Control of Robot Motion in Radial Mass Density Field

BRANKO NOVAKOVIC[a], DUBRAVKO MAJETIC JOSIP KASAC, DANKO BREZAK
Faculty of Mechanical Engineering and Naval Architecture,
University of Zagreb,
Ivana Lučića 5. Zagreb,
CROATIA

[a]ORCiD: https://orcid.org/0000-0003-0735-770X

Abstract: - T In this article, a new approach to control of robot motion in the radial mass density field is presented. This field is between the maximal and the minimal radial mass density values. Between these two limited values, one can use *n* points ($n = 1, 2, . . . n_{max}$) that can be included in the related algorithm for control of the robot motion. The number of the points n_{step} can be calculated by using the relation $n_{step} = n_{max} / n_{var}$, where n_{var} is the control parameter. The radial mass density is maximal at the minimal gravitational radius and minimal at the maximal gravitational radius. This is valid for Planck scale and for the scales that are less or higher of that one. Using the ratio of Planck mass and Planck radius it is generated the energy conservation constant $\kappa = 0.99993392118$.

Key-Words: - robot motion control; radial mass density field; maximal (minimal) radial mass density; energy conservation constant; macro (micro, nano) robot control; electrical robots; magnetic robots; chemical actuated robots; bio/soft robots.

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1 Introduction

Generally, the very large structures of the robots have a lot of application areas as in the precise production processes, in the medicine for cell manipulation, drug delivery, medical image acquisition, and non-invasive intervention. For those applications, one can use electrical, magnetic, chemical actuated robots and the bio/soft robots, [1], [2]. Genetic algorithms and unsupervised machine learning for predicting robotic manipulation failures for force-sensitive tasks discussed in [3]. An integrated design and fabrication strategy for entirely soft, autonomous robots is presented in [4]. Versatile soft-grippers with intrinsic electro-adhesion based on multifunctional polymer actuators is point out in [5]. Magnetic actuation methods in bio/soft robotics are discussed in [6]. Efficient constant-time addressing scheme for parallel-controlled assembly of stress-engineered MEMS micro-robots is present in [7].

In this article, the control of the robot's motion is described in the radial mass density field. This field is in the region from the minimal radius (with the maximal radial mass density ($\rho_{r\ max}$) and maximal radius (with the minimal radial mass density ($\rho_{r\ min}$). Between these two limited values, one can choose *n* points ($n=1,2,..n_{max}$). In the case of the precise robot motion the number n_{max} should be bigger. Contrary, for the less precise robot motion, the number n_{max} may be smaller. In that sense, one can introduce the related steps number (n_{step}) between maximal and minimal radiuses in a gravitational field. This value can be calculated by using the relation ($n_{step} = n_{max} / n_{var}$). If one uses the smaller parameter (n_{var}) than the number of the steps (n_{step}) is bigger and vice versa. In that way, one can obtain the most precise control of the robot's motion.

The very important consequence of the solution of the field equations by including gravitational energy-momentum tensor (*EMT*) on the right side of the field equation is that the gravitational field exhibits repulsive (positive) and attractive (negative) gravitational forces. The minimum time transition between quantum states in the gravitational field is present in [8]. To precisely follow the desired trajectory of the robot motion one can include the new Relativistic Radial Density Theory (RRDT), [9]. The particle transition and correlation in quantum mechanics are discussed in [10]. Independent position control of two identical magnetic micro-robots in a plane using permanent magnets and magnetically powerful microrobots is presented in [11]. This application represents the new approach to the medical revolution epoch.

Magnetically powered micro-robots are discussed in [12], [13].

Further, the robust control of micro-robot motion is presented in [14]. A conjugate gradient-based BPTT – like optimal control algorithm with vehicle dynamics control application is discussed in [15]. Robust motion control with anti-windup scheme for electromagnetic actuated micro-robot using time-delay estimation is presented in [16]. The two independent position controls of two equally micro-robots motion in a plane are realized by using rotating permanent magnets, [17]. Magnetically powered micro-robots and the robust motion control, with an anti-windup scheme for electromagnetic actuated micro-robots, are presented in [18] and [19], respectively. Robotic-assisted minimally invasive surgery is illustrated in [20]. The design of a novel haptic joystick for the teleoperation of continuum-mechanism-based medical robots is presented in [21]. In this reference, a novel mechanism with a series of coupled gears, that aims for the control of continuum robots for medical applications is pointed out. Positioning control of robotic manipulators subject to excitation from non-ideal sources is discussed in [22]. Further, tractor-robot cooperation is illustrated in [23]. Indoor positioning systems of mobile robots are present in [24]. A new single–leg lower-limb rehabilitation robot motion is presented in [25]. Multi-robot task scheduling for consensus-based fault resilient intelligent behavior in smart factories is discussed in [26]. A new single-leg lower limb rehabilitation robot with design, analysis, and experimental evolution is presented in [27]. It is also important to know how the portable surveillance robots can be used in IoT applications, [28]. The recent trends in robot learning and evolution for swarm robotics are presented in [29]. Finally, the proactivity of fish and leadership of self-propelled robotic fish during interaction and bio-inspiration with biomimietics is discussed in [30].

2 Dynamics of Autonomous Robot Motion in the Electromagnetic and Gravitational Radial Mass Density Field

The problem of the nonlinear control of robot motion is discussed as the function of the maximal radial mass density value. To simplify the related calculation, here it started with the concept of the external linearization of the nonlinear control of the robot motion in the radial mass density field. In that case, in the closed regulation loop, one obtains the linear behaviour of the hole-system. Thus, the problem of the robot position control in the radial mass density field can be started by the calculation of the control of the error vector, *e(t)*. This vector is a function of the radial mass density, ρ_r, and can be presented by the relations:

$$e = X_w - X\,,\quad \frac{d^2e}{dt^2} = r_w(t) - \frac{n}{\rho_{r\,max} r_{min}}\left[F_p + F_t + \frac{1}{c}NF_I\right], \tag{1}$$
$$r_w(t) = \frac{d^2X_w}{dt^2} = \frac{1/n}{\rho_{r\,max} r_{min}}\left[F_{pw} + F_{tw} + \frac{1}{c}NF_{Iw}\right].$$

Here $n=1,2,..,n_{max}$ and $n_{max} = \rho_{r\,max} / \rho_{r\,min}$, while the subscript w denotes the desired robot motion. The variables without this subscript present the real autonomous robot motion. Further, F_p is a potential force, F_t is a time - variation force, F_i is the interaction force and N is the related connection parameter. At the same time, the relations (1) also describe the canonical differential equations of the robot motion in the combination of the electromagnetic and gravitational fields. Vector $r_w(t)$ presents the desired (nominal) acceleration of the robot motion in the radial mass density field.

Now following the idea of external linearization, one can introduce the following substitution:

$$u(t) = \frac{d^2e}{dt^2} = r_w(t) - \frac{n}{\rho_{r\,max} r_{min}}\left[F_p + F_t + \frac{1}{c}NF_I\right], \tag{2}$$
$$u(t) = (u_x(t)\, u_y(t)\, u_z(t))^T.$$

Here $u(t)$ is the internal control vector of the robot motion in the radial mass density field. Further, one can apply the state-space phase variables, $(z_1\, z_2\, z_3)^T$, that from (1) gives the related state-space model of the robot motion in the radial mass density field:

$$e = (e_x\; e_y\; e_z)^T = Z_I = (z_1\; z_2\; z_3)^T,$$
$$\frac{de}{dt} = \left(\frac{de_x}{dt}\,\frac{de_y}{dt}\,\frac{de_z}{dt}\right)^T = Z_{II} = (z_4\; z_5\; z_6)^T, \tag{3}$$

and

$$dZ/dt = AZ(t) + Bu(t),$$
$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ I \end{bmatrix},\; I = diag[1, 1, 1]. \tag{4}$$

In (4), parameters A and B are constant matrices with dimension (6x6) and (6x3), respectively. Here, it is supposed that the disturbances in a state-space model of the robot motion in the radial mass density field (3) and (4) are of the initial condition types. To eliminate the control error of the robot motion in the radial mass density field, which is caused by the disturbances, one can introduce the following internal control:

$$F_p = \frac{1}{n}\rho_{r\,max} r_{min}\left[r_w(t) + K_I Z_I + K_{II} Z_{II}\right] - \left[F_t + \frac{N}{c}F_I\right], \quad (5)$$
$$u(t) = -KZ\,.$$

Here, K is the state space controller, Z is the control error, F_p is the potential force, F_t is the time variable force, F_I is an interaction force, N is a constant and c is the speed of the light in vacuum. Including the internal control relations (3) and (4) into (5), one obtains the related equation of the potential force as a function of the radial mass density value in the linear form:

$$F_p = \frac{1}{n}\rho_{r\,max} r_{min}\left[r_w(t) + K_I Z_I + K_{II} Z_{II}\right] - \left[F_t + \frac{N}{c}F_I\right]. \quad (6)$$

Now starting from the previous relations one can generate the new equations of the potential force F_p as the functions of the potential energies U_j an U_c :

$$F_{p_x} = -\left(\frac{\partial \Sigma U_j}{\partial x} + \frac{\partial U_c}{\partial x}\right), \quad F_{p_y} = -\left(\frac{\partial \Sigma U_j}{\partial y} + \frac{\partial U_c}{\partial y}\right), \quad (7)$$
$$F_{p_z} = -\left(\frac{\partial \Sigma U_j}{\partial z} + \frac{\partial U_c}{\partial z}\right).$$

Here $j = g$ for the gravitational field, $j = e$ for the electromagnetic field and U_c is the related control potential field. It is followed by the inclusion of the control potential force, F_{cp} , that is derived by the artificial control field with potential control energy U_c . After inclusion of the relation (7) into the relation (6), one obtains the nonlinear control of the robot motion in the multi-potential field as the function of the maximal radial mass density $\rho_{r\,max}$:

$$F_{cp} = \frac{1}{n}\rho_{r\,max} r_{min}\left[r(t) + K_I Z_I + K_{II} Z_{II}\right] - \left[F_{dp} + \mathrm{F}_t + \frac{N}{c}F_I\right]. \quad (8)$$

Now, using (8), the control of the nonlinear system is solved by employing the concept of external linearization in the radial mass density field. Here the obtained equations are functions of the radial mass density values.

The general approach to control the dynamics of the robot motion in radial mass density field for more potential fields, given in (8), can also be applied to the two potential electromagnetic and gravitational fields. In this sense, let a robot be an electrically charged particle with charge q and rest mass m_0 that is moving with a non-relativistic velocity ($v << c$) in combined electromagnetic and gravitational potential fields. Further, it is also assumed that the gravitational field is produced by the spherically symmetric (non-charged) body with mass M. In that case, the total potential energy U of the robot motion in the two potential radial mass density fields is described by the relation:

$$U = qV_e + m_0 V_g = qV_e + m_0\left(-\frac{GM}{r}\right),$$
$$\rho_{r\,max} = \frac{m_0}{r_{min}},\; U = qV_e + \frac{1}{n}\rho_{r\,max} r_{min}\left(-\frac{GM}{r}\right). \quad (9)$$

Here V_e and V_g are the related scalar potentials of the electromagnetic and gravitational radial mass density fields, respectively. Parameter G is the gravitational constant and r is the radius as the distance between the autonomous robot and the center of the mass M and $n=1,2,..,n_{max}$, $n_{max} = \rho_{r\,max} / \rho_{r\,min}$. Now applying (1) and using the notations, (E_e,H_e) for an electromagnetic field and (E_g,H_g) for the gravitational field, one can generate the vector equation as the explicit functions of the Lorentz forces:

$$\frac{1}{n}\rho_{r\,max} r_{min}\frac{d^2X}{dt^2} = q\left(E_e + \frac{1}{c}v \times H_e\right)$$
$$+ \frac{1}{n}\rho_{r\,max} r_{min}\left(E_g + \frac{1}{c}v \times H_g\right). \quad (10)$$

The parameters E_e, E_g, H_e and H_g are vectors described by the relations:

$$E_e = \begin{bmatrix} E_{e_x} \\ E_{e_y} \\ E_{e_z} \end{bmatrix}, \quad E_g = \begin{bmatrix} E_{g_x} \\ E_{g_y} \\ E_{g_z} \end{bmatrix}, \quad H_e = \begin{bmatrix} H_{e_x} \\ H_{e_y} \\ H_{e_z} \end{bmatrix}, \quad H_g = \begin{bmatrix} H_{g_x} \\ H_{g_y} \\ H_{g_z} \end{bmatrix}. \quad (11)$$

In this example, a robot is a particle with charge q and rest mass m_0 and, therefore, this robot interacts with both electromagnetic and gravitational radial mass density fields. In that sense, the relations (10) and (11) describe the dynamic of the robot motion in two – potential electromagnetic and gravitational field. The components of the vector E_e and E_g can be calculated by using the following equations:

$$E_{e_x} = -\frac{\partial V_e}{\partial x} - \frac{1}{c}\frac{\partial A_{e_x}}{\partial t}, \quad E_{g_x} = -\frac{\partial V_g}{\partial x} - \frac{1}{c}\frac{\partial A_{g_x}}{\partial t}, \quad (12)$$
$$E_{e_y} = -\frac{\partial V_e}{\partial y} - \frac{1}{c}\frac{\partial A_{e_y}}{\partial t},$$

and

$$E_{g_y} = -\frac{\partial V_g}{\partial y} - \frac{1}{c}\frac{\partial A_{g_y}}{\partial t}, \quad E_{e_z} = -\frac{\partial V_e}{\partial z} - \frac{1}{c}\frac{\partial A_{e_z}}{\partial t}, \quad E_{g_z} = -\frac{\partial V_g}{\partial z} - \frac{1}{c}\frac{\partial A_{g_z}}{\partial t}. \tag{13}$$

The components of vectors A_e, A_g, H_e, and H_g in (12) and (13) are given by the relations:

$$A_{e_i} = \left(\frac{v_i V_e}{c}\right), \; A_{g_i} = \left(\frac{v_i V_g}{c}\right), \; H_{e_x} = \frac{\partial A_{e_z}}{\partial y} - \frac{\partial A_{e_y}}{\partial z}, \quad H_{g_x} = \frac{\partial A_{g_z}}{\partial y} - \frac{\partial A_{g_y}}{\partial z}, \; i = x,y,z, \tag{14}$$

and

$$H_{e_y} = \frac{\partial A_{e_x}}{\partial z} - \frac{\partial A_{e_z}}{\partial x}, \; H_{g_y} = \frac{\partial A_{g_x}}{\partial z} - \frac{\partial A_{g_z}}{\partial x}, \quad H_{e_z} = \frac{\partial A_{e_y}}{\partial x} - \frac{\partial A_{e_x}}{\partial y}, \; H_{g_z} = \frac{\partial A_{g_y}}{\partial x} - \frac{\partial A_{g_x}}{\partial y}. \tag{15}$$

Applying (6) and (7) to the canonical differential equations of the autonomous robot motion in the two-potential radial mass density field, one obtains the control error model of the robot motion as a function of the maximal radial mass density value:

$$\ddot{e}(t) = r_w(t) - \frac{nq}{\rho_{r\,max} r_{min}}\left(E_e + \frac{1}{c}\mathrm{v}\times H_e\right) - \left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right), \tag{16}$$

and

$$r_w(t) = \frac{nq}{\rho_{r\,max} r_{min}}\left(E_{e_w} + \frac{1}{c}\mathrm{v}_w \times H_{e_w}\right) - \left(E_{g_w} + \frac{1}{c}\mathrm{v}_w \times H_{g_w}\right). \tag{17}$$

In (17) r_w is the vector of desired acceleration of the robot motion. The subscript w denotes the desired values of the related variables. The next step is the application of the concept of external linearization to transform the equation (16) into the new relation:

$$u(t) = r_w(t) - \frac{nq}{\rho_{r\,max} r_{min}}\left(E_e + \frac{1}{c}v\times H_e\right) - \left(E_g + \frac{1}{c}v\times H_g\right). \tag{18}$$

Here $u(t)$ is the internal control vector and $n=1,2,..n_{max}$ is the number of the robot steps from the minimal to the maximal radiuses in radial mass density field. From (17) and (18), one obtains the related equivalent of the linear control error model of the robot motion in the combined electromagnetic and gravitational radial mass density field, given by (6) and (7). Applying (18), one obtains the new relation as the function of the maximal radial mass density in the form:

$$E_e = \frac{\rho_{r\,max} r_{min}}{nq}\left[r_w(t) + K_I Z_I + K_{II} Z_{II}\right] - \left(\frac{1}{c}\mathrm{v}\times H_e\right) - \frac{\rho_{r\,max} r_{min}}{nq}\left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right). \tag{19}$$

Now, let the electric field E_e consist of the two electric components $E_e = E_{de} + E_{ce}$. Here E_{de} is a disturbance electric field that is caused by the influence of a two-potential field on the motion of the robot in the radial mass density field. The component E_{ce} is an artificial electric control field that should control robot motion in the two potential fields. Including $E_e = E_{de} + E_{ce}$ into (19), one obtains the nonlinear electric control of the robot motion in the two-potential radial mass density field as the function of the maximal radial mass density:

$$E_{ce} = \frac{\rho_{r\,max} r_{min}}{nq}\left[r_w(t) + K_I Z_I + K_{II} Z_{II}\right] \left(E_{de} + \frac{1}{c}\mathrm{v}\times H_e\right) - \frac{\rho_{r\,max} r_{min}}{nq}\left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right). \tag{20}$$

Taking into account the relation (10), the canonical differential equations of the robot motion in the two-potential radial mass density field can be rewritten as a function of the maximal radial mass density:

$$\frac{d^2X}{dt^2} = \frac{nq}{\rho_{r\,max} r_{min}}\left(E_{de} + E_{ce} + \frac{1}{c}\mathrm{v}\times H_e\right) + \left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right). \tag{21}$$

Applying the nonlinear control E_{ce} from (20) to the nonlinear dynamical model of the robot motion (21), one obtains the closed-loop system in the linear form:

$$\frac{d^2X}{dt^2} = r_w(t) + K_I Z_I + K_{II} Z_{II}. \tag{22}$$

Thus, equation (20) is the nonlinear control, which in the closed loop with the nonlinear canonical differential equations of the robot motion (21), results in the linear behaviour of the hole system (22). On that way the problem of controlling the robot motion in the combination of an electromagnetic and gravitational radial mass density field has been solved by employing the concept of external linearization. This is very important for application of the micro and nanorobots in the drag delivery across the human body.

3 The Other Methods of Application of the Radial Mass Density to Robot Control

The global positioning of robot manipulators with mixed revolute and prismatic joints is presented in [19]. In this section, it is illustrated how one can apply the maximal radial mass density theory to the mentioned class of the robots. In that sense, the dynamic model of the robot with the n-link rigid body can be described as the function of the maximal radial mass density:

$$m_0(q)\frac{d^2q}{dt^2}+C(q,\frac{dq}{dt})\frac{dq}{dt}+q(q)=U,$$
$$\rho_{r\,max}r_{min}(q)\frac{d^2q}{dt^2}+C(q,\frac{dq}{dt})\frac{dq}{dt}+q(q)=U, \quad (23)$$
$$m_0(q)=\rho_{r\,max}r_{min}.$$

Here q is $(nx1)$ vector of robot joints coordinates, dq/dt is the related vector of joints velocities, U is a vector of applied joint torques and forces, $m_0(q)$ is (nxn) inertia matrix, and $C(q,dq/dt)dq/dt$ is $(nx1)$ vector of centrifugal and Coriolis torques. Further $q(q)$ is the vector of gravitational torques and forces and $\rho_{r\,max}$ is the maximal radial mass density at the minimal radius. If the robot, described by (23), is in the closed loop with the nonlinear *PID* controller, described by the relation:

$$U(t)=-(K_p\frac{d^2q}{dt^2}+K_d\frac{dq}{dt}+K_I\frac{dq}{dt}), \quad (24)$$

Then the closed loop system of the relations (23) and (24) resulted in the form that is the function of the maximal radial mass density:

$$\rho_{r\,max}r_{min}(q)\frac{d^2q}{dt^2}+C(q,\frac{dq}{dt})\frac{dq}{dt}+q(q)$$
$$=-(K_p\frac{d^2q}{dt^2}+K_d\frac{dq}{dt}+K_I\frac{dq}{dt}). \quad (25)$$

The relation (25) can be applied for the parameter n =1,2,…, $\rho_{r\,max}/\rho_{r\,min}$. Now one can use the relation (25) in the new form:

$$\frac{d^2q}{dt^2}=-\frac{n_{step}}{\rho_{r\,max}r_{min}(q)}(C(q,\frac{dq}{dt})\frac{dq}{dt}+q(q)$$
$$+K_p\frac{d^2q}{dt^2}+K_d\frac{dq}{dt}+K_I\frac{dq}{dt}),\ n_{step}=n_{max}/n_{var}. \quad (26)$$

Thus, using the relation (26) it is possible to control the robot's acceleration by changing the numerical parameter n_{step}. In that way by changing the parameter n_{var} it is possible the realization of the most precise robot motion control. This means that the radial distance between two points should be minimal if the n_{var} is maximal.

The dynamics of the robot motion can also be described as the function of the alpha field parameters derived in the Relativistic Alpha Field Theory (*RAFT*), [7]. In this theory, one can start with the potential energy of the robot (particle) in the combination of the electromagnetic and gravitational fields, U_e and U_g, respectively. Now let q, m, V_e, and V_g are the robot's (particle's) charge, mass, electrical potential, and gravitational potential, respectively. Further, G is the gravitational constant, M is the mass of the gravitational field and c is the speed of light in a vacuum. The potential energy of the robot in combination with the electromagnetic and gravitational fields is given by the relations:

$$U=U_e+U_g=\pm qV_e-\frac{mGM}{r},$$
$$\frac{U}{mc^2}=\pm\frac{qV_e}{mc^2}-\frac{GM}{rc^2}, \quad (27)$$

and

$$\frac{U}{\rho_{r\,max}r_{min}c^2}=\pm\frac{qV_e}{\rho_{r\,max}r_{min}c^2}-\frac{GM}{rc^2},$$
$$m=\rho_{r\,max}r_{min}. \quad (28)$$

The relation (28) can also be described as the function of the parameter n:

$$\frac{nU}{\rho_{r\,max}r_{min}c^2}=\pm\frac{nqV_e}{\rho_{r\,max}r_{min}c^2}-\frac{GM}{rc^2},$$
$$n=1...,n_{max},\quad n_{step}=n_{max}/n_{var}. \quad (29)$$

If one wants to use *RAF* theory in robotics then it requires the introduction of the related alpha field parameters. The solution of the field parameters for an electron in the two-potential electromagnetic and gravitational fields are given as follows. In that sense parameters α_1 and α'_1 are given by the relations:

$$\alpha_1 = 1 + i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}\,,\quad \alpha'_1 = 1 - i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}\,. \tag{30}$$

For parameters α_2 and α'_2:

$$\alpha_2 = 1 - i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}\,,\quad \alpha'_2 = 1 + i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}\,. \tag{31}$$

For parameters α_3 and α'_3:

$$\alpha_3 = -1 + i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}\,,\quad \alpha'_3 = -1 - i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}\,, \tag{32}$$

and for parameters α_4 and α'_4:

$$\alpha_4 = -1 - i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}\,,\quad \alpha'_4 = -1 + i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}\,. \tag{33}$$

Now one can introduce the generalized Lorentz-Einstein parameters, for an electron in a two-potential electromagnetic and gravitational field. These parameters are described by the following equations:

$$H_{1,2} = \left[1 - \frac{v^2}{c^2 + \frac{nqV_e}{\rho_{r\,max} r_{min}} - \frac{GM}{r}} \pm \frac{2i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}\; c\cdot v}{c^2 + \frac{nqV_e}{\rho_{r\,max} r_{min}} - \frac{GM}{r}} \right]^{-1/2}. \tag{34}$$

The solutions of the $H_{3,4}$ are symmetric to the solutions of the parameters H_{12}.

The previously presented two potential fields can be generalized by the application of the multi-potential field as the function of the field parameters α and α'. Now, for derivation of a four-potential vector A of the related potential field, one can recall the general Hamilton function, H, for the weak potential fields:

$$H = -c\zeta_1\left(p_x - \frac{U_p v_x}{c^2}\right) - c\zeta_2\left(p_y - \frac{U_p v_y}{c^2}\right) - c\zeta_3\left(p_z - \frac{U_p v_z}{c^2}\right) - \frac{\beta\,\rho_{r\,max} r_{min}}{n} c^2 + U_p\,, \tag{35}$$
$$n = 1...,n_{max},\quad n_{step} = n_{max}/n_{var}.$$

Here U_p is a potential energy, (p_x, p_y, p_z) is a three-momentum vector, (v_x, v_y, v_z) is a three-velocity vector and $\zeta_1, \zeta_2, \zeta_3$ and β are the well-known Dirac's matrices. If an electron is moving with a constant velocity v << c in an electromagnetic field with a scalar potential, V, then one should use the following relations:

$$\frac{U_p v_x}{c^2} = \frac{q}{c}\frac{V v_x}{c} = \frac{q}{c}A_x\,,\quad \frac{U_p v_y}{c^2} = \frac{q}{c}\frac{V v_y}{c} = \frac{q}{c}A_y\,,\quad \frac{U_p v_z}{c^2} = \frac{q}{c}\frac{V v_z}{c} = \frac{q}{c}A_z\,. \tag{36}$$

Here q is an electric charge of an electron and (A_x, A_y, A_z) is a three-potential vector of the electromagnetic field. Including (36) into (35), one obtains the well-known Hamilton function for Dirac's electron in an electromagnetic field:

$$H = -c\zeta_1\left(P_x - \frac{q}{c}A_x\right) - c\zeta_2\left(P_y - \frac{q}{c}A_y\right) - c\zeta_3\left(P_z - \frac{q}{c}A_z\right) - \frac{\beta\,\rho_{r\,max} r_{min}}{n}c^2 + qV\,. \tag{37}$$

On the other hand, if a robot (particle) is moving with constant velocity v << c in a gravitational field, then, according to the previous procedure, one should use the following relations:

$$U_p = -\frac{\rho_{r\,max} r_{min} GM}{nr} = \frac{\rho_{r\,max} r_{min}}{n}V_g\,,\quad \frac{U_p v_x}{c^2} = \frac{\rho_{r\,max} r_{min}}{nc}\frac{V_g v_x}{c} = \frac{\rho_{r\,max} r_{min}}{nc}A_{g_x}\,. \tag{38}$$

and

$$\frac{U_p v_y}{c^2} = \frac{\rho_{r\,max} r_{min}}{nc}\frac{V_g v_y}{c} = \frac{\rho_{r\,max} r_{min}}{nc} A_{g_y},$$
$$\frac{U_p v_z}{c^2} = \frac{\rho_{r\,max} r_{min}}{nc}\frac{V_g v_z}{c} = \frac{\rho_{r\,max} r_{min}}{nc} A_{g_z}. \quad (39)$$

In the relations (38) and (39) G is a gravitational constant, M is a gravitational mass, V_g is a gravitational scalar potential and (A_{gx} , A_{gy} , A_{gz}) is a three-potential vector of the gravitational field. Including (39) into the equation (37), one obtains the Hamilton function H_g for the particle in a gravitational field:

$$H_g = -c\zeta_1\left(P_x - \frac{\rho_{r\,max} r_{min}}{nc} A_{g_x}\right) - c\zeta_2\left(P_y - \frac{\rho_{r\,max} r_{min}}{nc} A_{g_y}\right) - c\zeta_3\left(P_z - \frac{\rho_{r\,max} r_{min}}{nc} A_{g_z}\right) - \beta\frac{\rho_{r\,max} r_{min}}{n}c^2 + \frac{\rho_{r\,max} r_{min}}{n}V_g. \quad (40)$$

Generally, if the robot velocity v in a potential field is constant, then the four-potential vector A can be derived as a function of the field parameters α and α′:

$$A = \left[A^0, A^1, A^2, A^3\right], A^0 = \frac{c^2}{\eta}\left(\alpha\alpha' - 1\right), \quad A^1 = A_x = A^0\frac{v_x}{c}, \quad (41)$$

and

$$A^1 = A_x = A^0\frac{v_x}{c}, \quad A^2 = A_y = A^0\frac{v_y}{c}, \quad A^3 = A_z = A^0\frac{v_z}{c}. \quad (42)$$

Now, the components of the field tensor F_{ij} of the potential field can be calculated by using relations (41) and (42) and the well-known procedure:

$$F_{ij} = \frac{\partial A^j}{\partial x^i} - \frac{\partial A^i}{\partial x^j}, \quad i,j = 0,1,2,3, \quad X = \left[x^0, x^1, x^2, x^3\right] = \left[ct, x, y, z\right]. \quad (43)$$

As the result of this calculation, one obtains the well-known anti-symmetric tensor F_{ij} of the potential field in the following form:

$$F_{ij} = \begin{bmatrix} 0 & F_{01} & F_{02} & F_{03} \\ F_{10} & 0 & F_{12} & F_{13} \\ F_{20} & F_{21} & 0 & F_{23} \\ F_{30} & F_{31} & F_{32} & 0 \end{bmatrix} = \begin{bmatrix} 0 & F_{01} & F_{02} & F_{03} \\ -F_{01} & 0 & F_{12} & -F_{13} \\ -F_{02} & -F_{12} & 0 & F_{23} \\ -F_{03} & F_{13} & -F_{23} & 0 \end{bmatrix}. \quad (44)$$

This tensor can be employed for the derivation of the related Maxwell's like equations in a vacuum.

Following the previous consideration, one can introduce the normalized scalar potential A_m^0 of a multi-potential field in the dimension of specific potential energy:

$$A_m^0 = \Sigma\left(\eta_j A_j^0\right) = \left(\alpha\alpha'_j - 1\right)c^2, \; j = 1,2,..,n. \quad (45)$$

Here term αα′ has to be calculated by employing the relations (30) to (33):

$$\left(\alpha\alpha'\right) = \left(1 + \frac{n\Sigma U_{p_j}}{\rho_{r\,max} r_{min} c^2}\right), \quad j = 1,2,..,n. \quad (46)$$

The relations (45) and (46) tell us what the normalized scalar potential A_m^0 really is:

$$A_m^0 = \frac{n\Sigma U_{p_j}}{\rho_{r\,max} r_{min}}, \; j = 1,2,..,n, \; n = 1...,n_{max}, \quad n_{step} = n_{max} / n_{var}. \quad (47)$$

In recent decades, it has been created a wide range of robotic systems mostly inspired by animals. In that sense, engineers have created a wide range of robotic systems like four - legged robots, snake robots, insect robots, and fish robots, [37]. Following the previous consideration, it is possible to control that class of robots by using the radial mass density field theory.

4 Calculation of the Robot Motion in Radial Mass Density Field

Gravitational field with the mass M_g has the maximal and minimal gravitational radial mass densities given in [12]:

$$\rho_{rm\,\max} = \frac{M_g}{r_{\min}} = \frac{(1+\kappa)c^2}{G} = 2.693182\cdot 10^{27}\, kg/m, \quad (48)$$

and

$$\rho_{rm\,\min} = \frac{M_g}{r_{\max}} = \frac{(1-\kappa)c^2}{G} = 0.888779\cdot 10^{23}\, kg/m. \quad (49)$$

The numerical values in (48) and (49) are constant and are valued for all amounts of the gravitational mas M_g. In relations (48) and (49) the parameter κ is the energy conservation constant that has been calculated in the [12], by using Planck's mass and Planck's length:

$$L_p = \frac{2GM_p}{(1+\kappa)c^2}, \ \kappa = \frac{2GM_p}{L_p c^2} - 1 = 0.99993392118. \quad (50)$$

Thus, the value of κ is close to one but less than it. Using the combination of equations (21) and (48) one obtains the canonical differential equations of the robot motion at the minimal gravitational radius with the maximal radial mass density:

$$\frac{d^2X}{dt^2} = \frac{nq}{2.693182\cdot 10^{27}}\left(E_{de} + E_{ce} + \frac{1}{c}\mathrm{v}\times H_e\right) + \left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right) m/kg, \quad n_{step} = n_{max}/n_{var}. \quad (51)$$

By changing parameter $n = 1, 2, .., n_{max}$ and using the parameter n_{var}, one can obtain the desired acceleration and precise control of the robot motion in the related control region. Further, using the combination of (21) and (48) one obtains the canonical differential equations of the robot motion at the maximal gravitational radius with the minimal radial mass density:

$$\frac{d^2X}{dt^2} = \frac{nq}{0.888779\cdot 10^{23}}\left(E_{de} + E_{ce} + \frac{1}{c}\mathrm{v}\times H_e\right) + \left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right) m/kg, \quad n_{step} = n_{max}/n_{var}. \quad (52)$$

Now, by changing parameter $n = 1, 2, .., n_{max}$ and variable parameter n_{var}, one can obtain the desired acceleration and precise motion of the robot control in the related region. The ratio between the maximal and minimal radial mass densities can be calculated by using the relation:

$$n_{\max} = \frac{\rho_{rm\max}}{\rho_{rm\min}} = \frac{2.693182\cdot 10^{27}}{0.888779\cdot 10^{23}} = 3.030204\cdot 10^4. \quad (53)$$

This ratio is the constant and is valued for the all amounts of the gravitational masses.

Following the previous equations, one can calculate of the maximal steps, n_{step}, between maximal and minimal radiuses in a gravitational field. For the calculation of the precise motion of the robots in the gravitational radial direction, one can introduce the variable step of the robot motion, n_{var}. In that case, it is possible to select (change) the scale of the desirable step of the robot motion in the radial mass density field.

For an example, let the variable step of the robot motion in the radial direction be given by the amount n_{var} =100. In that case the number of the robot steps n_{step} from the minimal to the maximal radiuses has the value:

$$n_{step} = \frac{n_{\max}}{n_{\mathrm{var}}} = \frac{303.0204\cdot 10^2}{100} = 303.0204 steps. \quad (54)$$

In this calculation a robot needs 303 steps of the motion from the minimal to the maximal radiuses in the radial direction. In the case where the robot motion is not in the radial direction one should use the related projection of the radial trajectory to the desired robot trajectory.

The next example is related to the possibility that one wants to introduce the potential energies at the minimal and maximal gravitational radiuses, $U_{g\,max}$ and $U_{g\,min}$, respectively. In that case it is possible to calculate the minimal and the maximal radial lengths, L_{gmin} and L_{gmax}, respectively, by using the relations:

$$M_g = \rho_{rm\max}\, r_{\min},$$

$$L_{g\min} = \frac{2m_0 G\,\rho_{rm\max}\, r_{\min}}{U_{g\min}} = \frac{2m_0 G\,\rho_{rm\max}\, r_{p\min}}{(1+\kappa)c^2}, \quad (55)$$

and

$$L_{g\max} = \frac{2m_0 G\,\rho_{rm\min}\, r_{\max}}{U_{g\max}} = \frac{2m_0 G\,\rho_{rm\min}\, r_{p\min}}{(1-\kappa)c^2}. \quad (56)$$

From relations (55) and (56) one can see how the potential energies in the gravitational field can influence the robot's motion in that field.

5 Conclusion

This article is based on the new Relativistic Radial Density Theory (*RRDT*) that has been applied to the control of the robot motion in potential fields. The robot motion is calculated from the minimal to the maximal gravitational radiuses and vice-verse. In the case where the robot motion is not in the radial direction, it is necessary to transform the radial coordinates into the rectangular ones by using related projection. It is shown that the maximal radial mass density occurs at the minimal gravitational radius. On the other hand, the minimal radial mass density happens at the maximal gravitational radius. Furthermore, the both maximal and minimal radial mass densities can also be described as the functions of the energy conservation constant κ. In that sense, the related gravitational length, time, energy, and temperature can be represented as functions of the Planck length, time energy, and temperature, respectively.

Finally, it is concluded that the precise control of the robot motion in combination with the electromagnetic and gravitation fields can be controlled by the introduction of the variable step of the robot motion. In that sense, one can introduce the steps number, n_{step}, that is function of the variable term, $n_{step} = n_{max} / n_{var}$, between maximal and minimal radiuses in gravitational field. On that way, the smaller value of the parameter n_{var} gives the bigger number of the steps, n_{step}, and vice versa. Thus, the bigger n_{step} gives more precise control of the robot motion in the radial mass density field.

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Branko Novakovic was responsible for the concept of the research and for the organization of all contribution to this scientific article. Dubravko Majetic carried out for the computer simulation. Josip Kasac and Danko Brezak have implemented the computer algorithm for the simulation of the presented article.

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Conflict of Interest

The authors have no conflicts of interest to declare.

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