

Cost-Effective Joint Placement of μ PMU and AMI in Active Distribution Networks: A MILP Formulation with ANOVA-Validated Performance

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Abstract: The integration of distributed energy resources (DERs) into active distribution networks (ADNs) aggravates existing observability limitations, demanding advanced measurement infrastructure beyond traditional SCADA systems. Micro-phasor measurement units (μ PMUs) provide synchronized, high-resolution measurements at sampling rates between 30 and 120 samples per second. However, their high cost—approximately 22 times that of advanced metering infrastructure (AMI) devices—limits large-scale deployment. This paper formulates a mixed-integer linear programming (MILP) model for the Joint Placement of μ PMUs and AMI meters that minimizes capital expenditure (CAPEX) while guaranteeing complete topological observability, DER coverage, and channel constraints. Seven optimization methods (exact, heuristic, and metaheuristic approaches) are evaluated on four IEEE benchmark systems (13–240 buses) under 80 scenario combinations of DER penetration (0–80%) and pre-existing SCADA coverage (0–80%), yielding 560 experiments.

Key-Words: μ PMU placement, AMI, joint placement, MILP, Active Distribution Networks (ADN), DER, ANOVA, observability.

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1 Introduction

Historically, distribution networks have functioned with very limited real-time observability beyond the substation level, depending instead on pseudo-measurements based on load profiles and historical data, [1], [2], [3]. This paradigm is becoming increasingly unsustainable as distributed energy resources (DERs)—such as photovoltaic systems, battery energy storage systems (BESS), and electric vehicles—convert traditional feeders into active distribution networks (ADNs) characterized by bidirectional power flows, fast voltage variations, and stochastic generation, [4], [5], [6].

Advanced metering infrastructure (AMI) has improved data availability at the distribution level, providing voltage magnitude, active and reactive power at 15–60 minute intervals, [7], [8]. However, AMI measurements lack phase-angle information and time synchronization, limiting their utility for dynamic state estimation and fault detection. Micro-phasor measurement units (μ PMUs) address these limitations by providing GPS-synchronized voltage and current phasors at 30–120 samples per second with total vector error (TVE) below 0.05%, [9], [10]. However, the total installed cost of a μ PMU—including hardware, communications, and installation—is about \$6,200 USD per device, whereas an AMI meter costs roughly \$280 USD per unit, corresponding to a cost ratio of 22.1:1. This disparity makes massive

μ PMU deployment economically infeasible for most distribution system operators (DSOs), [11].

1.1 Literature Review

The optimal PMU placement (OPP) problem has been extensively studied for transmission networks. Exact methods, particularly integer linear programming (ILP) formulations, guarantee global optimality and have been the predominant approach since the foundational work presented in: [12], who demonstrated that binary ILP can solve the OPP in polynomial time for practical-sized systems.

Extensions include zero-injection bus attacks, [13], two-stage approaches that first minimize the required number of PMUs and subsequently maximize measurement redundancy, [14], as well as the incorporation of machine learning techniques to accelerate solution times, [15]. Recent contributions have further refined ILP-based OPP by incorporating normalized degree of centrality into the objective function, demonstrating improved SORI metrics while maintaining minimum PMU count across standard IEEE benchmarks, [16].

Heuristic methods comprise incremental placement algorithms, [17], the connectivity analysis method (CMA), [18], as well as the modified connectivity analysis method (MCMA), [19]. Despite being computationally efficient, these methods can lead to device overprovisioning in larger networks, given their lack of global optimization guarantees.

Table 1: Positioning relative to prior work.

Reference	Net. type	Joint PMU+AMI	DER cstr.	Ch. cstr.	ANOVA valid.	Syst. (buses)
[12]	T	–	–	–	–	14–300
[13]	T	–	–	–	–	14–118
[23]	T	✓	–	✓	–	14–118
[24]	D	✓	–	–	–	33–69
[27]	D	–	✓	–	–	33–119
[14]	D	–	–	–	–	34–123
This work	D	✓	✓	✓	✓	13–240

Source: created by the authors.

T = transmission; D = distribution; Ch. = channel; cstr. = constraint.

On the other hand, metaheuristic approaches such as genetic algorithms (GA), particle swarm optimization (PSO), and their hybrid variants have been extensively used for the OPP, providing flexibility in handling multi-objective formulations, [20], [21]. However, their stochastic nature introduces solution variability, and scalability remains a concern for networks exceeding 100 buses, [16].

Although there is extensive literature on the placement of individual technologies, the *joint placement* of heterogeneous measurement devices (μ PMU, AMI, and SCADA) in distribution networks is still an unresolved issue. Recent efforts have addressed multi-source state estimation architectures, [22], μ PMU co-optimization with communication constraints, [23], [24], and joint PMU placement with fault location, [25], [26].

Nevertheless, a comprehensive review, [16], [27], confirms that no existing formulation simultaneously addresses: (i) joint μ PMU–AMI allocation with mutual exclusion, (ii) DER coverage as a mandatory constraint, (iii) pre-existing SCADA integration, (iv) channel capacity limits, and (v) statistical validation of algorithm performance. Table 1 summarizes the positioning of this work relative to representative prior studies.

1.2 Proposed Approach and Contributions

Accordingly, this paper presents a fundamentally different approach for ADN measurement infrastructure planning. Rather than treating μ PMU placement as an isolated optimization problem, [28], [29], we formulate a joint placement model that simultaneously allocates μ PMUs and AMI meters, leveraging the complementary strengths of both technologies. Specifically, μ PMUs provide sub-second, GPS-synchronized phasor measurements for dynamic monitoring at DER-hosting buses, [5], [30], while AMI enables cost-effective topological observability across the rest of the feeder, [31], [32]. This hybrid strategy, together with the explicit incorporation of existing SCADA infrastructure and DER coverage requirements, results in a comprehensive MILP formulation that is both cost-optimal and operationally

practical.

To clarify the novelty of our approach against both recent literature and our previous investigations, it is worth noting that while recent studies have explored the operational integration of AMI and state estimation, including consumption analytics, demand response, and data-fusion techniques, [33], [34], none explicitly addresses the joint physical planning problem of heterogeneous measurement infrastructure in active distribution networks. Recent optimal-placement works either treat pre-existing metering strictly as fixed input data without co-optimizing capital expenditure, [34], [35], or rely on metaheuristic approaches such as NSGA-II, [36], whose stochastic behavior may affect scalability and repeatability in large feeders. Furthermore, this study is independent of previous efforts by the authors on scheduling and uncertainty quantification, [37]; it introduces an original MILP formulation specifically designed to arbitrate the 22.1:1 cost differential between μ PMUs and AMI meters while guaranteeing complete topological observability, DER coverage, and channel-feasible placement through an exact optimization framework.

The main contributions of this paper can be summarized as follows:

- A novel MILP formulation for the joint placement of μ PMUs and AMI meters in active distribution networks, incorporating mutual exclusion constraints, hybrid observability (direct, adjacent, and pre-existing SCADA), mandatory DER coverage, and channel capacity limits—the first model to address all five aspects simultaneously.
- A systematic benchmark of seven optimization algorithms spanning three paradigms (exact, heuristic, metaheuristic) across four IEEE test systems (13 to 240 buses), demonstrating that the joint model achieves an 87.9% CAPEX reduction relative to μ PMU-only placement.
- A systematic experimental study of 560 experiments spanning 80 scenario combinations of DER penetration and SCADA coverage (both ranging 0–80%), evidencing that DER integration amplifies micro-PMU requirements while SCADA pre-instrumentation reduces reliance on AMI deployment.
- A clear methodological separation between hybrid observability (supported by μ PMU, AMI, and SCADA) and strict DER coverage (enforced by direct or adjacent μ PMU support), which determines the device mix selected by the optimizer and explains why GA_NSGA2 may lose full observability in large-scale scenarios even when DER coverage is satisfied.

- Practical guidelines for distribution system operators (DSOs) on measurement infrastructure planning, including quantified trade-offs between device types, cost sensitivity to SCADA coverage, and conditions requiring selective μ PMU investment.

The remainder of this paper is organized as follows. Section 2 presents the mathematical formulation of the OPP baseline and the proposed joint μ PMU–AMI placement model, including the hybrid observability constraint and the DER coverage requirement. Section 3 describes the four IEEE test systems, the seven optimization algorithms, and the 80-scenario experimental protocol. Section 4 presents consolidated results from 560 experiments, including CAPEX comparisons, sensitivity analyses with respect to SCADA coverage and DER penetration, and ANOVA-based statistical validation. Section 5 discusses practical implications for distribution system operators and identifies study limitations. Finally, Section 6 summarizes the key findings and outlines directions for future work.

2 Problem Formulation

2.1 OPP Baseline (Single-Technology Placement)

Let $G = (V, E)$ be the graph of a distribution network with $n = |V|$ buses and $|E|$ branches. Define the binary adjacency-plus-identity matrix $\mathbf{A} \in \{0, 1\}^{n \times n}$ where $a_{ij} = 1$ if bus i is directly connected to bus j or $i = j$, and $a_{ij} = 0$ otherwise. The OPP is formulated as a binary integer linear program (BILP):

$$\min F(\mathbf{x}) = \sum_{i=1}^n x_i \quad (1)$$

$$\text{s.t.} \quad \sum_{j=1}^n a_{ij} x_j \geq 1, \quad \forall i = 1, \dots, n \quad (2)$$

where $x_i \in \{0, 1\}$ indicates whether a μ PMU is installed at bus i . Constraint (2) ensures that every bus is observed by at least one μ PMU—either directly or through an adjacent placement.

The System Observability Redundancy Index (SORI) quantifies measurement redundancy as:

$$\text{SORI} = \sum_{i=1}^n f_i(\mathbf{x}), \quad f_i(\mathbf{x}) = \sum_{j=1}^n a_{ij} x_j \quad (3)$$

where $f_i(\mathbf{x}) \geq 1$ after observability is satisfied; higher values indicate greater redundancy at bus i . It is worth noting that SORI is only meaningful for comparing solution sets with the same number of PMUs on a given network; when device counts

differ, the index becomes an unreliable comparator, [16]. From the definition of $f_i(\mathbf{x})$ and constraint (2), $\text{SORI} \geq N_{\mu\text{PMU}}$, with equality only when every installed μ PMU monitors a leaf bus in $\mathbf{A}$ (no neighbors observed beyond itself); the SORI excess above $N_{\mu\text{PMU}}$ therefore quantifies the topological redundancy obtained at no additional device cost.

A μ PMU installed at bus i measures the voltage phasor at i plus the current phasor on each incident branch, subject to a channel limit L :

$$\sum_{j \in \mathcal{N}(i)} w_{ij} \leq L, \quad \forall i \in V \quad (4)$$

where $\mathcal{N}(i)$ is the set of neighbors of bus i and $w_{ij} = 1$ when the branch (i, j) is monitored. Commercial μ PMUs typically support $L = 4$ to $L = 8$ channels.

2.2 Joint Placement Model

The joint placement model extends the OPP by introducing a second technology—AMI meters—and optimizing the allocation of both device types simultaneously. Two binary decision variables are defined for each bus:

$$x_i \in \{0, 1\}, \quad y_i \in \{0, 1\}, \quad \forall i \in V \quad (5)$$

where $x_i = 1$ denotes μ PMU installation and $y_i = 1$ denotes AMI assignment at bus i . Since both technologies provide bus observability, mutual exclusion prevents redundant co-location:

$$x_i + y_i \leq 1, \quad \forall i \in V \quad (6)$$

The objective function minimizes total CAPEX:

$$\min Z = \sum_{i=1}^n (C_{\text{PMU}} \cdot x_i + C_{\text{AMI}} \cdot y_i) \quad (7)$$

where $C_{\text{PMU}} = \$6,200$ USD/unit (hardware \$5,000 + communication \$1,000 + installation \$200) and $C_{\text{AMI}} = \$280$ USD/unit (meter \$150 + installation \$100 + communication module \$30), yielding a cost ratio $C_{\text{PMU}}/C_{\text{AMI}} = 22.1$.

The hybrid observability constraint ensures that every bus is observed by at least one technology— μ PMU (directly or adjacently), AMI (locally), or pre-existing SCADA:

$$\sum_{j=1}^n a_{ij} x_j + y_i + s_i \geq 1, \quad \forall i = 1, \dots, n \quad (8)$$

where $s_i \in \{0, 1\}$ is a fixed parameter indicating pre-existing SCADA coverage at bus i .

Table 2: Optimization algorithms implemented.

Algorithm	Category	Solver	Guarantee
MILP	Exact	PuLP/CBC	Global optimum
MILP+Med	Exact	PuLP/CBC	Global optimum
2-Stage	Exact	PuLP/CBC	Global optimum
Greedy	Heuristic	Custom	Local approx.
CM	Heuristic	Custom	Upper bound
MCM	Heuristic	Custom	Upper bound
GA (NSGA-II)	Metaheur.	Custom	Pareto approx.

Source: created by the authors.

Buses hosting DERs require μ PMU observability for dynamic monitoring—AMI sampling rates (15–60 min) cannot capture sub-second transients from inverter-based resources:

$$\sum_{j=1}^n a_{kj} x_j \geq 1, \quad \forall k \in \mathcal{D} \quad (9)$$

where $\mathcal{D} \subseteq V$ is the set of DER-hosting buses.

The channel constraint (4) applies unchanged to μ PMU placements in the joint model.

The complete joint placement formulation is:

$$\begin{aligned} \min \quad & Z = \sum_{i=1}^n (C_{\text{PMU}} \cdot x_i + C_{\text{AMI}} \cdot y_i) \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij} x_j + y_i + s_i \geq 1, \quad \forall i \\ & \sum_{j=1}^n a_{kj} x_j \geq 1, \quad \forall k \in \mathcal{D} \\ & x_i + y_i \leq 1, \quad \forall i \\ & \sum_{j \in \mathcal{N}(i)} w_{ij} \leq L, \quad \forall i \\ & x_i, y_i \in \{0, 1\}, \quad \forall i \end{aligned} \quad (10)$$

2.3 Solution Algorithms

Seven algorithms are implemented to solve both the baseline OPP (μ PMU-only placement) and the proposed Joint PMU-AMI Placement (JPA) model, spanning three paradigms. Table 2 summarizes their characteristics.

Exact methods. The base MILP solves (1)–(2) directly, without any prior instrumentation assumption. Building on this, the MILP+Med variant incorporates pre-existing meters through an additional measurement vector $\mathbf{m}$ in the constraint set. The 2-Stage approach takes a different direction: it first determines the minimum N_{PMU}^* via the base MILP, and subsequently maximizes SORI under the budget constraint $\sum x_i \leq N_{\text{PMU}}^* + \Delta$. In the JPA framework, these formulations extend to MILP_Joint,

which minimizes (7) subject to (10), and MILP_MultiObj, which optimizes a weighted objective $\alpha Z - (1 - \alpha) \text{SORI}$.

Heuristic methods. The Greedy algorithm builds a placement set iteratively, at each step selecting the bus that maximizes new observability coverage: $i^* = \arg \max_{i \in V \setminus S} |\mathcal{N}_{\text{new}}(i)|$. For the JPA model, this strategy gives rise to four variants depending on device priority: cost-effective (AMI-first, with μ PMUs restricted to DER buses), PMU-first, AMI-first, and hybrid. Regarding covering-based methods, the Covering Method (CM) assigns one device per unobserved bus, while its extension, the Modified Covering Method (MCM), introduces device-type selection as an additional optimization layer.

Metaheuristic method. The GA follows an NSGA-II scheme that simultaneously minimizes cost and maximizes SORI. Each individual uses ternary encoding $g_i \in \{0, 1, 2\}$ to represent no device, μ PMU, or AMI at bus i . The search is configured with crossover probability $p_c = 0.9$, per-gene mutation probability $p_m = 1/n$, a population of 100 individuals, and a stopping criterion of 200 generations.

Complexity and convergence guarantees.

The MILP formulations (1)–(2) and (7)–(10) are NP-hard in general, but for the radial IEEE feeders considered in this paper the LP relaxation is tight in practice and CBC solves every instance in under one second. The greedy heuristic terminates in $O(n^2)$ operations and provides a logarithmic approximation guarantee for the underlying set-cover-like coverage problem; the Covering Method (CM) and its modified variant (MCM) are upper-bound heuristics that converge in $O(n \cdot d_{\max})$ steps and produce feasible but generally non-optimal placements. The metaheuristic method (NSGA-II) explores a search space of 3^n configurations under the ternary encoding and offers no convergence guarantee to the global optimum within a finite number of generations; consequently, in this paper it is reported as a metaheuristic baseline rather than as a primary comparator.

2.4 Scenario Design

The experimental campaign evaluates algorithm performance across two independent scenario dimensions, summarized in Table 3.

The full experimental matrix comprises 5 DER levels $\times$ 4 SCADA levels $\times$ 4 IEEE systems $\times$ 7 algorithms = 560 experiments.

3 Test Systems and Experimental Setup

Four IEEE benchmark feeders with radial (or near-radial) topology and increasing size are used in the

Table 3: DER penetration and SCADA coverage scenarios.

Scenario	PV (%)	BESS (%)	Description
<i>DER Penetration Levels</i>			
Base	0	0	No DERs
Low	10	5	Initial adoption
Medium	30	15	Energy transition
High	50	25	High penetration
Extreme	80	40	Colombia 2035 target
<i>Pre-existing SCADA Coverage</i>			
0%	Greenfield (no instrumentation)		
20%	Basic (substations only)		
50%	Partial (main feeder nodes)		
80%	Extensive coverage		

Source: created by the authors.

Table 4: IEEE test system characteristics.

System	Buses (n)	Branches ($ E $)	$d_{\max}$	Topology
IEEE 13	13	12	4	Radial
IEEE 37	37	36	4	Radial
IEEE 123	123	122	4	Radial w/ laterals
IEEE 240	240	239	4	Extended radial

Source: created by the authors.

experiments. Table 4 summarizes their key topological characteristics.

According to Table 4, a key topological observation is that the maximum node degree is $d_{\max} = 4$ across all four systems. Since commercial μ PMUs support at least $L = 4$ channels, the channel constraint (4) is never binding in these benchmarks. This is consistent with the predominantly radial nature of distribution feeders, where node degrees rarely exceed 3–4.

Computational tools. All algorithms are implemented in Python 3.11 using PuLP 2.8 with the CBC 2.10 solver for MILP formulations, pandas 2.14 for network modeling, and Pandas/NumPy for data processing. The GA uses a custom NSGA-II implementation.

Experimental protocol. Each experiment follows a five-step procedure: (1) load the IEEE system topology and construct the adjacency matrix $\mathbf{A}$; (2) assign DER locations according to the penetration scenario; (3) set SCADA coverage vector $\mathbf{s}$; (4) execute the optimization algorithm; (5) record $N_{\mu\text{PMU}}$, N_{AMI} , total cost, SORI, execution time, and observability percentage.

Hardware. All experiments execute on a Beelink SER8 mini-PC (AMD Ryzen 7 8845HS, 32 GB RAM, NVMe SSD) running Python under Windows 11.

4 Results

4.1 Baseline OPP Results (μ PMU-Only)

A distribution network is said to be *topologically observable* when every bus voltage can be determined—

either by direct measurement or through Ohm’s law applied to an adjacent monitored branch—given the installed set of measurement devices [12]. Table 1 (Appendix) presents the consolidated baseline OPP results across all four IEEE systems, where all algorithms achieve 100% topological observability.

Fig. 1 (Appendix) compares the number of μ PMUs selected by each algorithm across the four IEEE test systems under the baseline OPP setting (μ PMU-only). The results show a clear scalability gap as network size increases: the exact methods and the greedy heuristic remain near the optimum, whereas CM, MCM, and GA increasingly over-provision devices in the larger feeders. The dashed red line marks the optimal μ PMU count for each system.

As shown in table 1 (Appendix), the exact methods (MILP, MILP+Med, 2-Stage) attain or closely approach the global minimum $N_{\mu\text{PMU}}^*$ across all systems. In particular, MILP+Med reduces the required placements by one unit on IEEE 37 (12 vs. 13) and IEEE 123 (46 vs. 47) by leveraging pre-existing meters. The Greedy heuristic matches the MILP optimum on three of four systems and differs by only one unit on IEEE 240 (88 vs. 87), while executing three orders of magnitude faster.

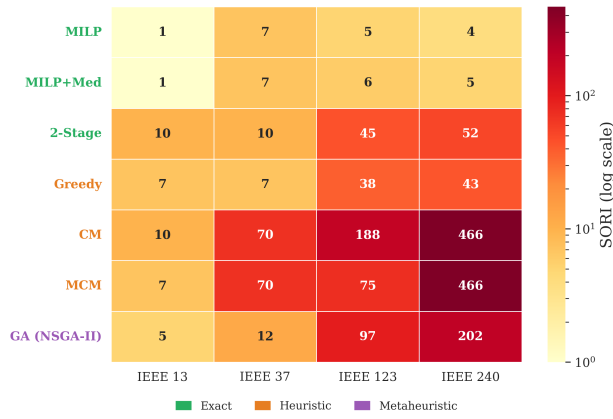
In contrast, CM and MCM lead to severe over-provisioning: on IEEE 240, both assign 240 μ PMUs (one per bus), yielding $\text{SORI} = 466$ and a total cost of \$1,488,000. Likewise, the GA (NSGA-II) places 147 μ PMUs on IEEE 240—70% above the optimum—with 38.9 s computation time, illustrating the difficulty of exploring a search space of 2^{240} configurations.

Fig. 1 complements this analysis by summarizing the SORI values across algorithms and test systems. The heatmap highlights a clear trade-off: methods that over-provision devices (e.g., CM and MCM, and to a lesser extent GA) achieve higher measurement redundancy, but at the cost of deploying an excessive number of μ PMUs.

4.2 Joint Placement (JPA) vs. μ PMU-Only Baseline

Table 5 compares the minimum-cost solutions from the baseline OPP (μ PMU-only, using MILP+Med) and the JPA model (joint placement, using MILP_Joint) under the base scenario (no DERs, no SCADA).

The JPA model achieves an average CAPEX reduction of **87.9%** across all systems. Under the topological observability criterion and in the absence of DERs, the MILP_Joint optimizer allocates exclusively AMI meters (zero μ PMUs) because AMI provides equivalent bus-level observability at roughly 1/22 of the unit cost of a μ PMU. Fig. 2 illustrates



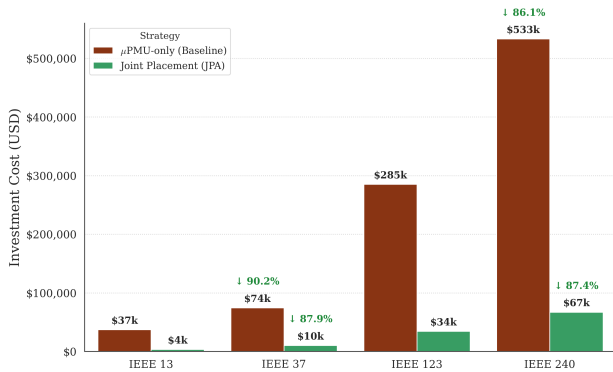
Source: created by the authors.

Figure 1: SORI heatmap for baseline OPP (μ PMU-only) across algorithms and IEEE test systems.

Table 5: Cost comparison: μ PMU-only baseline vs. Joint Placement (JPA), base scenario, SCADA = 0%.

System	μ PMU-only		Joint (JPA)		Saving (%)
	$N_{\mu PMU}$	Cost	N_{AMI}	Cost	
IEEE 13	6	\$37,200	13	\$3,640	90.2
IEEE 37	12	\$74,400	37	\$10,360	86.1
IEEE 123	46	\$285,200	123	\$34,440	87.9
IEEE 240	86	\$533,200	240	\$67,200	87.4
Average saving					87.9

Source: created by the authors.



Source: created by the authors.

Figure 2: Investment comparison: μ PMU-only baseline vs. Joint Placement (JPA) across IEEE test systems.

this pronounced cost gap: the 22.1:1 cost ratio largely explains the observed 87.9% average CAPEX reduction.

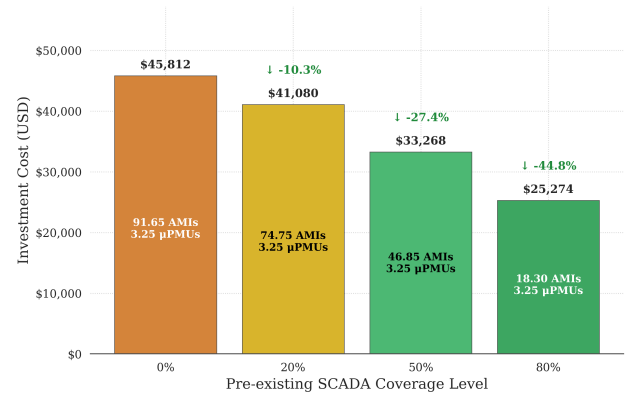
4.3 Effect of SCADA Coverage

Table 6 reports the effect of pre-existing SCADA coverage on total investment cost for the MILP_Joint algorithm (averaged across the four IEEE systems and

Table 6: Effect of pre-existing SCADA coverage on investment cost (MILP_Joint, average across systems).

SCADA (%)	Avg Cost (USD)	Avg $N_{\mu PMU}$	Avg N_{AMI}	Reduction vs. 0%
0	45,812	3.25	91.65	—
20	41,080	3.25	74.75	10.3%
50	33,268	3.25	46.85	27.4%
80	25,274	3.25	18.30	44.8%

Source: created by the authors.



Source: created by the authors.

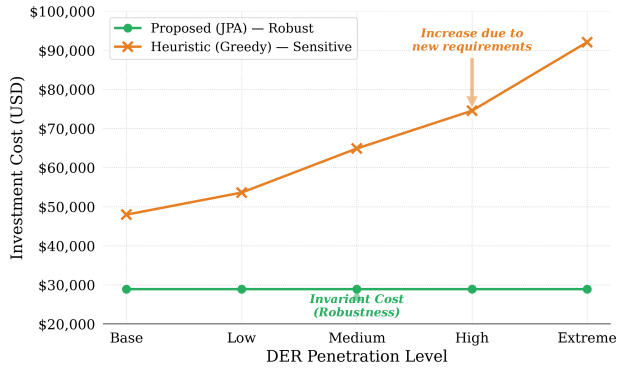
Figure 3: Effect of pre-existing SCADA coverage on total investment cost.

all DER scenarios).

Pre-existing SCADA infrastructure substantially reduces the required AMI investment, but it does not eliminate the micro-PMUs imposed by DER coverage. The average MILP_Joint cost decreases from \$45,812 at 0% SCADA to \$25,274 at 80% SCADA, which corresponds to a **44.8%** reduction across all DER scenarios. The average number of μ PMUs remains constant at 3.25, confirming that SCADA mainly substitutes new AMI devices rather than the micro-PMUs required for DER monitoring. Fig. 3 should therefore be interpreted as a monotonic cost-reduction trend driven by pre-existing infrastructure, not as evidence that DER-related micro-PMU needs disappear.

4.4 Effect of DER Penetration

Under zero pre-existing SCADA coverage, the benchmark shows that the MILP_Joint investment cost is **not invariant** to DER penetration. Averaged across the four IEEE systems, the MILP_Joint cost increases from \$28,910 in the base scenario to \$36,530, \$45,700, \$52,400, and \$65,520 for the low, medium, high, and extreme DER scenarios, respectively. Over the same progression, the average number of μ PMUs rises from 0 to 1.5, 3.25, 4.5, and 7.0, while the average number of AMIs decreases from 103.25 to 79.00.



Source: created by the authors.

Figure 4: Cost sensitivity to DER penetration for MILP_Joint and Greedy_hybrid (SCADA = 0%, averaged across systems).

Table 7: JPA results for IEEE 240 (base scenario, SCADA = 0%).

Algorithm	$N_{\mu\text{PMU}}$	N_{AMI}	Cost (USD)	SORI
MILP_Joint	0	240	67,200	0
MILP_MultiObj_alpha08	0	240	67,200	0
Greedy_cost_eff	0	240	67,200	0
Greedy_ami_first	0	240	67,200	0
Greedy_hybrid	24	147	189,960	11
Greedy_pmu_first	93	0	576,600	71
GA_NSGA2	110	78	703,840	181

Source: created by the authors.

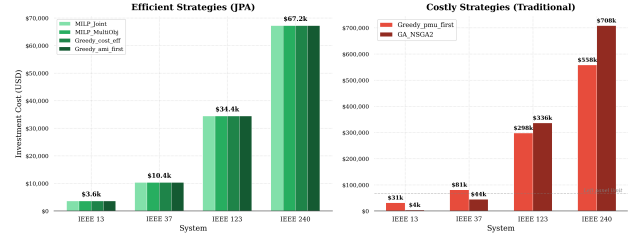
This behavior follows directly from the formulation. Hybrid observability is supported by μPMUs , AMI, and SCADA, but DER coverage itself is enforced strictly through direct or adjacent μPMU support, as expressed by constraint (9). As a result, higher DER penetration moves the optimizer out of a purely AMI-dominated regime and requires a progressively larger micro-PMU investment. Fig. 4 illustrates this increasing trend across the five DER levels.

4.5 Algorithm Performance Comparison (JPA)

Table 7 presents the JPA results for the largest test system (IEEE 240), which best discriminates algorithm performance.

According to Table 7, a distinct divide remains visible in the base IEEE 240 scenario. The AMI-oriented strategies (MILP_Joint, MILP_MultiObj_alpha08, Greedy_cost_eff, and Greedy_ami_first) all reach the minimum cost of \$67,200, whereas the μPMU -oriented strategies (Greedy_pmu_first and GA_NSGA2) are almost one order of magnitude more expensive. Greedy_hybrid occupies an intermediate position at approximately $2.83 \times$ the optimum.

However, this exact coincidence does *not* persist across the full set of 80 scenarios. Greedy_ami_first and Greedy_cost_eff match MILP_Joint in 51 cases



Source: created by the authors.

Figure 5: JPA cost comparison across IEEE systems: lower-cost strategies versus higher-cost strategies.

and deviate marginally in the remaining 29, while GA_NSGA2 additionally fails to maintain full observability in 11 IEEE 240 cases even though it still satisfies DER coverage. Fig. 5 therefore shows a qualitative separation between low-cost and high-cost strategies that is consistent with the homogeneous groups identified by the Tukey HSD test in Section 4.6.

4.6 Statistical Validation of Algorithm Performance

The 560 experiments—spanning 80 scenario combinations across 7 algorithms—provide the basis for a formal statistical assessment of whether algorithm choice exerts a significant effect on four response variables: total cost, number of μPMUs deployed, SORI, and execution time. Given that the distribution of total cost exhibits heavy-tailed behavior driven by μPMU -oriented strategies, the assumptions underlying one-way ANOVA cannot be taken for granted; accordingly, they are examined explicitly prior to drawing any inferential conclusions.

Assumption tests. The Shapiro–Wilk test on within-algorithm residuals of total cost rejects normality for all seven algorithms ($p < 10^{-3}$ in every case). Levene’s test on the seven groups also rejects homogeneity of variances ($W = 34.03$, $p < 10^{-34}$). Both ANOVA assumptions are therefore violated. Consequently, the inferential conclusions of this section rely primarily on the non-parametric Kruskal–Wallis test, while the ANOVA F -statistics and η^2 effect sizes are reported as parametric reference values that are internally consistent with the Kruskal–Wallis outcome.

Joint significance across algorithms. Table 8 reports the joint test results for the four metrics. All four metrics show highly significant differences across algorithms ($p < 10^{-15}$ in every test). The largest effect is observed for execution time ($\eta^2 = 0.62$), followed by number of μPMUs ($\eta^2 = 0.32$),

Table 8: Joint significance tests across the seven algorithms ($N = 560$). ANOVA F and η^2 are reported for reference; Kruskal–Wallis H is the primary inferential test given the violation of normality.

Metric	ANOVA			Kruskal–Wallis		
	F	p	η^2	H	p	
Total Cost (USD)	32.43	$< 10^{-30}$	0.26	81.10	$< 10^{-14}$	
Number of μ PMUs	42.90	$< 10^{-40}$	0.32	162.18	$< 10^{-30}$	
SORI	36.51	$< 10^{-35}$	0.28	238.50	$< 10^{-45}$	
Execution Time (s)	151.47	$< 10^{-100}$	0.62	479.61	$< 10^{-95}$	

Source: created by the authors.

Table 9: Mean and standard deviation of total cost across the 560 experiments.

Algorithm	Mean Cost (USD)	Std Dev (USD)
MILP_Joint	36,358.5	26,646.4
Greedy_cost_effective	38,495.0	26,730.7
Greedy_ami_first	38,495.0	26,730.7
MILP_MultiObj_alpha08	48,150.0	32,988.1
Greedy_hybrid	62,315.5	54,355.4
GA_NSGA2	160,854.5	190,841.6
Greedy_pmu_first	192,122.5	177,446.4

Source: created by the authors.

SORI ($\eta^2 = 0.28$) and total cost ($\eta^2 = 0.26$). Algorithm choice therefore explains between 26% and 62% of the observed variance, depending on the metric.

Per-algorithm summary. Table 9 reports the mean and standard deviation of total cost for each algorithm. MILP_Joint exhibits the lowest mean cost ($\$36,358.5 \pm \$26,646.4$), followed closely by Greedy_cost_eff and Greedy_ami_first, which share an identical mean of $\$38,495.0 \pm \$26,730.7$. MILP_MultiObj_alpha08 is more expensive because it trades cost for redundancy, and Greedy_hybrid forms an intermediate tier. GA_NSGA2 and Greedy_pmu_first are the most expensive strategies and also exhibit the largest dispersion.

Tukey HSD post-hoc. A Tukey HSD test on total cost ($\alpha = 0.05$) groups the seven algorithms into two homogeneous sets. The first set (**Group A**) contains the two exact methods, the three AMI-oriented heuristics and the hybrid heuristic (MILP_Joint, MILP_MultiObj_alpha08, Greedy_cost_eff, Greedy_ami_first and Greedy_hybrid), with no statistically significant pairwise differences in total cost across these five algorithms. The second set (**Group B**) contains the two μ PMU-oriented strategies (GA_NSGA2 and Greedy_pmu_first), which are not significantly different from each other ($p = 0.47$). Every cross-group pair is significant at $\alpha = 0.05$. The summary is reported in Table 10.

Table 10: Homogeneous groups from Tukey HSD on total cost ($\alpha = 0.05$). Algorithms inside the same group are not significantly different.

Algorithm	Mean Cost (USD)	Group
MILP_Joint	36,358.5	A
Greedy_cost_effective	38,495.0	A
Greedy_ami_first	38,495.0	A
MILP_MultiObj_alpha08	48,150.0	A
Greedy_hybrid	62,315.5	A
GA_NSGA2	160,854.5	B
Greedy_pmu_first	192,122.5	B

Source: created by the authors.

Table 11: Observability compliance across the 80 scenarios.

Algorithm	Obs. 100% Cases	Min Obs. (%)	Mean $N_{\mu\text{PMU}}$
MILP_Joint	80 / 80	100.00	3.25
MILP_MultiObj_alpha08	80 / 80	100.00	3.25
Greedy_cost_effective	80 / 80	100.00	3.64
Greedy_ami_first	80 / 80	100.00	3.64
Greedy_hybrid	80 / 80	100.00	8.00
Greedy_pmu_first	80 / 80	100.00	30.99
GA_NSGA2	69 / 80	95.83	24.54

Source: created by the authors.

Observability compliance. Total cost alone is not a sufficient performance indicator: the GA also fails to satisfy full observability in 11 out of 80 scenarios on the IEEE 240-bus system, with a minimum observability of 95.83 %. All exact methods and all greedy heuristics maintain 100 % observability across the 80 scenarios. This is reported in Table 11 and is the basis for treating GA_NSGA2 as a metaheuristic baseline rather than as a primary comparator.

Sensitivity to feasibility. To assess the robustness of these findings, the analysis is repeated on the 549 fully observable cases obtained after excluding the 11 GA_NSGA2 outcomes with $\text{obs}\% < 100$. Both parametric and non-parametric tests yield qualitatively identical conclusions: statistical significance is preserved across every metric ($p < 10^{-14}$), and the Tukey HSD partition separating the AMI-oriented Group A from μ PMU-oriented strategies remains intact. Notably, the mean total cost of GA_NSGA2 in this restricted set decreases from $\$160,854$ to $\$102,906$ —a reduction consistent with the interpretation that the elevated cost observed in the main analysis is jointly attributable to device over-provisioning and to infeasible solutions arising in the largest test system.

Reading of the validation. The combined evidence from the Shapiro–Wilk, Levene, ANOVA, Kruskal–Wallis, and Tukey HSD tests, together with the observability compliance summary, supports three conclusions of practical relevance for distribution system operators. First, exact methods and AMI-

oriented heuristics are statistically indistinguishable in total cost over the 560 experiments and guarantee complete observability in every scenario evaluated. Second, μ PMU-oriented strategies incur significantly higher costs and, in the particular case of GA_NSGA2, do not consistently ensure feasibility on the largest system. Third, the μ PMU support induced by the DER coverage constraint is clearly reflected in the mean μ PMU count of AMI-oriented heuristics, which rises from zero in DER-free scenarios to non-trivial levels as DER penetration increases—precisely as predicted by the formulation presented in Section 2.

5 Discussion

5.1 Practical Implications for DSOs

The 87.9% CAPEX reduction offered by the joint placement model has direct implications for distribution system modernization. A DSO managing a 240-bus feeder would invest \$67,200 in AMI-based observability versus \$533,200 for μ PMU-only placement—a difference of \$466,000 per feeder. At the utility scale, where there are hundreds of feeders, this amounts to investment deferrals worth tens of millions of dollars.

The benefit of pre-existing SCADA coverage provides a clear upgrade pathway: averaged across all DER scenarios, MILP_Joint cost is reduced by 44.8% between the 0% and 80% SCADA cases, mainly by reducing the number of new AMIs that must be installed.

However, three conditions justify selective μ PMU investment even when AMI suffices for topological observability:

- 1) **DER presence:** Buses that include inverter-based DERs need monitoring at sub-second intervals to capture transients, maintain voltage within limits, and prevent unintentional islanding—functions that cannot be achieved with AMI sampling intervals of 15–60 minutes.
- 2) **Data loss resilience:** Solutions with $\text{SORI} = 0$ (purely AMI-based) exhibit no redundancy in their measurements; if even one meter fails, the associated bus becomes unobservable.
- 3) **Dynamic state estimation:** Applications requiring synchronized phasor data—fault location, oscillation detection, and real-time voltage stability monitoring—inherently require μ PMU measurements.

5.2 Limitations

The main limitation of this study is its reliance on *topological* observability, which only verifies

whether a bus is covered by at least one measurement device and does not, by itself, guarantee numerical state-estimation accuracy. In particular, a bus observed indirectly through a distant AMI meter may exhibit larger estimation errors than a bus directly monitored by a μ PMU. Future work should integrate distribution state estimation (DSSE) to evaluate whether the topologically optimal placements also deliver acceptable estimation accuracy under realistic noise and data loss conditions.

Additionally, the IEEE test systems used are standard academic benchmarks with radial topologies ($d_{\max} = 4$). Real-world distribution networks may include meshed sections, longer feeders, and heterogeneous load patterns that could alter the optimal placement.

Sensitivity to cost ratio and channel limit.

The economic conclusions of this paper are governed by two parameters: the unit-cost ratio $r = C_{\mu\text{PMU}}/C_{\text{AMI}} = 22.1$ and the per-device channel limit L . The 87.9% CAPEX reduction reported in Section 4.2 is approximately proportional to $1 - 1/r$ when DER coverage is inactive; if the cost ratio falls to $r = 10$ (a more aggressive future μ PMU price), the proportional saving relative to μ PMU-only placement contracts to roughly 70%, while a higher ratio of $r = 40$ widens the saving above 92%. The channel limit $L = 4$ is not binding in any of the four IEEE feeders studied, since $d_{\max} = 4$. For operationally constrained μ PMU hardware with $L = 2$, constraint (4) would become active in buses with degree three or four and would force additional μ PMU placements at adjacent buses, raising the total cost; conversely, $L = 6$ would not change the present results because no bus exceeds four neighbors in the studied systems.

5.3 Future Work

Ongoing research extends this framework in two directions: (1) DSSE validation under data loss, precision degradation, and N-1 contingency scenarios to quantify the accuracy trade-off between AMI-dominated and mixed μ PMU–AMI configurations; and (2) application to real distribution circuits to validate transferability beyond IEEE benchmarks.

6 Conclusions

The joint MILP formulation introduced in this paper allocates μ PMUs and AMI meters simultaneously, exploiting their complementary roles to reduce capital expenditure while preserving full topological observability and DER coverage. The model was evaluated on four IEEE radial feeders (13, 37, 123 and 240 buses) under 80 scenario combinations and seven

competing algorithms, yielding 560 individual experiments.

- 1) In the base scenario without pre-existing SCADA, the joint model attains an average CAPEX reduction of 87.9% relative to μ PMU-only placement. Because AMI provides equivalent bus-level observability at roughly 1/22 of the unit cost, the optimizer chooses AMI wherever the DER coverage constraint does not force a μ PMU.
- 2) Pre-existing SCADA at 80% coverage reduces the average MILP_Joint cost by 44.8% relative to the greenfield case. The reduction is achieved entirely by displacing AMI meters: the average μ PMU count remains constant at 3.25, since DER monitoring cannot be substituted by either AMI or SCADA.
- 3) Investment cost is not invariant to DER penetration. As the strict coverage constraint (9) activates, the average μ PMU count grows from 0 (base scenario) to 7.0 (extreme penetration), and the total cost rises from \$28,910 to \$65,520 across the four feeders.
- 4) The Tukey HSD post-hoc test partitions the seven algorithms into two homogeneous groups at $\alpha = 0.05$. Within the low-cost group, MILP_Joint, the AMI-oriented greedy heuristics and Greedy_hybrid are statistically indistinguishable; the high-cost group contains the μ PMU-oriented strategies. NSGA-II, used here as a metaheuristic baseline, fails to satisfy full observability in 11 of 80 IEEE 240 cases.
- 5) The benchmark separates two notions that the prior literature often conflates: *hybrid observability*, which AMI and SCADA can satisfy at the bus level, and *strict DER coverage*, which only μ PMU support—direct or adjacent—can guarantee.

The formulation is therefore positioned as a planning aid for distribution system operators that face a binding capital budget but cannot relax DER monitoring. Two limitations call for further work: the topological observability criterion adopted here does not by itself guarantee a target accuracy in distribution state estimation, and the experimental campaign assumes nominal communications. Both questions—accuracy under realistic measurement noise and the impact of latency or packet loss on the chosen placement—are left for a subsequent study.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

The authors wrote, reviewed and edited the content as needed and they have not utilised artificial intelligence (AI) tools. The authors take full responsibility for the content of the publication.

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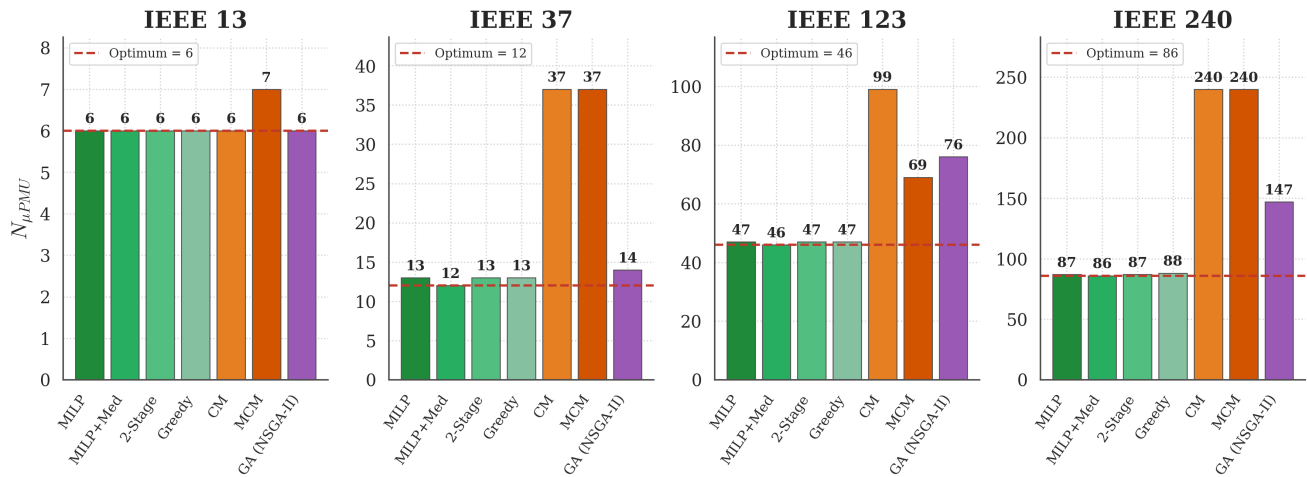
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Appendix



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Figure 1: Number of μ PMUs by algorithm and IEEE test system (baseline OPP, μ PMU-only).

Table 1: Consolidated baseline OPP results (μ PMU-only): number of μ PMUs ($N_{\mu PMU}$) and SORI by algorithm and test system.

Algorithm	IEEE 13		IEEE 37		IEEE 123		IEEE 240	
	$N_{\mu PMU}$	SORI	$N_{\mu PMU}$	SORI	$N_{\mu PMU}$	SORI	$N_{\mu PMU}$	SORI
MILP	6	1	13	7	47	5	87	4
MILP+Med	6	1	12	7	46	6	86	5
2-Stage	6	10	13	10	47	45	87	52
Greedy	6	7	13	7	47	38	88	43
CM	6	10	37	70	99	188	240	466
MCM	7	7	37	70	69	75	240	466
GA (NSGA-II)	6	5	14	12	76	97	147	202

Source: created by the authors.