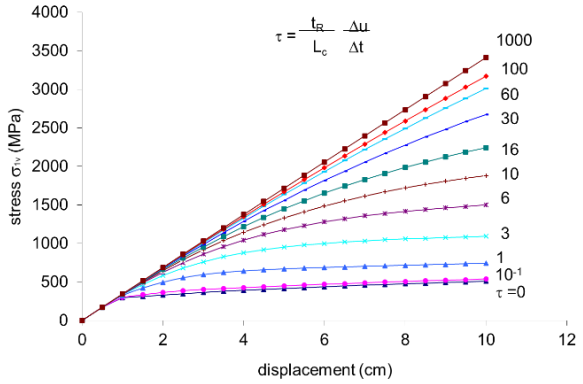

Fig. 2: Plot of stress vs imposed upper edge displacement for different values of the prescribed loading rate parameter τ
Source: created by the author

4 Conclusion

In this research analysis, a suitable computational approach has been adopted and discussed for the numerical simulation of evolutionary problems in inelastic solid materials.

A numerical integration procedure has been illustrated for simulating evolutive flow laws associated with constitutive models for rate-independent and rate-dependent problems in plasticity, by implementing a useful algorithmic scheme.

The present integration scheme has the advantage of holding for different types of constitutive relations and models in which arbitrary flow functions can be assumed and suitably specified as associated with the adopted constitutive model.

The computational solution procedure is implemented further by the derivation of a consistent tangent operator that can be suitably

specialized for arbitrary expressions of the yield criteria.

Computational applications and algorithmic results are illustrated by showing the effectiveness of the adopted computational approach and by properly illustrating the usefulness of the discussed computational algorithmic scheme.

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Conflict of Interest

The author has no conflicts of interest to declare that are relevant to the content of this article.

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