

The Method for Assessing the Magnitude of Air Gap Torque Ripple of an Induction Motor Powered by an Autonomous Voltage Source Inverter or Current Source Inverter

MIKHAIL PUSTOVETOV

Department of Electrical and Electronics Engineering,
Don State Technical University,
344000, Rostov region, Rostov-on-Don, Gagarin sq., 1,
RUSSIA

Abstract: - The magnitude of the air gap torque (electromagnetic torque) ripple is an important characteristic of the operating mode of an induction motor powered by a voltage source inverter or a current source inverter. Its empirical determination is quite difficult (the torque sensor is expensive and requires precise installation). The air gap torque of the IM can be calculated using the harmonic analysis method. This method involves decomposing the supply voltage or current of the converter into a truncated Fourier series into time harmonics and calculating the parameters of the T-shaped equivalent circuit of the induction motor for each harmonic. After that it is possible to calculate in this way power, efficiency, the average value of the air gap torque, and the magnitudes of currents or voltages. So, the ripple factor of the air gap torque, which is the magnitude of torque pulsations in relative units. The disadvantage is the large amount of initial data on the machine parameters and operating mode characteristics required for the calculation. A simple method is proposed by the author for approximately calculating (assessing) the magnitude of air gap torque ripple in a quasi-steady state using two oscillograms: phase current and phase voltage of an induction motor. The relationship between the harmonic composition of the voltage and the magnitude of the air gap torque ripple has been revealed.

Key-Words: - current source inverter, harmonic composition, induction motor, method for calculating, phasor diagram, pulse-width modulation, ripple factor, torque ripple magnitude, voltage source inverter

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1 Introduction

One of the features and problems of induction motors powered by pulse converters is the air gap torque ripple. A number of publications are devoted to the issues of their reduction [1], [2], [3], [4], [5], [6], [7], [8], [9]. As is well known, in order to improve something, you need to be able to calculate it. Let us consider one of the methods for calculating the magnitude of the air gap torque ripple.

Assumptions adopted in the article.

1. A 3-phase IM with a squirrel-cage rotor is considered.

2. The operation of an induction motor (IM) in a quasi-steady state is considered, that is, at a constant rotor speed and the absence of a transient process caused by starting, changing the supply frequency, etc.

3. The three-phase IM is structurally symmetrical.

4. Stator winding connection diagram – wye without a neutral wire.

5. An IM is powered by a symmetrical system of non-sinusoidal voltages in case of an autonomous voltage source inverter (AVSI) or non-sinusoidal currents in case of an autonomous current source inverter (ACSI) [10].

6. The voltage or current spectrum of the IM phase contains only harmonic components, the ordinal number of which is odd and not a multiple of three.

7. For each operating point, the IM equivalent circuit is considered linear with a certain value of the saturation coefficient of the magnetic system. When deriving equations for a T-shaped equivalent circuit, the superposition method is used, valid for linear electric and magnetic circuits.

The presence in the spectrum of the voltage or current of the IM phase when powered from an AVSI or ACSI of higher time harmonics (HTH) leads to a decrease in its efficiency, increased heating, and air gap torque ripples. The magnitude of torque ripples is especially important at low frequencies f_1 of the fundamental harmonic of the

IM supply voltage (at few Hz or less frequencies), which occur during frequency starting of the IM, since the minimum frequency of the air gap torque ripple is $f_r = 6 \cdot f_1$ [11], [12], [13], [14], [15], [16], [17] and with significant ripples, the step mode of operation of the IM can result and the impossibility of IM acceleration. At higher frequencies, torque ripples can be a source of vibration and noise. Thus, the magnitude of the air gap torque ripples, along with the efficiency and power factor, is an important criterion for the quality of operation of an IM powered by an AVSI or ACSI, but determining it empirically is more difficult compared to oscillography of non-sinusoidal voltages and currents.

The air gap torque of the IM can be calculated using the harmonic analysis method, the main provisions of which are set out in [18] by Doctor of Engineering Ivan Talya, Soviet and Russian scientist. The author of this article, under the supervision of I. Talya, took part in the development of the method.

Dr. Ivan Talya is perhaps best known as one of the principal developers of a linear electric drive and magnetic levitation system for a passenger vehicle with a travel speed of up to 300 km/h [19] (he was Chief designer of the Research and Development Enterprise "Transport with Magnetic Levitation"), See Figure 1.

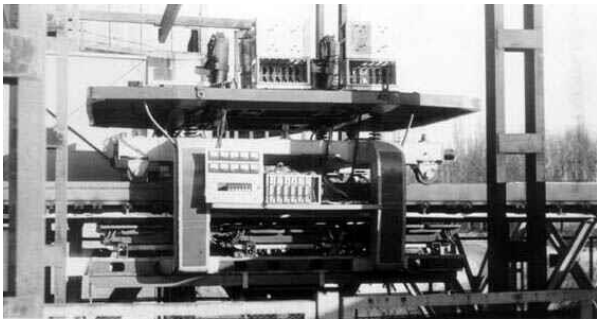


Fig. 1: - The 6-tonne prototype of a vehicle with linear induction traction and levitation modules on the test bench in Novocherkassk (Russia, 1990s), developed under the supervision of Dr. Ivan Talya
Source: [19]

The harmonic analysis method involves decomposing the supply voltage in the case of AVSI or current in the case of ACSI into a truncated Fourier series into time harmonics and calculating the parameters of the T-shaped equivalent circuit of the IM for each harmonic. After that it is possible to calculate in this way power, efficiency, the average value of the air gap torque $T_{\delta avg}$, and the amplitudes of the IM currents.

2 Brief Description of the Harmonic Analysis Method

The provisions of the described method are similar to those published in [11], [20], [21], [22].

When considering the case of the IM supply from the AVSI, the instantaneous value of the stator phase voltage, if the Fourier cosine series is used, can be written as

$$v = \sum_{k=1}^{\infty} V_{km} \cdot \cos(k \cdot \omega_1 \cdot t + \Psi_{v0k}), \quad (1)$$

where V_{km} is the amplitude of the k-th time harmonic of the voltage;

Ψ_{v0k} is the initial phase angle of the k-th time harmonic of the voltage.

In the case of setting the voltage by the Fourier sine series in (1), the cosines are replaced by sines.

The Kirchhoff equations for the k-th time harmonic in accordance with the IM equivalent circuit are as follows:

$$\begin{cases} \dot{V}_k = r_{1k} \cdot \dot{I}_{1k} + jL_{\sigma 1k} \cdot k \cdot \omega_1 \cdot \dot{I}_{1k} + r_{11k} \cdot \dot{I}_{\mu k} + \\ + jL_{11k} \cdot k \cdot \omega_1 \cdot \dot{I}_{\mu k}; \\ 0 = r_{11k} \cdot \dot{I}_{\mu k} + jL_{11k} \cdot k \cdot \omega_1 \cdot \dot{I}_{\mu k} + \frac{r'_{2k}}{s_k} \cdot \dot{I}'_{2k} + \\ + jL'_{2k} \cdot k \cdot \omega_1 \cdot \dot{I}'_{2k}; \\ \dot{I}_{1k} + \dot{I}'_{2k} = \dot{I}_{\mu k}. \end{cases} \quad (2)$$

In this case, the magnetization branch is represented by a series connection of r_{11k} and X_{11k} or a parallel connection of $r_{\mu k}$ and $X_{\mu k}$. For the RMS value of each harmonic of the voltage V_k , the RMS values of the harmonics of the phase current of the stator winding I_{1k} and the reduced current of the rotor winding I'_{2k} are determined according to the T-shaped equivalent circuit of IM. From equations (2) the expressions for the currents are:

$$i_1 = \sum_{k=1}^{\infty} I_{1km} \cdot \cos(k \cdot \omega_1 \cdot t + \Psi_{01k}), \quad (3)$$

$$i'_2 = \sum_{k=1}^{\infty} I'_{2km} \cdot \cos(k \cdot \omega_1 \cdot t + \Psi_{02k}). \quad (4)$$

For the case of the IM power supply from the ACSI, the initial expression is (3), that is, we are given by the harmonic spectrum (amplitudes I_{1km}

and initial phases Ψ_{01k} of the time harmonics) of the stator phase current. The components of expressions (1) and (4) are calculated. If the stator phase current is given by a sine series, that is, in expression (3) sines are given instead of cosines, then in (1) and (4) it is also necessary to replace the cosines with sines.

The resistance of the stator phase winding r_{1k} and the reduced resistance of the squirrel cage r'_{2k} , the leakage inductances $L_{\sigma 1k}$ and L'_{2k} for each harmonic are determined taking into account the current displacement in the slot part of the winding conductors [23], [24], respectively, at frequencies (5) and (6).

$$f_{1k} = k \cdot f_1, \quad (5)$$

$$f_{2k} = k \cdot f_1 \cdot s_k = k \cdot f_1 \cdot \left(1 \pm \frac{1-s_1}{k}\right) \approx (k \pm 1) \cdot f_1. \quad (6)$$

Rotor slip relative to the electromagnetic field's HTH

$$s_k = 1 \pm \frac{1-s_1}{k}. \quad (7)$$

Absolute value of impedance of the equivalent circuit for the k-th harmonic

$$Z_k = \sqrt{(r_{1k} + r_{12k})^2 + (X_{\sigma 1k} + X_{12k})^2}. \quad (8)$$

where:

$$r_{12k} = \frac{A_k \cdot B_k + C_k \cdot D_k}{\left(r_{11k} + \frac{r'_{2k}}{s_k}\right)^2 + (X_{11k} + X'_{2k})^2}, \quad (9)$$

$$X_{12k} = \frac{C_k \cdot B_k - A_k \cdot D_k}{\left(r_{11k} + \frac{r'_{2k}}{s_k}\right)^2 + (X_{11k} + X'_{2k})^2}, \quad (10)$$

$$A_k = r_{11k} \cdot \frac{r'_{2k}}{s_k} - X_{11k} \cdot X'_{2k}, \quad (11)$$

$$B_k = r_{11k} \cdot \frac{r'_{2k}}{s_k}, \quad (12)$$

$$C_k = X_{11k} \cdot \frac{r'_{2k}}{s_k} - r_{11k} \cdot X'_{2k}, \quad (13)$$

$$D_k = X_{11k} + X'_{2k}, \quad (14)$$

$$r_{11k} = \frac{X_{\mu k}^2 \cdot r_{\mu k}}{r_{\mu k}^2 + X_{\mu k}^2}, \quad (15)$$

$$X_{11k} = \frac{X_{\mu k} \cdot r_{\mu k}^2}{r_{\mu k}^2 + X_{\mu k}^2}. \quad (16)$$

Series connection in the magnetization branch r_{11k} and X_{11k} is obtained by transforming the parallel connection $r_{\mu k}$ and $X_{\mu k}$.

Regardless of the inverter type, the relationships between the phase angles and the amplitudes of the equivalent circuit voltage and current HTHs remain the same. The amplitudes of the k-th HTH of the phase current and voltage are related to each other by the expression

$$I_{1km} = V_{km} / Z_k. \quad (17)$$

The initial phase of the k-th HTH of the stator current when powered from the AVSI

$$\begin{aligned} \Psi_{01k} &= \Psi_{v0k} - \varphi_k = \\ &= \Psi_{v0k} - \arctg\left(\frac{X_{\sigma 1k} + X_{12k}}{r_{1k} + r_{12k}}\right). \end{aligned} \quad (18)$$

where φ_k is the phase shift angle between the corresponding HTHs of the stator phase voltage.

When powered from the ACSI, Ψ_{01k} is a known value, and it can be used to determine the initial phase of the k-th HTH of the phase voltage of the IM:

$$\Psi_{v0k} = \Psi_{01k} + \varphi_k. \quad (19)$$

The RMS value of the phase-to-phase voltage

$$V_{\text{phase-to-phase}} = \sqrt{\sum_{k=1}^{\infty} V_{k_ \text{phase-to-phase}}^2}. \quad (20)$$

The RMS value of the phase current

$$V_{1\text{phase}} = \sqrt{\sum_{k=1}^{\infty} I_{1k}^2}. \quad (21)$$

The expression for the air gap torque of the IM through the interaction of spatial induction waves in the air gap and the rotor current also has the same form for the case of the AVSI or ACSI. In order to determine the air gap torque caused by the interaction of all time harmonics, we write an expression for it on an elementary section of the rotor circumference [25], [11]:

$$dT_{\delta} = \frac{D_2}{2} \cdot l_{\delta} \cdot B_{\delta} \cdot i_{2s} \cdot dx. \quad (22)$$

where

D_2 - rotor diameter;

$B_{\delta v}$ - normal component of the resulting magnetic field induction in the gap;

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HTH of the positive sequence ($v = k = 2 \cdot m \cdot n + 1$), and the signs “-” refer to the HTH of the negative sequence ($v = k = 2 \cdot m \cdot n - 1$), $n = 1, 2, 3, \dots$; $m = 3$. Different notations for the ordinal number of HTH of air gap field and HTH of the rotor spatial current (v and k) are adopted to avoid confusion during further transformations of expression (23).

The magnetic induction in the air gap can be represented as a sum of forward and backward rotating waves with the same pole divisions τ :

$$B_{\delta} = \sum_{v=1}^{\infty} B_{\delta v} \cdot \cos\left(\pm \frac{\pi}{\tau} \cdot x + k \cdot \omega_1 \cdot t + \Psi_v\right). \quad (30)$$

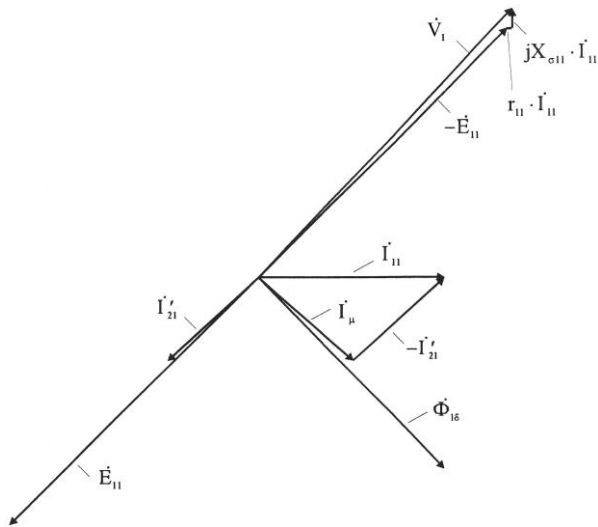


Fig. 3: - Phasor diagram for fundamental time harmonics of voltages and currents in case of traction IM of type NTA-350 powered by ACSI, rectangular current pulse 120 wide (meander with platform), rated mode, $= 0.0054$, $= 72.55$ Hz, RMS phase stator current fundamental time harmonic is $229.4 \text{ A} \cdot s_1 \cdot f_1$
 $I_{11} =$

Source: created by the author.

Average value of the air gap torque

$$T_{\delta \text{avg}} = \frac{p \cdot m}{\omega_1} \times \sum_{k=1}^{\infty} \pm \frac{1}{k} \cdot I_{1k}^2 \cdot \frac{r_{12k}^2 + X_{12k}^2}{X_{2k}'^2 + \left(\frac{r_{2k}'}{s_k}\right)^2} \cdot \frac{r_{2k}'}{s_k}, \quad (31)$$

here and in (32) the sign “+” in the motoric mode is taken for the positive sequence HTH and the sign “-” for the negative sequence HTH.

Average value of the torque on the IM's shaft

$$T_{\text{shaft}} = \frac{p \cdot m}{\omega_1 \cdot (1 - s_1)} \times \sum_{k=1}^{\infty} \pm \frac{1}{k} \cdot I_{1k}^2 \cdot \frac{r_{12k}^2 + X_{12k}^2}{X_{2k}'^2 + \left(\frac{r_{2k}'}{s_k}\right)^2} \cdot \frac{r_{2k}'}{s_k} \cdot (1 - s_k). \quad (32)$$

Let us integrate expression (23).

$$\begin{aligned} T_{\delta} &= 2 \cdot p \cdot \frac{D_2}{2} \cdot l_{\delta} \times \\ &\times \int_0^{\tau} \left[B_{\delta 1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_1\right) + \right. \\ &+ B_{\delta 5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + \omega_5 \cdot t + \Psi_5\right) + \\ &+ B_{\delta 7} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_7 \cdot t + \Psi_7\right) + \\ &+ B_{\delta 11} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + \omega_{11} \cdot t + \Psi_{11}\right) + \\ &+ B_{\delta 13} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_{13} \cdot t + \Psi_{13}\right) + \dots \left. \right] \times \\ &\times \left[I_{2S1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_{021}\right) + \right. \\ &+ I_{2S5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + \omega_5 \cdot t + \Psi_{025}\right) + \\ &+ I_{2S7} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_7 \cdot t + \Psi_{027}\right) + \\ &+ I_{2S11} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + \omega_{11} \cdot t + \Psi_{0211}\right) + \\ &+ I_{2S13} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_{13} \cdot t + \Psi_{0213}\right) + \dots \left. \right] dx = \\ &= 2 \cdot p \cdot \frac{D_2}{2} \cdot l_{\delta} \cdot \left\{ \int_0^{\tau} B_{\delta 1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_1\right) \times \right. \\ &\times \left[I_{2S1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_{021}\right) + \right. \\ &+ I_{2S5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + \omega_5 \cdot t + \Psi_{025}\right) + \\ &+ I_{2S7} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_7 \cdot t + \Psi_{027}\right) + \end{aligned}$$

$$\begin{aligned}
 &+I_{2S11} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + \omega_{11} \cdot t + \Psi_{0211}\right) + \\
 &+I_{2S13} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_{13} \cdot t + \Psi_{0213}\right) + \dots \Bigg] + \\
 &+B_{\delta 5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + \omega_5 \cdot t + \Psi_5\right) \times \\
 &\times \left[I_{2S1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_{021}\right) + \right. \\
 &+I_{2S5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + \omega_5 \cdot t + \Psi_{025}\right) + \\
 &+I_{2S7} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_7 \cdot t + \Psi_{027}\right) + \\
 &+I_{2S11} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + \omega_{11} \cdot t + \Psi_{0211}\right) + \\
 &+I_{2S13} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_{13} \cdot t + \Psi_{0213}\right) + \dots \Bigg] dx. (33)
 \end{aligned}$$

Let's consider products and integrals separately. First, let's look at expressions involving the 1st harmonic of magnetic induction.

$$\begin{aligned}
 &\int_0^{\tau} B_{\delta 1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_1\right) \times \\
 &\times I_{2S1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_{021}\right) dx = \\
 &= \frac{B_{\delta 1} \cdot I_{2S1}}{2} \cdot \int_0^{\tau} \left[\cos\left(\frac{2 \cdot \pi}{\tau} \cdot x + 2 \cdot \omega_1 \cdot t + \Psi_1 + \Psi_{021}\right) + \right. \\
 &+ \cos(\Psi_1 - \Psi_{021}) \Bigg] dx = \frac{B_{\delta 1} \cdot I_{2S1}}{2} \times \\
 &\times \left[\frac{\tau}{2 \cdot \pi} \cdot \sin\left(\frac{2 \cdot \pi}{\tau} \cdot x + 2 \cdot \omega_1 \cdot t + \Psi_1 + \Psi_{021}\right) \right]_0^{\tau} + \\
 &+ \tau \cdot \cos(\Psi_1 - \Psi_{021}) \Bigg] = \frac{B_{\delta 1} \cdot I_{2S1}}{2} \tau \cdot \cos(\Psi_1 - \Psi_{021}) (34)
 \end{aligned}$$

$$\begin{aligned}
 &\int_0^{\tau} B_{\delta 1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_1\right) \times \\
 &\times I_{2S5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 5 \cdot \omega_1 \cdot t + \Psi_{025}\right) dx = \\
 &= \frac{B_{\delta 1} \cdot I_{2S5}}{2} \cdot \int_0^{\tau} \left[\cos(0 + 6 \cdot \omega_1 \cdot t + \Psi_1 + \Psi_{025}) + \right. \\
 &+ \cos\left(-\frac{2 \cdot \pi}{\tau} \cdot x + 4 \cdot \omega_1 \cdot t + \Psi_{025} - \Psi_1\right) \Bigg] dx = \\
 &= \frac{B_{\delta 1} \cdot I_{2S5}}{2} \cdot \left[\cos(6 \cdot \omega_1 \cdot t + \Psi_1 + \Psi_{025}) \cdot x - \right. \\
 &- \frac{\tau}{2 \cdot \pi} \cdot \sin\left(-\frac{2 \cdot \pi}{\tau} \cdot x + 4 \cdot \omega_1 \cdot t + \Psi_{025} - \Psi_1\right) \Bigg]_0^{\tau} = \\
 &= \frac{B_{\delta 1} \cdot I_{2S5}}{2} \cdot \tau \cdot \cos(6 \cdot \omega_1 \cdot t + \Psi_1 + \Psi_{025}). (35)
 \end{aligned}$$

$$\begin{aligned}
 &\int_0^{\tau} B_{\delta 1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_1\right) \times \\
 &\times I_{2S7} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + 7 \cdot \omega_1 \cdot t + \Psi_{027}\right) dx = \\
 &= \frac{B_{\delta 1} \cdot I_{2S7}}{2} \cdot \int_0^{\tau} \left[\cos\left(\frac{2 \cdot \pi}{\tau} \cdot x + 8 \cdot \omega_1 \cdot t + \Psi_1 + \Psi_{027}\right) + \right. \\
 &+ \cos(6 \cdot \omega_1 \cdot t + \Psi_{027} - \Psi_1) \Bigg] dx = \\
 &= \frac{B_{\delta 1} \cdot I_{2S7}}{2} \cdot \tau \cdot \cos(6 \cdot \omega_1 \cdot t + \Psi_{027} - \Psi_1). (36)
 \end{aligned}$$

$$\begin{aligned}
 &\int_0^{\tau} B_{\delta 1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_1\right) \times \\
 &\times I_{2S11} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 11 \cdot \omega_1 \cdot t + \Psi_{0211}\right) dx = \\
 &= \frac{B_{\delta 1} \cdot I_{2S11}}{2} \cdot \int_0^{\tau} \left[\cos(12 \cdot \omega_1 \cdot t + \Psi_1 + \Psi_{0211}) + \right. \\
 &+ \cos\left(\frac{2 \cdot \pi}{\tau} \cdot x - 10 \cdot \omega_1 \cdot t + \Psi_1 - \Psi_{0211}\right) \Bigg] dx = \\
 &= \frac{B_{\delta 1} \cdot I_{2S11}}{2} \cdot \tau \cdot \cos(12 \cdot \omega_1 \cdot t + \Psi_1 + \Psi_{0211}). (37)
 \end{aligned}$$

$$\begin{aligned}
 &\int_0^{\tau} B_{\delta 1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_1\right) \times \\
 &\times I_{2S13} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + 13 \cdot \omega_1 \cdot t + \Psi_{0213}\right) dx = \\
 &= \frac{B_{\delta 1} \cdot I_{2S13}}{2} \cdot \int_0^{\tau} \left[\cos(12 \cdot \omega_1 \cdot t + \Psi_{0213} - \Psi_1) + \right. \\
 &+ \cos\left(\frac{2 \cdot \pi}{\tau} \cdot x + 14 \cdot \omega_1 \cdot t + \Psi_1 + \Psi_{0213}\right) \Bigg] dx =
 \end{aligned}$$

$$= \frac{B_{\delta 1} \cdot I_{2S13}}{2} \cdot \tau \cdot \cos(12 \cdot \omega_1 \cdot t + \Psi_{0213} - \Psi_1). \quad (38)$$

Taking into account the obtained results, we have

$$\begin{aligned} & \int_0^{\tau} B_{\delta 1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_1\right) \times \\ & \times \left[I_{2S1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_{021}\right) + \right. \\ & + I_{2S5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 5 \cdot \omega_1 \cdot t + \Psi_{025}\right) + \\ & + I_{2S7} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + 7 \cdot \omega_1 \cdot t + \Psi_{027}\right) + \\ & + I_{2S11} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 11 \cdot \omega_1 \cdot t + \Psi_{0211}\right) + \\ & + I_{2S13} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + 13 \cdot \omega_1 \cdot t + \Psi_{0213}\right) + \\ & + \dots + I_{2Sk} \cdot \cos\left(\pm \frac{\pi}{\tau} \cdot x + k \cdot \omega_1 \cdot t + \Psi_{02k}\right) \Big] dx = \\ & = \frac{B_{\delta 1} \cdot \tau}{2} \cdot \left[I_{2S1} \cdot \cos(\Psi_1 - \Psi_{021}) + \right. \\ & + I_{2S5} \cdot \cos(6 \cdot \omega_1 \cdot t + \Psi_1 + \Psi_{025}) + \\ & + I_{2S7} \cdot \cos(6 \cdot \omega_1 \cdot t + \Psi_{027} - \Psi_1) + \\ & + I_{2S11} \cdot \cos(12 \cdot \omega_1 \cdot t + \Psi_1 + \Psi_{0211}) + \\ & + I_{2S13} \cdot \cos(12 \cdot \omega_1 \cdot t + \Psi_{0213} - \Psi_1) + \dots + \\ & \left. + I_{2Sk} \cdot \cos((k \pm 1) \cdot \omega_1 \cdot t + \Psi_{02k} \pm \Psi_1) \right], \quad (39) \end{aligned}$$

where the "+" sign stands for negative-sequence harmonics, and the "-" sign stands for positive-sequence harmonics.

Now let's consider separately the expressions containing the 5th HTH of magnetic induction.

$$\begin{aligned} & \int_0^{\tau} B_{\delta 5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 5 \cdot \omega_1 \cdot t + \Psi_5\right) \times \\ & \times \left[I_{2S1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_{021}\right) + \right. \\ & + I_{2S5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 5 \cdot \omega_1 \cdot t + \Psi_{025}\right) + \\ & + I_{2S7} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + 7 \cdot \omega_1 \cdot t + \Psi_{027}\right) + \\ & + I_{2S11} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 11 \cdot \omega_1 \cdot t + \Psi_{0211}\right) + \\ & \left. + I_{2S13} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + 13 \cdot \omega_1 \cdot t + \Psi_{0213}\right) + \right. \end{aligned}$$

$$\left. + \dots + I_{2Sk} \cdot \cos\left(\pm \frac{\pi}{\tau} \cdot x + k \cdot \omega_1 \cdot t + \Psi_{02k}\right) \right] dx = \dots \quad (40)$$

Let's consider each term separately.

$$\begin{aligned} & \int_0^{\tau} B_{\delta 5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 5 \cdot \omega_1 \cdot t + \Psi_5\right) \times \\ & \times I_{2S1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_{021}\right) dx = \\ & = \frac{B_{\delta 5} \cdot I_{2S1}}{2} \cdot \int_0^{\tau} \left[\cos\left(\frac{2 \cdot \pi}{\tau} \cdot x - 4 \cdot \omega_1 \cdot t + \Psi_{021} - \Psi_5\right) + \right. \\ & \left. + \cos(6 \cdot \omega_1 \cdot t + \Psi_{021} + \Psi_5) \right] dx = \\ & = \frac{B_{\delta 5} \cdot I_{2S1}}{2} \cdot \tau \cdot \cos(6 \cdot \omega_1 \cdot t + \Psi_{021} + \Psi_5). \quad (41) \end{aligned}$$

$$\begin{aligned} & \int_0^{\tau} B_{\delta 5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 5 \cdot \omega_1 \cdot t + \Psi_5\right) \times \\ & \times I_{2S5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 5 \cdot \omega_1 \cdot t + \Psi_{025}\right) dx = \\ & = \frac{B_{\delta 5} \cdot I_{2S5}}{2} \cdot \int_0^{\tau} \left[\cos\left(-\frac{2 \cdot \pi}{\tau} \cdot x + 10 \cdot \omega_1 \cdot t + \Psi_5 + \Psi_{025}\right) + \right. \\ & \left. + \cos(\Psi_5 - \Psi_{025}) \right] dx = \\ & = \frac{B_{\delta 5} \cdot I_{2S5}}{2} \cdot \tau \cdot \cos(\Psi_5 - \Psi_{025}). \quad (42) \end{aligned}$$

$$\begin{aligned} & \int_0^{\tau} B_{\delta 5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 5 \cdot \omega_1 \cdot t + \Psi_5\right) \times \\ & \times I_{2S7} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + 7 \cdot \omega_1 \cdot t + \Psi_{027}\right) dx = \\ & = \frac{B_{\delta 5} \cdot I_{2S7}}{2} \cdot \int_0^{\tau} \left[\cos\left(\frac{2 \cdot \pi}{\tau} \cdot x + 2 \cdot \omega_1 \cdot t + \Psi_{027} - \Psi_5\right) + \right. \\ & \left. + \cos(12 \cdot \omega_1 \cdot t + \Psi_{027} + \Psi_5) \right] dx = \\ & = \frac{B_{\delta 5} \cdot I_{2S7}}{2} \cdot \tau \cdot \cos(12 \cdot \omega_1 \cdot t + \Psi_5 + \Psi_{027}). \quad (43) \end{aligned}$$

$$\begin{aligned} & \int_0^{\tau} B_{\delta 5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 5 \cdot \omega_1 \cdot t + \Psi_5\right) \times \\ & \times I_{2S11} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 11 \cdot \omega_1 \cdot t + \Psi_{0211}\right) dx = \\ & = \frac{B_{\delta 5} \cdot I_{2S11}}{2} \cdot \int_0^{\tau} \left[\cos\left(-\frac{2 \cdot \pi}{\tau} \cdot x + 16 \cdot \omega_1 \cdot t + \Psi_5 + \Psi_{0211}\right) + \right. \\ & \left. + \cos(6 \cdot \omega_1 \cdot t + \Psi_{0211} - \Psi_5) \right] dx = \\ & = \frac{B_{\delta 5} \cdot I_{2S11}}{2} \cdot \tau \cdot \cos(6 \cdot \omega_1 \cdot t + \Psi_{0211} - \Psi_5). \quad (44) \end{aligned}$$

$$\begin{aligned} & \int_0^{\tau} B_{\delta 5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 5 \cdot \omega_1 \cdot t + \Psi_5\right) \times \\ & \times I_{2S13} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + 13 \cdot \omega_1 \cdot t + \Psi_{0213}\right) dx = \\ & = \frac{B_{\delta 5} \cdot I_{2S13}}{2} \cdot \int_0^{\tau} \left[\cos\left(\frac{2 \cdot \pi}{\tau} \cdot x + 8 \cdot \omega_1 \cdot t + \Psi_{0213} - \Psi_5\right) + \right. \\ & \left. + \cos(18 \cdot \omega_1 \cdot t + \Psi_{0213} + \Psi_5) \right] dx = \\ & = \frac{B_{\delta 5} \cdot I_{2S13}}{2} \cdot \tau \cdot \cos(18 \cdot \omega_1 \cdot t + \Psi_{0213} + \Psi_5). \quad (45) \end{aligned}$$

As a result, we can write

$$\begin{aligned} & \int_0^{\tau} B_{\delta 5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 5 \cdot \omega_1 \cdot t + \Psi_5\right) \times \\ & \times I_{2Sk} \cdot \cos\left(\pm \frac{\pi}{\tau} \cdot x + k \cdot \omega_1 \cdot t + \Psi_{02k}\right) dx = \\ & = \frac{B_{\delta 5} \cdot I_{2Sk}}{2} \cdot \int_0^{\tau} \left\{ \cos\left[\left(\pm \frac{\pi}{\tau} \cdot x\right) - \right. \right. \\ & \left. \left. - \frac{\pi}{\tau} \cdot x + (k+5) \cdot \omega_1 \cdot t + \Psi_{02k} + \Psi_5\right] + \right. \\ & \left. + \cos\left[\left(\pm \frac{\pi}{\tau} \cdot x\right) + \frac{\pi}{\tau} \cdot x + \right. \right. \\ & \left. \left. + (k-5) \cdot \omega_1 \cdot t + \Psi_{02k} - \Psi_5\right] \right\} dx. \quad (46) \end{aligned}$$

For positive-sequence harmonics ($k = 1, 7, 13 \dots$) the expression (46) is transformed to the form

$$\frac{B_{\delta 5} \cdot I_{2Sk}}{2} \cdot \tau \cdot \cos[(k+5) \cdot \omega_1 \cdot t + \Psi_{02k} + \Psi_5]. \quad (47)$$

For negative-sequence harmonics ($k = 5, 11, 17 \dots$) the expression (46) is transformed to the form

$$\frac{B_{\delta 5} \cdot I_{2Sk}}{2} \cdot \tau \cdot \cos[(k-5) \cdot \omega_1 \cdot t + \Psi_{02k} - \Psi_5]. \quad (48)$$

In general, the expression (46) is transformed to the form

$$\frac{B_{\delta 5} \cdot I_{2Sk}}{2} \cdot \tau \cdot \cos[(k \mp 5) \cdot \omega_1 \cdot t + \Psi_{02k} \mp \Psi_5], \quad (49)$$

where the signs “-” refer to the HTH of the negative sequence, and the signs “+” refer to the first harmonic and HTH of the positive sequence.

Then, for expressions containing the 5th HTH of magnetic induction, we have the opportunity to continue the equality at the end of the expression (40) as follows

$$\begin{aligned} & = \frac{B_{\delta 5} \cdot \tau}{2} \cdot [I_{2S1} \cdot \cos(6 \cdot \omega_1 \cdot t + \Psi_5 + \Psi_{021}) + \\ & + I_{2S5} \cdot \cos(\Psi_5 - \Psi_{025}) + \\ & + I_{2S7} \cdot \cos(12 \cdot \omega_1 \cdot t + \Psi_{027} + \Psi_5) + \\ & + I_{2S11} \cdot \cos(6 \cdot \omega_1 \cdot t - \Psi_5 + \Psi_{0211}) + \\ & + I_{2S13} \cdot \cos(18 \cdot \omega_1 \cdot t + \Psi_{0213} + \Psi_5) + \dots + \\ & + I_{2Sk} \cdot \cos((k \mp 5) \cdot \omega_1 \cdot t + \Psi_{02k} \mp \Psi_5)]. \quad (50) \end{aligned}$$

For expressions containing the 7th HTH of magnetic induction we have

$$\begin{aligned} & \int_0^{\tau} B_{\delta 7} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + 7 \cdot \omega_1 \cdot t + \Psi_7\right) \times \\ & \times \left[I_{2S1} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + \omega_1 \cdot t + \Psi_{021}\right) + \right. \\ & + I_{2S5} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 5 \cdot \omega_1 \cdot t + \Psi_{025}\right) + \\ & + I_{2S7} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + 7 \cdot \omega_1 \cdot t + \Psi_{027}\right) + \\ & + I_{2S11} \cdot \cos\left(-\frac{\pi}{\tau} \cdot x + 11 \cdot \omega_1 \cdot t + \Psi_{0211}\right) + \\ & + I_{2S13} \cdot \cos\left(\frac{\pi}{\tau} \cdot x + 13 \cdot \omega_1 \cdot t + \Psi_{0213}\right) + \\ & + \dots + I_{2Sk} \cdot \cos\left(\pm \frac{\pi}{\tau} \cdot x + k \cdot \omega_1 \cdot t + \Psi_{02k}\right) \Big] dx = \\ & = \frac{B_{\delta 7} \cdot \tau}{2} \cdot [I_{2S1} \cdot \cos(6 \cdot \omega_1 \cdot t + \Psi_7 - \Psi_{021}) + \\ & + I_{2S5} \cdot \cos(12 \cdot \omega_1 \cdot t + \Psi_{025} + \Psi_7) + \\ & + I_{2S7} \cdot \cos(\Psi_7 - \Psi_{027}) + \\ & + I_{2S11} \cdot \cos(18 \cdot \omega_1 \cdot t + \Psi_7 + \Psi_{0211}) + \\ & + I_{2S13} \cdot \cos(6 \cdot \omega_1 \cdot t + \Psi_{0213} - \Psi_7) + \dots + \\ & + I_{2Sk} \cdot \cos((k \mp 7) \cdot \omega_1 \cdot t + \Psi_{02k} \mp \Psi_7)], \quad (51) \end{aligned}$$

where the signs “-” refer to the first harmonic and HTH of the positive sequence, and the signs “+” refer to the HTH of the negative sequence.

As a result of integration, expression (23) takes the form:

$$T_{\delta} = p \cdot l_{\delta} \cdot \tau \cdot \frac{D_2}{2} \cdot \sum_{v=1}^{\infty} B_{\delta v} \times \quad (52)$$

$$\times \sum_{k=1}^{\infty} I_{2Sk} \cdot \cos((k \pm v) \cdot \omega_1 \cdot t + \Psi_{02k} \pm \Psi_v),$$

similarly for the Fourier sine series we obtain (if the supply voltage or current is given by a Fourier sine series)

$$T_{\delta} = \mp p \cdot l_{\delta} \cdot \tau \cdot \frac{D_2}{2} \cdot \sum_{v=1}^{\infty} B_{\delta v} \times \times \sum_{k=1}^{\infty} I_{2Sk} \cdot \cos((k \pm v) \cdot \omega_1 \cdot t + \Psi_{02k} \pm \Psi_v), \quad (53)$$

where are the

$$\begin{aligned} \text{upper signs at } & \begin{cases} v = 1; 7; 13 \dots \\ k = 5; 11; 17 \dots \end{cases} \text{ or at } \begin{cases} v = 5; 11; 17 \dots \\ k = 1; 7; 13 \dots \end{cases}, \\ \text{lower signs at } & \begin{cases} v = 1; 7; 13 \dots \\ k = 1; 7; 13 \dots \end{cases} \text{ or at } \begin{cases} v = 5; 11; 17 \dots \\ k = 5; 11; 17 \dots \end{cases}. \end{aligned}$$

It should be noted that when integrating equation (23), the asynchronous components of the air gap torque (which do not vary with time), created by the interaction of negative sequence harmonics of the air gap field and the rotor current of the same ordinal number ($v = k = 5; 11; 17 \dots$), give the result in the form of positive torque values. But, since they are directed towards the rotation of the rotor, then in expressions (52) and (53) such torques must be taken with a “-” sign.

3 Forming Time-Varying Components of the Air Gap Torque

The time-varying components of the air gap torque are caused by the interaction of time harmonics of air gap field and rotor squirrel cage currents of different ordinal numbers ($v \neq k$), that is, created independently of each other. In the theory of electrical machines, such torques are called synchronous; they create ripples of the air gap torque of the IM.

By calculating the curve of the air gap torque of the IM for 1/6 of the period of the supply voltage, we can determine the ripple factor of the air gap torque [26] (similarly to current ripple factor [27]):

$$k_{Tr} = \frac{T_{\max} - T_{\min}}{2 \cdot T_1}, \quad (54)$$

where $T_{\max}$ and $T_{\min}$ - the largest and smallest values of the air gap torque; T_1 is the value of the air gap torque from the interaction of the first (fundamental) harmonics of the air gap field and the rotor current. In (4), it is legitimate to use T_1 instead of $T_{\delta av}$, since T_1 and $T_{\delta av}$ differ by less than $0.03 \cdot T_1$ due to the influence of asynchronous torques from the HTH (52). Ripple factor of the air gap torque characterizes in relative units the magnitude of torque pulsations.

Analyzing expressions (52) and (53), it is easy to notice that each synchronous component of the air gap torque is a cosine wave with an oscillation frequency $f_T = 2 \cdot m \cdot n$ that is a multiple of $6 \cdot f_1$. There are at least a pair of synchronous torques with the same f_T . Such a pair can be considered synchronous torques obtained from the interaction of the time harmonic of air gap field with number V with the time harmonics of the spatial wave of the rotor current with numbers $k = 2 \cdot m \cdot n - 1$ and $k = 2 \cdot m \cdot n + 1$. For example, the fundamental harmonic of air gap field with the fifth and seventh HTH of the rotor current - in this case we have a pair of synchronous torques with $f_T = 6 \cdot f_1$. The phase difference between the cosines of synchronous torques of the same frequency is constant:

in the case of specifying the supply voltage (current) by the Fourier cosine series:

$$\varphi_{coh} = 2 \cdot \Psi_v + \Psi_{02k_{2mn-1}} - \Psi_{02k_{2mn+1}}, \quad (55)$$

in the case of specifying the supply voltage (current) by the Fourier sine series (taking into account $-\cos \alpha = \cos(\alpha + \pi)$):

$$\varphi_{coh} = 2 \cdot \Psi_v - \Psi_{02k_{2mn-1}} + \Psi_{02k_{2mn+1}} + \pi. \quad (56)$$

Thus, both components of the torque are coherent waves and have the ability to interfere in time. In other words, the result of adding two simple harmonic oscillations of the same frequency is a simple harmonic oscillation of the same frequency. For the amplitude of the resulting torque wave resulting from the addition of a pair of synchronous torques, the following relation is valid:

$$T_{cohv_{2mn}} = \sqrt{T_{v_{2mn+1}}^2 + T_{v_{2mn-1}}^2 + 2 \cdot T_{v_{2mn+1}} \cdot T_{v_{2mn-1}} \cdot \cos \varphi_{coh}}, \quad (57)$$

where $T_{cohv_{2mn}}$ is minimum at $\varphi_{coh} = \pi$ and maximum at $\varphi_{coh} = 0$.

Let us illustrate (7) with an example. Figure 4 shows vectors $\overline{AB}$ and $\overline{AC}$ rotating in the same direction with the same angular frequency $6 \cdot \omega_1 \cdot t$, the projections of each of which on the horizontal axis symbolize, respectively, the synchronous torques from the interaction of the 1st harmonic of induction and the 5th HTH of the spatial rotor current T_{15} and the 1st harmonic of induction and the

7th HTH of the spatial rotor current T_{17} . Vector $\overline{AD}$ is the sum of vectors $\overline{AB}$ and $\overline{AC}$, and its projection on the horizontal axis symbolizes the pulsating torque formed by the joint action of T_{15} and T_{17} [11], [28].

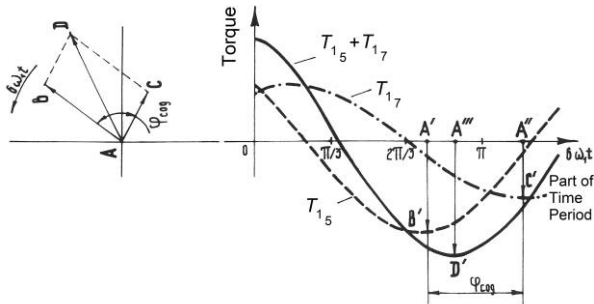


Fig. 4: - Interference in time of coherent waves of a ir gap torque
Source: created by the author.

Since the main torque of the IM is T_1 , we will take it as the base unit for each specific operating mode. Time-varying components of the air gap torque are created primarily by the interaction of the fundamental harmonic of the air gap field and the k -th time harmonics of the spatial wave of the rotor current. Then, based on (52), (53) and (57), when determining the relative amplitude of the resulting torque wave, we can use the exact relation

$$T_{cohv2mn}^* = \frac{\sqrt{I_{2mn+1}^2 + I_{2mn-1}^2 + 2 \cdot I_{2mn+1} \cdot I_{2mn-1} \cdot \cos \varphi_{coh}}}{I_{21}}, \quad (58)$$

where I_{2k} - amplitude of the time harmonic of the IM rotor current.

When determining the relative amplitude of the resulting torque wave, you can also use an approximate relationship that allows you to draw conclusions about synchronous torques, having at your disposal only oscillograms of the phase current and voltage of the IM and their expansion into the Fourier series into time harmonics, without knowing anything about slip, the design parameters of the IM, temperature of the windings, saturation coefficient of the magnetic core, necessary for carrying out a complete calculation using the harmonic analysis method and using expressions (52) and (53)

$$T_{cohv2mn}^* \geq \frac{\sqrt{I_{2mn+1}^2 + I_{2mn-1}^2 + 2 \cdot I_{2mn+1} \cdot I_{2mn-1} \cdot \cos \varphi_{coh}}}{I_{l1}} \quad (59)$$

Considering that $\Psi_{02mn-1} - \Psi_{02mn+1} = \Psi_{v02mn-1} - \Psi_{v02mn+1} = \Psi_{012mn-1} - \Psi_{012mn+1}$, the phase

difference in the HTH of the rotor current, voltage and stator current of the IM with numbers $k = 2 \cdot m \cdot n - 1$ and $k = 2 \cdot m \cdot n + 1$, respectively, is the same (the calculated discrepancy in the equality of the phase differences for thirty-three time harmonics with numbers from 1st to 99-th is less than 1°), $\Psi_v \approx \Psi_{v0v} - \pi$ (the largest calculated difference in equality for time harmonics from the 1st to the 99th is 15.1° , and values more than 8° were obtained for the frequency start mode of the IM, when f_1 is a few Hz, the rotor is braked). I_{lk} is the amplitude of the time harmonic of the IM stator current.

Thus, (59) allows us to obtain an answer to the question, no less than what value should be expected for the ripples of the air gap torque of the IM at the f_T frequency of interest for a given phase voltage or current shape. And having calculated the magnitude of the ripple of the air gap torque from a pair of the most pronounced HTH currents, we can conclude that no less than what value should be expected for the total ripple of the air gap torque of the IM.

From expressions (57) - (59) one can get an idea of the magnitude of the synchronous torque from one HTH of the supply voltage (current), assuming the paired one is equal to zero. Then equality (58) and inequality (59) will be reduced to

$$T_{vk}^* = \frac{T_{vk}}{T_1} = \frac{I_{2k}}{I_{21}} \geq \frac{I_{lk}}{I_{l1}}. \quad (60)$$

The Table 1 shows the comparative results of calculations of the air gap torque ripple for IM of types NB-607 (6-pole, 900 kW), ANE-225 (similar to NVA-55 4-pole, 55 kW [29]) and NTA-350 (6-pole, 350 kW) in the presence of only three time harmonics in the voltage (current) spectrum: the 1st and such a pair of harmonics which creates waves of synchronous torques of the same frequency f_T .

By the way, in Figure 5 we can see, that compared to the one obtained as a result of computer simulation, the experimental current rises and falls faster when the phase voltage is switched, and has a slightly larger amplitude [30].

When powered from an AVSI, it is possible to assess the suitability of a particular voltage form for powering the IM from the point of view of the magnitude of the air gap torque ripples, without resorting to analysis of the phase current oscillogram. This is relevant when evaluating various PWM or PWR algorithms - you can choose an algorithm that, for example, minimizes torque ripple at $f_T = 6 \cdot f_1$. The dependence of the ripple factor of the air gap torque k_{Tr} on the modulation index (for PWM) or the pulse duty cycle (for PWR) is explained by the difference in the harmonic composition of the phase voltage curve at different modulation depths, both in the sense of the HTH amplitudes and in the sense of their phase angles.

Table 1. The air gap torque ripple of IM, calculated by different methods

The magnitude of the air gap torque ripple of IM			
by (52 - 54)	by (58)	by (59)	by (52 - 54)*
IM of type NB-607, powered by AVSI, 6-step operation, starting mode, $S_1 = 1$, $f_1 = 1$ Hz, 1st, 5th and 7th time harmonics			
0.111	0.09	0.065	0.132
IM of type ANE-225, powered by AVSI, 6-step operation, $S_1 = 0.039$, $f_1 = 50$ Hz, 1st, 5th and 7th time harmonics			
0.228	0.223	0.178	0.226; 0.228**
IM of type ANE-225, powered by AVSI, pulse width regulation (PWR), $S_1 = 0.34$, $f_1 = 33.3$ Hz (Figure 5), 1st, 17th and 19th time harmonics			
0.275	0.234	0.230	0.348
IM of type ANE-225, powered by AVSI, PWR, $S_1 = 0.0667$, $f_1 = 25$ Hz, 1st, 23rd and 25th time harmonics			
1.919	1.91	1.446	2.756
IM of type NTA-350, powered by ACSI, rectangular current pulse 120° wide (meander with platform), starting mode, $S_1 = 1$, $f_1 = 1$ Hz (Figure 6), 1st, 5th and 7th time harmonics			
0.204; 0.201***	0.166	0.085	0.32; 0.201***

Notes: * - calculation based on thirty-three time harmonics (from the 1st to the 97th);

** - calculation based on nine time harmonics (from the 1st to the 25th);

*** - only synchronous torques pulsating with a frequency of $6 \cdot f_1$ are taken into account;

S_1 - slip for the 1st harmonic.

Source: created by the author.

The total ripple factor of the air gap torque is directly proportional to the absolute value of the sum of the complexes of the pair of the most pronounced phase voltage HTHs of ordinal numbers $k = 2 \cdot m \cdot n - 1$ and $k = 2 \cdot m \cdot n + 1$, related to the absolute value of the 1st harmonic of the phase voltage in the case of specifying a non-sinusoidal voltage using a Fourier cosine series. The case of specifying a non-sinusoidal voltage using a sine Fourier series is somewhat more complicated. One should add the HTH complex of the positive sequence voltage with a fictitious complex value of the negative sequence HTH having a phase angle $\Psi_{V_{02mn-1}} + \pi$ or vice versa, add the complex HTH of the negative sequence voltage with the fictitious complex value of the positive sequence HTH, having a phase angle $\Psi_{V_{02mn+1}} + \pi$.

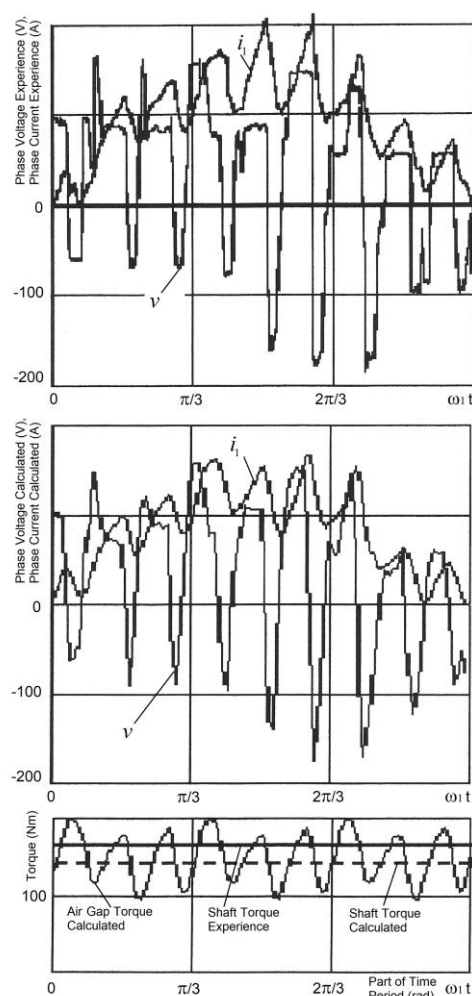


Fig. 5: - Experience and calculated curves of phase voltage, current and torques for IM of type ANE-225 @ $= 0.34$, $= 33.3$ Hz $s_1 f_1$

Source: created by the author.

In this case, attention should be paid to the fact that for all HTHs it is their initial phases at time $t = 0$ that are considered. This is important, since we are dealing with voltage vectors rotating at different angular speeds, the absolute value of their vector sum at each moment of time has a different value (the absolute value of the vector sum has maxima and minima 4 times during the period of the first harmonic).

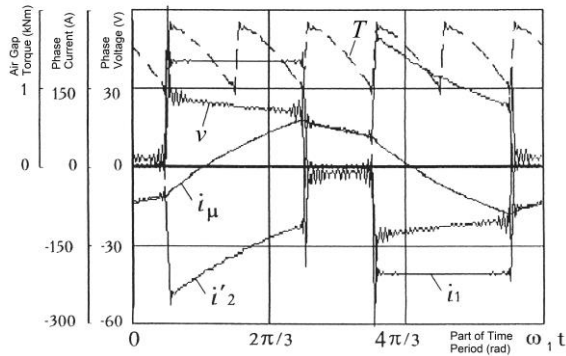


Fig. 6: - Calculated curves of phase voltage, currents, and air gap torque for traction IM of type NTA-350 @ $\omega = 1$, $f_1 = 1$ Hz, RMS phase stator current fundamental time harmonic is 229.4 A. The calculation is based on thirty-three time harmonics (from the 1st to the 97th) $s_1 f_1 I_{11} =$

Source: created by the author.

However, the vectors of synchronous torques created by each of the paired HTH voltages with the participation, for example, of the first harmonic of air gap field (magnetic induction), have the same angular velocity and a constant phase shift in time, which is ultimately determined by the phase shift at time $t = 0$ between the paired HTH of voltage (57). All statements in this paragraph are also valid for the air gap torque ripple factor for a specific frequency $f_T = 2 \cdot m \cdot n$, with the only caveat that it is necessary to consider a suitable pair of voltage HTH. Thus, we can write an expression for the case of specifying a non-sinusoidal voltage using the Fourier cosine series, where the ripple factor of the air gap torque is directly proportional to the calculation result on the right side:

$$k_{Tr} \square \frac{\sqrt{V_{2mn+1}^2 + V_{2mn-1}^2 + 2 \cdot V_{2mn+1} \cdot V_{2mn-1} \times}}{V_1} \times \cos(\Psi_{V_{02mn+1}} - \Psi_{V_{02mn-1}}) \quad (61)$$

For the case of specifying a non-sinusoidal voltage using a Fourier sine series an expression similar to (61) will be

$$k_{Tr} \square \frac{\sqrt{V_{2mn+1}^2 + V_{2mn-1}^2 + 2 \cdot V_{2mn+1} \cdot V_{2mn-1} \times}}{V_1} \times \cos(\Psi_{V_{02mn+1}} - \Psi_{V_{02mn-1}} - \pi) \quad (62)$$

Note that the numerical results obtained in the right-hand sides of (61) and (62), being taken for a separate mode, are of no practical value, but it is advisable to compare them, for example, for different pulse duty cycles for the same PWR scheme. In Figure 7 are shown for the case of phase-to-phase voltage at the output of the AVSI, which has the form of a single rectangular pulse per half-cycle of fundamental frequency (the most pronounced HTHs are the 5th and 7th): 1 - dependence characterizing the k_{Tr} according to (62); 2 - for IM of type ANE-225 k_{Tr} , taking into account air gap torque ripples only at $f_T = 6 \cdot f_1$; 3 - for IM of type ANE-225 k_{Tr} , taking into account all air gap torque ripples from the voltage HTH till the 97th. All curves in Figure 7 are given as a function of the pulse duty cycle; curves 2 and 3 correspond to the mode $f_1 = 50$ Hz; RMS value $V_{\text{phase-to-phase1}} = 380$ V; $s_1 = 0.039$ and were calculated according to (53) and (54).

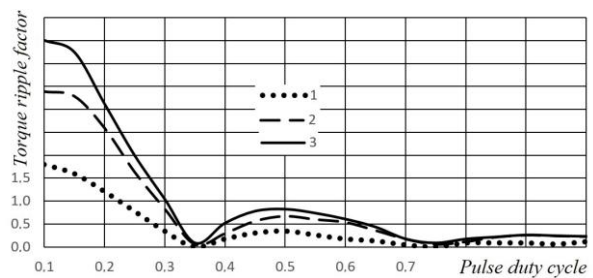


Fig. 7: - Comparison of the values of a ir gap torque ripples for IM of type ANE-225, calculated in different ways

Source: created by the author.

4 Conclusion

The article considers the calculation of the air gap torque of a 3-phase IM with a squirrel-cage rotor in a steady-state operating mode, as well as the torque ripple factor using the harmonic analysis method. It is shown that a comparative assessment of the air gap torque ripple magnitude for different operating modes of the IM can be given without having a

complete set of data on the IM parameters. As a first approximation, it is sufficient to analyze the oscillogram of the phase current or phase voltage. Thus, it becomes possible to judge, for example, which PWM algorithm of several compared ones provokes smaller ripples of the air gap torque. Or at what modulation index should the greatest torque ripple magnitude be expected?

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

The authors wrote, reviewed and edited the content as needed and they have not utilised artificial intelligence (AI) tools. The authors take full responsibility for the content of the publication.

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