On Modelling of Autonomous Cooperative Robotics in Monoidal Category of Binary Relations

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Abstract: - The game theoretical approach is applied to cooperative robotics simulation. Robots' team is regarded as a multiagent system. Robots are universal intellectual players that might maximize their preferences and relations. Acting autonomously, they are to service the given set of requirements submitted to some precedence relation. To achieve this, robots use information exchange to build optimal communications networks on the base of natural coalitions. The latter is the result of their rational strategies application. The agents' activity leans on a distributed algorithm to find an optimal scheduling. Relational Bellman's method contributes to the search for an equilibrium of the game. The approach to cooperative robotics admits scalable realization due to the polynomial complexity of the algorithm.

Key-Words: - Monoidal category, preferences, precedence, dynamic, game results, and Bellman's relations, relational scheduling, natural ordering, coalition, communications structure, equilibrium.

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1 Introduction

Robots' team activity is regarded as an optimized multiagent system functioning. A game statement of the problem is applied, [1], [2], [3]. Robots are to distribute given tasks among themselves and solve some scheduling optimization problems, [4], [5]. Different schedules are situations in the dynamic game. For coordinated concurrent assignment and execution of the given requirements, robots lean on information exchange. Their aim is to find an equilibrium solution. Instead of cost functions usage [3], the simulation is made in the category of binary relations *REL*, [6], [7]. Players' interests are presented in the form of preference relations on the set of all tasks. The relations can be introduced according to the tasks' processing times by robots. Incoming requirements are also ordered with the help of a precedence relation. The generalized makespan problem with decentralized control can be stated in this way. Similar multi-objective problem formulations with unit cost functions are given in [8], [9].

In the traditional scheduling problem, there are many parameters such as release and start times, due dates, and so on. It makes it NP-hard forcing to apply heuristic algorithms and stochastic approaches, [5]. As for robots, they are intelligent agents making decisions themselves. They issue from their own preferences relations to make rational decisions leaning on messaging. They are able to coordinate their actions under cooperative behavior. They can form effective coalitions based on the natural orderings of their partners. It allows them to pick out optimal network structures and assign tasks autonomously under decentralized control. Robots' team functioning is similar to the trading and load balancing control method applied to distributed systems, [8], [10]. However, it seems to be better than having a leader in the multiagent system, [11]. To assess the efficiency of the contributed decentralized relational approach, agents' preferences relations and tasks precedence relation must be additionally equipped with numerical parameters to express their features, [4], [5], [9]. Agents' preferences relations could be defined functionally in terms of task execution times. But they are not known precisely such as tasks arrival moments, delay times, and so on. It is impossible to determine task precedence relation in the exact numerical form due to the factor's stochastic nature. In the model under consideration, special studies are needed.

Binary relations are regarded as morphisms of the monoidal category $\rho_i \in REL(J, J)$, [6], [7]. This reveals the compositionality of the approach, [11]. It allows equivalent transformations of the game resulting and of coalitions characteristic relations thus facilitating the game study. As for robotics, the formulation of the problem significantly differs from the general case of relational games, [12]. Firstly, instead of tuples of players' strategies [3], [12], situations are given tasks. Secondly, players themselves build natural communication structures as the result of their strategy's application. The structure is used in the game reduction process. Thirdly, further generalization of Bellman's method is contributed to finding a balanced solution, [4], [12].

2 Problem Formulation

Let set of the requirements $j \in J = \{A_1, ..., A_K\}$ be ordered by means of the given precedence relation:

$$\tau : J \to J. \tag{1}$$

All robots $i \in I = \{1, 2, ..., N\}$ might maximize their preferences relations $\rho_i : J \to J$ picking out tasks $A_i \in J$ following the order (1). Each requirement $A_i \in J$ is to be done by only one of them. To coordinate the activity, robots' team is able to exchange data. They can be assigned tasks, and created agents' coalitions C along with their interests ρ^C expressed by means of characteristic relations.

Agents' preference relations $\rho_i, i \in I,$ are supposed to be some orders specified on the set of game situations [3]. Therefore, if a relation ρ^C is to be built for a coalition C, transitive and reflexive closure operations may be used to achieve it.

The rational behaviour of the robots' team consists of an equilibrium search in some dynamic game in which players exchange data to bring their awareness. The balanced solution of the game is to be found as the result of an optimization problem solution:

$$\tilde{A}_i^* \subset MAX\, \tilde{\rho}_i, i \in I. \tag{2}$$

In the formula (2), $\tilde{\rho}_i, i \in I = \{1, ..., N\},$ are dynamic relations to be defined afterward. They are inherited from robots' preferences relations and task ordering (1). Situations of the dynamic game are different schedules

$$(\tilde{A}_1, ..., \tilde{A}_N), \tilde{A}_i = \{A_{j_1(i)}, ..., A_{j_s(i)}\}, i \in I. \tag{3}$$

So, the next partitioning property has to take place:

$$J = \cup_{i \in I} \tilde{A}_i, \forall_{i \neq j} \tilde{A}_i \cap \tilde{A}_j = \varnothing. \tag{4}$$

Data exchange engenders communications network which defines classes of admissible strategies, [3], [12].

Requirements processing in the schedule (3), (4) occurs in the linear order $A_{j_1(i)} \to ... \to A_{j_s(i)}$ keeping the precedence relation (1). Depicted in the scale of time $t = t_1, t_2, ...,$ the desired schedule is:

$$A_{j_k(i)} = A_{j(i, t_k)}, t_k \leq t_{k+1}, k = 1, 2, ..., s.$$

"Release" dates partition the tasks set J on the segments $J_t, t = t_k, k = 1, 2, ..., T,$ as follows:

$$\begin{aligned} &J = \cup_t J_t, \\ &J_1 = MIN\tau, J_2 = MIN\tau\,|_{J \setminus J_1}, ..., \\ &J_T = MIN\tau\,|_{J \setminus (J_1 \cup J_2 \cup ... \cup J_{T-1})}, J_{T+1} = \varnothing. \end{aligned} \tag{5}$$

In the formula (5), relation τ is restricted $\tau|_{J'} \equiv \tau \cap J' \times J'$ on the subsets $J', J' \subset J.$ Starting in the time instance t all requirements $A_i \in J_t$ can be serviced independently one from another. Thus, the next linear order on the set $\{J_t\}$ is generated:

$$J_1 \to J_2 \to ... \to J_T. \tag{6}$$

Finite number of robots cannot always do its work segment wise according to the sequence (6). In reality, another partitioning $J = \cup_t \tilde{J}_t$ will occur:

$$\tilde{J}_1 \to \tilde{J}_2 \to ... \to \tilde{J}_{\tilde{T}}, \tilde{J}_1 \subset J_1, \tilde{T} \geq T. \tag{7}$$

All tasks $j \in \tilde{J}_t$ can be considered to be serviced simultaneously having release dates $t = t_k, k = 1, 2, ..., \tilde{T}$. It depends on number N_t of available robots, amount K_t of tasks in the set $\tilde{J}_t$, of requirements service times $\Delta_j, j \in J,$ and other factors. On the sets $\tilde{J}_t$, static games G_t are played supported by auxiliary messaging including information about already distributed tasks and organized coalitions, [12].

2.1 Dynamic Relation

Let's consider relations $\rho_i(t) = \rho_i\,|_{\tilde{J}_t}, t = 1, ..., T,$ and the next functors, [6], [7]:

$$\tau(t) : REL(J_t, J_t) \to REL(J_{t+1}, J_{t+1}), t = 1, ..., T-1,$$

engendered by the precedence relation (1).

Definition 1. *Relation* $\tau^{-1} \circ \rho_i(t+1)$ *is co-image of the morphism* $\rho_i(t+1)$ *over the functor* $\tau = \tau(t)$ *if the diagram from the Figure 1 is commutative.*

So as $\rho_i(t+1)$ is an order then every co-image $\tau^{-1} \circ \rho_i(t+1)$ is a preorder [6], (Figure 1). Transform it into an order taking reflexive and transitive closure.

$$\begin{array}{ccc} J_t & \xrightarrow{\tau^{-1}\circ\rho_i(t+1)} & J_t \\ \tau\downarrow & & \downarrow\tau \\ J_{t+1} & \xrightarrow{\rho_i(t+1)} & J_{t+1} \end{array}$$

Fig. 1: Co-image of relation $\rho_i(t+1)$ over τ

In the problem (2), dynamic relations are tuples

$$\tilde{\rho}_i = (\tilde{\rho}_i(1), ..., \tilde{\rho}_i(T)), i \in I,$$

defined inductively in the Def. 2.

Definition 2. *Let it be* $\tilde{\rho}_i(T) = \rho_i(T).$ *For all* $t = T-1, ..., 1$ *relations* $\tilde{\rho}_i(t)$ *are built according to the next recursive formulae:*

$$\tilde{\rho}_i(t) = \rho_i(t) \circ \tau^{-1} \circ \tilde{\rho}_i(t) \coprod \coprod_{r=1,...,\tilde{T}-t} \tilde{\rho}_i(t+r)|_{J_{t+r} \cap \tilde{J}_t}, i \in I. \quad (8)$$

In (8), monoidal operations of product and conjunctive sum are used [6] presented in the Figure 2:

$$\left[\begin{array}{l} \tilde{J}_t \setminus J_t^0 \xrightarrow{\tau^{-1}\circ\tilde{\rho}_i(t+1)} \tilde{J}_t \setminus J_t^0 \xrightarrow{\rho_i(t)} \tilde{J}_t \setminus J_t^0, J_t^0 = \cup_{r=1}^{T-1} J_{t+r}, \\ \qquad \tilde{J}_t \cap J_{t+1} \xrightarrow{\tilde{\rho}_i(t+1)|_{J_{t+1}\cap\tilde{J}_t}} \tilde{J}_t \cap J_{t+1}, \\ \qquad \qquad ... \\ \qquad \tilde{J}_t \cap J_T \xrightarrow{\tilde{\rho}_i(\tilde{T})|_{J_T\cap\tilde{J}_t}} \tilde{J}_t \cap J_T. \end{array} \right.$$

Fig. 2: Dynamic relation $\tilde{\rho}_i(t) : \tilde{J}_t \to \tilde{J}_t$

They are transmitted as $\tilde{\rho}_i \cup \rho_i|_{\tilde{J}_t}$ on the sets $\tilde{J}_t, 1, ..., \tilde{T},$ to search an equilibrium of the game

$$\tilde{A}_i^*(t) = \{A_{j(i,t)}\}, A_{j(i,t)} \in \tilde{J}_t, i \in I, t = 1, 2, ..., \tilde{T}, \quad (9)$$

with the help of sequential maximization (2). It gives a solution of the relational scheduling problem.

2.2 Robots' Communications Network

Optimal task distribution among robots can be performed using communications structures $S_t, t = 1, ..., \tilde{T},$ defining agents' awareness and admissible classes of strategies, [3], [12].

It is a family of oriented graphs in which vertices are robots $i \in I$ and edges $(i,k) \in E$ are agents' communicating pairs. All edges are marked by transmitted data. The whole network structure $S = \cup_t S_t$ contains all players' moves, [3], [12]. Due to the structure S, robots' team can act coordinately lessening decision-making uncertainty.

In accordance with ordering S, each player picks out one of the unserved requirements $A_{j(i,t)} \in \tilde{J}_t$ ready to be processed in a moment $t = t_k, k = 1, 2, ..., \tilde{T}.$ The network structure can be found by agents themselves with the help of distributive algorithm which is based on coalition creation. Each player $i \in I$ can select a partner $k \in I$ to inform him about his strategy choice, [3]. In this way, edges $C : i \xrightarrow{A_{j(i,t)}} k$ of the structure $S(t)$ emerge. In other words, a sequential hierarchical 2-persons' coalition $C = \{i, k\}$ is created. From now on coalition C acts as one player which interests are expressed by means of *characteristic* relation $\rho^C : [J] \to [J]$. The latter is factorization of the transitive closure $\rho^C = \overline{\rho_i \circ \rho_k}$ [6], [7]. The set $[J]$ consists of equivalence classes $[r], r \in J,$

$$r, s \in [r] \Leftrightarrow (r\rho s) \wedge s\rho r, \rho = \overline{\rho_k \circ \rho_i}.$$

For easier recording, ρ^C is denoted as $\rho_i \circ \rho_k$.

So as task choice $A_{j(i)} = A_{j(i,t)}$ of the player $i \in C$ becomes known to the partner $k \in C$, the class of admissible strategies of the latter consists of functions $A_{j(k)} = A_{j(k)}(A_{j(i,t)}).$ In the case of $\rho^C = \rho_i \circ \rho_k,$ rational decision-making dynamics look like this [3]:

$$\rho_i|_{J^C} \to MAX \Rightarrow A_{j(i)}; J^C = \uparrow^{\rho_i} MAX_{\rho_k} \Rightarrow A_{j(k)}. \quad (10)$$

In (10), the upper cone $\uparrow^{\rho} J'$ over a subset $J' \subset J$ of the ordered set (J, ρ) is considered [7].

Besides, players can form parallel coalitions D having the following characteristic relation ρ^D :

$$D : \frac{\boxed{i}}{\boxed{k}} \square \rho^D = \rho_i \coprod \rho_k. \quad (11)$$

The conjunctive sum in (11) does not diminish uncertainty in decision-making process. Though coalition participants $i, k \in D$ do not exchange any data, they ensure noncontradictory choice of executed requirements, e.g. by means of ancillary messaging or technical vision use. Distribution of tasks as the result of optimization $MAX\rho^D$ means a choice

$$A_{j(i)}, A_{j(k)} \in MAX\rho_i \coprod MAX\rho_k. \qquad (11')$$

Rules (10), (11') allow assigning tasks to all partners in any coalition. Communications network contains different combinations of 2-persons' coalitions of both types with characteristic relations expressed in the category *REL*. If all players are united in one coalition, the characteristic relation of the latter is called *game resulting* relation.

Lemma 1. *Characteristic relations are orders.*

Proof. Firstly, conjunctive sum, see (11), transfers orders ρ_i, ρ_k on the set $J \coprod J$. Secondly, characteristic relation of the sequential coalition ρ^C is an order according to its building.

Cooperative subgames G_t, see (2), (3), (7), are to be solved on each current segment of works $\tilde{J}_t, t = 1, ..., \tilde{T}$. Transition from initial game to the game of coalitions is called *reduction* process. Hereinafter, formulae $\rho^C = \rho_i \circ \rho_k$ and (11) are used in the game reduction process.

Formulae (10), (11') reflect game theory presentations about agents' rational behavior, [1], [2], [3]. Data exchange restricts corresponding relations diminishing decision-making uncertainty, see (10).

Coalitions building must be grounded on the application of effectiveness and equilibrium principles, [1], [2], [3], [12]. They are incarnated in the notion of natural coalition.

3 Optimal Relational Scheduling

Problem (1) – (8) solution supposes the choice and use of *optimal* communications structures $S_t^*, t = 1, ..., \tilde{T}$, to govern requirements distribution. It can be done by robots themselves leaning on *natural* orderings $\Theta_{\rho_i} = \Theta_{\rho_i}(t)$ on the preferences relations sets $R = R_{\rho_i} = \{\rho_j : \rho_j \neq \rho_i\}$. They are inherited from the Cartesian product $\rho_1 \times \rho_2$ [6], [7].

3.1 Natural Ordering of the Set *R*

A morphism ρ_1 is *pushed* through a morphism ρ_2 with the help of ρ if the next diagram is commutative, [6], [7]:

$$\begin{tikzcd} R \arrow[dr, "\rho"] & \\ & R. \\ R \arrow[uu, "\rho_1"] \arrow[ur, "\rho_2"] & \end{tikzcd}$$

Fig. 3: Relation $\rho_1\Theta_\rho\rho_2$ on the set $R = \{\rho_j : \rho_j \neq \rho\}$.

Notation $\rho_1 \prec_\rho \rho_2$ will be used instead of $\rho_1\Theta_\rho\rho_2$.

Definition 3. *Relation $\rho_1 \prec_\rho \rho_2$ on the set R takes place if the diagram from Figure* 3 *is commutative.*

Lemma 2. *Every relation $\rho_1 \prec_\rho \rho_2$ is a preorder on the set R.*

Proof. Due to the reflexivity of the order ρ, $\rho_1 \subset \rho \circ \rho_1$. It proves that Θ_ρ is reflexive relation. Let now $\rho_1 \prec_\rho \rho_2$ and $\rho_2 \prec_\rho \rho_3$ be true. Compile two diagrams with ρ_1, ρ_2 and ρ_2, ρ_3, (Figure 3), to obtain the next one:

$$\begin{tikzcd} R \arrow[dr, "\rho"] & & & & R \arrow[dr, "\rho^2"] & \\ & R \arrow[dr, "\rho"] & & \Rightarrow & & R. \\ R \arrow[uu, "\rho_1"] \arrow[ur, "\rho_2"] \arrow[rr, "\rho_3"] & & R & & R \arrow[uu, "\rho_1"] \arrow[ur, "\rho_3"] & \end{tikzcd}$$

Fig. 4: Transitivity property proof

As transitive relation, every order satisfies to the property $\rho^2 = \rho$. Hence, (Figure 4), $\rho_1 \prec_\rho \rho_3$.

If a player $i \neq 1, 2$ has preferences relation $\rho = \rho_i$ and $\rho_1 \prec_\rho \rho_2$ then coalitions $K_{i.1} = \{i \to 1\}$ and $K_{i.2} = \{i \to 2\}$ can be compared in the following sense. So as their characteristic relations are connected by the next inclusion $\rho \circ \rho_1 \subset \rho \circ \rho_2$, coming from the Figure 3, then coalition $K_{i.1}$ is *less effective* than $K_{i.2}$. Agent i is more interested to communicate with player 2 rather than with player 1. Being in the sequential coalition $K_{i.2}$, agent i makes his move earlier than 2. Picking out a requirement to execute agent i transmits the choice to his partner 2.

Due to lemmas 1, 2, the relation Θ_ρ is an order on the set R. Otherwise, it is sufficient to get to the factor relation $\Theta_\rho / \sim$ bearing in mind the next equivalence relation [7]:

$$\rho_1 \sim \rho_2 \Leftrightarrow \rho_1 \prec_\rho \rho_2 \wedge \rho_2 \prec_\rho \rho_1$$

The most effective or *natural* hierarchical coalition $K = \{k \to j^*\}$ for an agent k can be found as the result of relation Θ_ρ maximization:

$$\rho_{j^*} \in \underset{\rho_j}{MAX}\, \Theta_{\rho_k}, \rho_j \in REL(R_{\rho_k}, R_{\rho_k}). \qquad (12)$$

It may happen that all problems (12) have no solutions if the orders Θ_ρ are trivial. In this case agents can form *parallel natural* coalitions $i \to i_1 \coprod i_2$ issuing from the obvious property:

$$\forall_{\rho_i, \rho_{i_1}, \rho_{i_2}} \rho_{i_1} \Theta_{\rho_i} (\rho_{i_1} \coprod \rho_{i_2}). \qquad (13)$$

Robots can search his effective coalitions autonomously. Some natural coalitions may have common players. It means that, in reality, there are natural coalitions with many participants. On the basis of natural coalitions search, robots' optimal communications network $S^* = (S_t^*, t = 1, ..., \tilde{T})$, emerges, see Def. 4, in the next paragraph. Game-resulting relation will be also defined.

3.2 Optimal Communications Structure

Natural coalitions engender optimal structure. S^*.

Definition 4. *A communications network S^* is called optimal if all data are transmitted within natural coalitions.*

In any current static game, optimal structure S_t^* is not the only one. In order to build it, the next game *reduction* process is used. All partners from every emerging natural coalition C are to be replaced by only one new player. The latter has preferences relation ρ^C. Its strategy is the tuple of strategies of all C participants. In the reduced game, the search of natural coalitions can be continued by applying the rules (12), (13). The process is finite. It is terminated when the general coalition is built having so called game *resulting* relation $\tilde{\rho}^g(t)$.

3.3 Complexity of the Network Structure Building and Scheduling

Let $N_t, N_t \le N,$ and K_t be the numbers of available agents $i \in \tilde{I}_t$ and tasks $A_k \in \tilde{J}_t$ in a static game G_t. A distributed algorithm is described below. On its $O(N_t)$ iterations, every player generates his m persons' coalition. Let n be the average power of the situations sets $\tilde{J}_t, t = 1, ..., \tilde{T}$.

Lemma 3. *An optimal communications network S_t^* building requires $O(K_t^6)$ binary operations and $O(N_t^3)$ messages transmissions. The complexity of the scheduling problem* (1) – (8) *solution has the order*

$$O\left(\frac{(K^6 + N^3)K}{\min(n, N)} + n^2 m\right) \log_m N. \qquad (14)$$

Proof. Each preferences relation ρ is presented by means of $K_t \times K_t$ binary incidences matrix. Binary multiplication $B_\rho = B_{\rho_1} \cdot B_{\rho_2}$ answers to the product $\rho = \rho_1 \circ \rho_2$ of relations. Matrix

$$B_\rho = \begin{pmatrix} B_{\rho_1} & 0 \\ 0 & B_{\rho_2} \end{pmatrix}$$

corresponds to subjunctive sum $\rho = \rho_1 \coprod \rho_2$. Fulfilling the relationship $\rho_1 \prec_\rho \rho_2$ can be established by $O(K), K = K_t^3,$ binary operations. So as every agent i has to compare $O(N^2), N = N_t,$ pairs of preferences relations, incidences matrix $\tilde{B}_{\prec_\rho}$ building will require $O(K^5)$ operations. Every agent $i \in \tilde{I}_t$ must construct the set of all sequential natural coalitions $\{i \to i_r \to ... \to i_2, i \in I\}$. He picks out the largest chains $\rho_{i_1} \prec \rho_{i_2} \prec ... \prec \rho_{i_r}$. In total, building of interconnected bundles $C = C^1, ..., C^{N_t'}, N_t' < N_t,$ of the coalitions takes $O(K^6)$ of binary operations. In the reduced game, players C have preferences in relations

$$\rho^C = \coprod_i \mu_i, \mu_i = \rho_i \circ \rho_{i_r} \circ ... \circ \rho_{i_2},$$

called coalitions *characteristic* relations.

All agents must have awareness of emerging coalitions and their interests ρ^C. On the whole, distributed method must be supported by messaging with the time consumption of the order $O(N_t^3)$. The needed structure S_t^* will be obtained when $N_t' = 1$.

In the reduced game all preferences relations ρ^C might be trivial. Then natural parallel coalition $\tilde{D}^i = (C^i \to C^{i_1} \coprod C^{i_2})$ can be formed by players, see (12), (13). The fact is that elements of the sets

$$\{\rho_{C^{i_1}} \coprod \rho_{C^{i_2}}, C^{i_1}, C^{i_2} \ne C\}$$

are not pairwise comparable whatever you take a natural ordering. New coalitions $\tilde{D}^i, \rho_{\tilde{D}^i}$ may be also interconnected and, in reality, are greater ones:

$$\forall_k D^k = \tilde{D}^{i_1} \cup ... \cup \tilde{D}^{i_s} : (D^1, \rho^{D^1}), ..., (D^{N''}, \rho^{D^{N''}}).$$

For $m \ge 3$ there are not more than $N'' \le [N'/3]$. unlinked coalitions of the kind. After that, the game can be reduced and needed network S_t^* building continued. Under final reduction, the general coalition C produces the resulting relation $\tilde{\rho}^g(t) = \rho^C$.

On average, to find optimal scheduling it is necessary to solve additionally $K/\min(n,N)$ static games. Relation $\tilde{\rho}^g(t)$ optimization, see (10), (11'), requires $O(n^2 m)$ operations in case of m persons' coalition. There are no more than $O(\log_m N)$ static games G_t. To summarize, the scheduling algorithm has computational complexity (14).

Besides, note the following corollary is proved:

Lemma 4. *Game-resulting relation exists.*

3.4 Relational Dynamic Programming

Using natural orders $\Theta_{\rho_i}(t)$ on the sets of dynamic relations (8) $\tilde{R}|_{\tilde{J}_t} = \{\tilde{\rho}_k|_{\tilde{J}_t}, k \neq i, k \in \tilde{I}\}$ agents $i \in \tilde{I}_t$ build communications structures $S_t^*, t = 1,...,\tilde{T}$, to solve static games $G_1,...,G_{\tilde{T}}$, see (2). *Bellman's* resulting relations $\tilde{\rho}^{G_1},...,\tilde{\rho}^{G_{\tilde{T}}}$ are sought with the help of the distributive algorithm, see lemma 4.

Sequential maximization (2) corresponds to the following one:

$$\tilde{\rho}^{G_1} \to MAX_{\tilde{J}_1},...,\tilde{\rho}^{G_{\tilde{T}}} \to MAX_{\tilde{J}_{\tilde{T}}}. \qquad (15)$$

Each static game problem in (15) is solved by applying rules (10), and (11') to distribute tasks among players $i \in \tilde{I}_t$ and find optimal scheduling (2), (9).

The scheme (15) answers dynamic programming approach to the relational scheduling problem (1) – (8) solution.

Example 1. Let in the next game there are two agents with preference relations

$\rho_1 = \{(1,2),(3,2),(3,4),(4,5),(3,5)\}, \rho_2 = \{(1,4),(2,3),(2,5)\}$,

and tasks are ordered by relation (1) $\tau = \{(1,4),(2,4),(3,4),(3,5)\}$. Entry $(s,r) \in \rho$ means $A_s \rho A_r$. It gives partitioning (5) $J_1 = \{1,2,3\}, J_2 = \{4,5\}$. Then Bellman's dynamic relations (8) are reflexive closures of the next ones:

$$\tilde{\rho}_1(1) = \{(1,2),(3,2)\}, \tilde{\rho}_2(1) = \{(2,3)\},$$
$$\tilde{\rho}_1(2) = \tilde{\rho}_2(2) = \{(1,1),(5,5)\}.$$

Hence (7), $\tilde{J}_1 = \{2,3\}, \tilde{J}_2 = \{1,5\}, \tilde{J}_3 = \{4\}$. Robots apply here only parallel natural coalitions. Relational dynamic programming method constructs the optimal scheduling $\tilde{A}_1^* = (A_2, A_1, A_4), \tilde{A}_2^* = (A_3, A_5)$.

4 Conclusion

The optimal behavior of the robot team is modeled in the category *REL*. A game theoretic approach is used. Robots solve autonomously relational scheduling problems leaning on messaging. For this purpose, they form natural coalitions to build optimal communications networks. The relational distributed dynamic programming method contributes to find an equilibrium solution to the scheduling problem. The algorithm has polynomial complexity.

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