Method of optimisation of parameters of control systems of driving control systems of a semi-trailer truck in reverse

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Abstract: - The questions of the possibility to improve the operational properties of a semi-trailer truck in combination with non-rotating wheels in the infrastructure of a Smart City system, which is an unstable system when reversing, by introducing feedback of motion control with the help of an automated data processing system are considered. As the results of the research have shown, the use of such systems can significantly improve their manoeuvrability and directional stability. The results of using the previously developed method of determining the conditions for ensuring the stable movement of the semi-trailer truck in reverse with non-rotating wheels of the semi-trailer allowed solving the problems of manoeuvring control algorithmisation described by the mathematical model of differential nonlinear equations. The substantiated additive criterion and algorithm for determining the optimal parameters of the motion control system of a tractor-trailer combination with non-turning wheels of a semi-trailer are presented, which is used for the development of the improved optimisation method of its parameters. It is shown that the application of the obtained results will reduce the area and time of vehicle manoeuvring when performing transport tasks in the system Smart City.

Key-Words: - Smart City, Semi-trailer truck, Maneuverability, Method of Optimisation, Manoeuvring, Non-rotating wheels

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1 Introduction

1.1 Motivation and goal of the paper

As humanity has evolved, the world has seen an increase in the population and the vehicles it uses for its various transport activities. The constant increase in the number of vehicles and transport volumes leads to more complex road conditions and reduced road capacity, requires more people to operate vehicles, and requires the development and automation of systems to manage traffic flows and individual vehicles [1], [2], [3], [4], [5]. In road transport vehicles as a part of the infrastructure of Smart City, non-automatic traffic control is still mostly used, but automatic devices and drives that facilitate the driver's labour are increasingly being used. Automated traffic control of motor vehicles can be carried out through the use of automatic or automated traffic control systems for various machines and units in the Smart city infrastructure.

A special place in traffic flows is occupied by large-sized vehicles, which have manoeuvring properties worse than those of single vehicles. Thus, one of the estimated manoeuvrability indicators is the difficulty of controlled reversing movement. The evaluation of this indicator for road trains can be carried out with the help of two experimental indicators of the manoeuvrability index in reverse. One of the indicators is the length of the distance travelled by the road train with the steering wheel fixed before the tractor and semi-trailer start folding. Another indicator is the number of turns of the steering wheel per unit of travel during the controlled movement of the semi-trailer truck relative to a straight reference line. The specified evaluation indicators of manoeuvrability are used for comparative evaluation of the studied road trains for this operational property [2], [3], [4], [5]. It should be noted that the limiting influence of manoeuvring properties on the technical speed is strongly manifested in urban conditions for the system Smart City", and on the operational speed - in unloading and loading places, which, as a rule, are characterised by inconvenient and small areas.

1.2 Work related analysis

There are scientific publications devoted to the study of semi-trailer truck stability. The authors of [1] analyse the complexity of the task for a driver who is not trained in reversing a trailer with a dolly and propose a planning and motion control framework for use in the automatic planning and execution of complex manoeuvres. Paper [2] also shows how important it is to study driver behaviour patterns for decision-making and planning using traffic data. The authors of [3] propose a comprehensive structure of autonomous traffic management systems in the IoV system supporting a multi-level architecture that is built autonomously without any centralised control and can be used to collect, exchange, and transmit information in a multi-link mode. Paper [4] describes a mathematical model of a two-link road train movement that allows, to a first approximation, to simulate the "shuffle" manoeuvre on rigid wheels and to give a preliminary assessment of the controllability and stability of movement at the design stage of a vehicle without taking into account the instability of the road train scheme when driving in reverse.

The mathematical model of the overtaking manoeuvre on two-lane rural roads proposed by the authors in [5] does not allow for assessing the manoeuvring properties when driving in reverse. In [6], a unified approach to local trajectory planning and control is proposed for autonomous driving from a guided trajectory. However, it does not take into account the peculiarities of the trajectories of road trains with fixed semi-trailer wheels. A review of existing approaches and methods for planning traffic in autonomous road traffic, their critical assessment in terms of their advantages/disadvantages, inherent limitations, feasibility, optimality, obstacle overcoming, and testing of the working environment is given in [7]. In [8], the author presents mathematical models of a road train developed by the author to study its forward and reverse movement and synthesises the laws of turning the wheels of a car in the desired direction when driving in reverse. In his dissertation [9], he developed optimisation schemes for nonlinear control for trajectory tracking and trajectory following in the presence of model prediction constraints (NMPC) for both classes of problems. The stability conditions of NMPC for trajectory tracking based on time-varying terminal regions and the introduction of positively invariant sets of time-varying Lyapunov function levels as terminal regions are derived, which allow for expanding the area of attraction of the proposed control schemes and thus improve the control efficiency.

Paper [10] solves the optimisation problem (OP) based on the optimal control method using the principle of maximum and linearisation with the help of Taylor expansion, proposes new algorithms, and proves convergence. Paper [11] presents an iterative method of steering with longitudinal, lateral, and yaw motion, including coupled nonlinear dynamics between them, providing robustness to uncertainty and external perturbations. The authors of the paper [12] have been investigating the issues of optimisation of motion control for a long time: optimisation of lateral and longitudinal force distributions in tyres as a function of tyre load, minimax optimisation for several configurations of independently controlled vehicles based on numerically stable combinations of the algebraic approach, the bisection method and the golden section method, minimax optimisation and consideration of the possibility of using the complementarity of lateral and longitudinal forces in the formation of yaw moment for long-distance vehicles. In [13], a control strategy for electric power steering based on gear angular velocity control is proposed.

However, the conducted studies and proposed laws do not take into account the conditions for ensuring the stability of such a road train scheme with a semi-trailer when driving in reverse and in a given direction. The manoeuvrability of a vehicle is one of its main operational properties, which is described in a number of works. Paper [14] developed mathematical models of a road train moving in a given direction in reverse. Paper [15] modelled decision-making on driver behaviour when receiving recommendations on vehicle speed using vehicle-infrastructure communication based on energy consumption and determining a safe speed. The authors of [16] analysed the results of modelling the movement of a trailer road train in reverse, a technical solution for automatic reversal prevention for automated parallel parking in conjunction with a trajectory planning algorithm. In [17], the authors analyse the reversing movement of an articulated vehicle, namely a tractor-trailer with one trailer on the axle, and develop a fully autonomous driving system that allows reversing in the presence of static obstacles. In [18], it is shown how a vehicle with passive trailers can be easily driven using the proposed driver assistance system and a push-pull manual steering scheme. In [19], it is shown that the complexity of road train control is due to significantly pronounced nonlinearities, as well as instability of the control object when driving in reverse, which often leads to the phenomenon of a 'folding knife'.

Paper [20] proposes three control approaches for dynamic stabilisation in a road train configuration and a methodology for setting control gains using kinematic and dynamic models of three actuators. In [21], a path tracking system and a cascade controller scheme are developed for controlling an unmanned truck tractor when driving in reverse. In [22], a deep learning method based on a car model is proposed to determine the parameters of the front and rear stiffness of a car when cornering. Paper [23] calculates the cornering balance using a model of a front-wheel drive vehicle with rear wheels 'locked 'at zero angular velocity with a handbrake applied by a controller, taking into account drift methods applicable to front-wheel drive (FWD) configurations. In [24], a V2I architecture is proposed that can operate in both autonomous and manually driven vehicles and coordinate their movement based on V2I communication. In [25], the reversing motion of a vehicle-trailer combination is studied using a single-vehicle trajectory model with a quasi-static tyre model to develop a simple linear feedback controller that can ensure stable reversing motion along a straight path. Paper [26] proposes technical solutions for motion control reversing a trailed road train. For this purpose, two feedback controllers have been developed that provide the driver with inputs for automatic control in different situations. The problem of controlling the trajectory of a mobile robot with two trailers was considered in [27], for which a trajectory control algorithm was proposed (using the LOS (Line of Sight) and PID (Proportion, Differential, and Integral) control algorithms).

In [28] and [29], the results obtained by scientists using kinematic differential equations made for the "bicycle scheme", which investigated the manoeuvrability of tractor-trailers with non-rotating wheels of semi-trailers moving in reverse, were published. The study revealed that the loss of controllability and stability of rectilinear motion is associated with exceeding a given folding angle.

In [30], the necessity of taking into account the peculiarities of the construction of vehicles with different overall dimensions and designs, which determine their operational properties: manoeuvrability and directional stability, when organising freight transport in the "Smart City" system is shown. It is especially important for road trains with semi-trailers with non-turning wheels. On the basis of the developed mathematical model of the rearward movement of the road train and the proposed law of control of the front wheels of the tractor, the conditions for bringing the road train to stable directional movement in reverse have been obtained. The results of theoretical research and simulation modelling allowed us to develop a method for determining the conditions for ensuring stable reversing movement of a semi-trailer combination with non-turning semi-trailer wheels.

The aim of our research is to develop a new method for determining the optimal parameters of the control systems for the rearward movement of a semi-trailer truck combination with non-turning semi-trailer wheels for the system infrastructure "Smart City", which will allow us to select and justify the law of controlling the rotation of the tractor wheels at the stage of design and development work.

2 Problem Formulation

To date, the issue of assessing the manoeuvrability of a fifth-wheel combination with non-turning semi-trailer wheels moving in reverse remains insufficiently investigated. The issue of ensuring a stable rearward movement of this type of combination along the reference line (collision line) remains unresolved [6], [16].

In order to turn the combination on narrow roads in the "Smart City" system and to accurately deliver it to the object, it is necessary to manoeuvre it using the front and rear wheel drive. It is difficult to steer when reversing, as even a slight turn of the steering wheel can cause the combination to fold. When reversing, the position of the trailed link must be monitored using the rear-view mirrors, the tractor cab door, or the rear-view camera of the vehicle navigation system. In the event of even a slight change in the direction of travel of the semi-trailer, the trailer must be immediately levelled in relation to its axis of travel. To align the semi-trailer, turn the steering wheel in the direction of the change. It can also be levelled with a short forward movement. However, this manoeuvring control of the road train does not allow it to reverse accurately and quickly to the object or in the chosen direction.

In non-automated systems, as well as in all automated systems, the driver is involved in traffic control. The driver uses the information received by the senses to act on certain controls to perform a manoeuvre that he or she determines. Most of the information is obtained visually. In a non-automated system, the main source of information is the road. The driver uses the information about the position of the road train relative to the road to make a decision about the manoeuvre required. The time taken by the driver to assess the situation, the position of the road train relative to the approach object, the required direction of travel, and to make and implement the decision is usually referred to as the driver's reaction time. Tasks solved by the driver in the process of driving a vehicle can be divided into logical and reflex tasks. In the thinking process, logical decisions are known to arise. In reflex actions, the role of conditioned reflex, behavioural automatism, which is formed on the basis of knowledge, experience, and training, is great. In non-automated control systems, the reaction of a road train driver is complex. He has to evaluate the situation of road conditions and choose the right decision from several possible ones in order to perform the necessary motor action. Significant reaction time leads to untimely or erroneous control actions by the driver, which may result in the vehicle not approaching or leaving the roadway correctly or causing an accident.

It is particularly difficult to control the movement of a road train if the semi-trailer is long and its width is greater than the width of the tractor. In such conditions, the driver has limited opportunities to assess the position of the road train relative to the road, to detect other vehicles and obstacles, which is especially important in poor visibility conditions. In heavy rain, fog, or snowfall, the visibility range is reduced to several metres, limiting the permissible speed. At the same time, the accuracy of estimating the distance to objects on the road and the speed of other vehicles is low and depends on the speed and direction of movement. All these factors lead to low accuracy of trajectory prediction and, consequently, to poor road train control.

In automated on-board systems, the driver receives not only visual information about the position of the road train on the road, but also about its movement parameters from various sensors. Also, to improve driver awareness, rear-view cameras of the car navigation system can be used, interactive algorithms for constructing transformations of images received by the rear-view camera into images corresponding to such virtual cameras as wide-angle, normal top view, and others. For them, guidelines are also constructed, indicating how the car will move at a given position of the steering wheel. Such transformations are used in car navigation systems to improve the driver's perception of the situation behind the car. This visualisation of guidance lines helps to ensure accident-free manoeuvres when parking.

Computing devices included in automated control systems can produce not only informative, but also command information on the need to brake, increase speed, or change the course of the road train, greatly facilitating the driver's work, thereby reducing his fatigue and increasing safety.

The efficiency of the driver's actions in an automated system depends to a large extent on the amount and type of information delivered to the driver.

To date, the issue of evaluating the controllability and manoeuvring properties of a semi-trailer combination with the wheels of the semi-trailer not rotated relative to the frame, moving in reverse, has not been sufficiently investigated. In such a movement, it is required to ensure the stability of the rearward movement of this type of semi-trailer truck along the collision line and the necessary accuracy of its delivery to the object with a minimum manoeuvring area in a short time.

In order to turn the combination on narrow roads and to deliver it accurately to the object, it is necessary to manoeuvre using the forward and reverse gears. It is difficult to steer when backing up, as even a slight turn of the steering wheel can cause the combination to fold up.

3 Problem Solution

Earlier studies have shown the theoretical possibility of creating automated systems for controlling the rearward movement of a tractor-trailer to an object, capable of providing the necessary directional stability and accurate delivery of the road train under its loading or unloading, ensuring accident-free movement [30]. In the Figure 1 [30], the tractor frame is labelled *l* and the semi-trailer frame is labelled *L*.

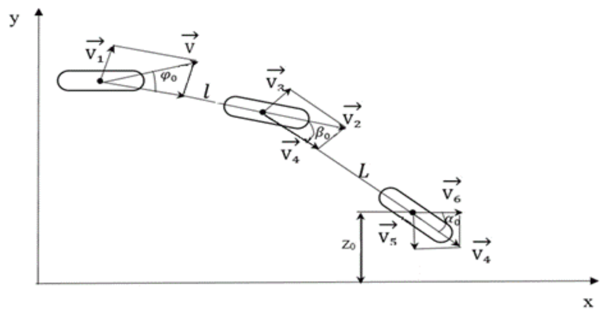

Fig. 1: Schematic of the vehicle position before reversing (X-axis coincides with the collision line). Source: [30]

Mathematical models describing the reversing motion of a road train under different laws of controlling the rotation of the tractor wheels have been developed. The complete system of equations of the road train motion, taking into account the control, takes the following form [30]:

$$\dot{\beta}(t) = \frac{V \cdot sin\varphi}{l} + \frac{V \cdot cos\varphi \cdot sin\beta}{L};$$
$$\dot{\alpha}(t) = \frac{V \cdot cos\varphi \cdot sin\beta}{L}; \quad (1)$$

$$\dot{Z}(t) = V \cdot cos\varphi \cdot cos\beta \cdot sin\alpha;$$
$$\dot{X}(t) = V \cdot cos\varphi \cdot cos\beta \cdot cos\alpha;$$
$$\varphi(t) = K_1 \cdot \beta(t) + K_2 \cdot \alpha(t) + K_3 \cdot Z(t).$$

Having transformed the given system of equations using the Laplace transform with non-zero initial conditions and then excluding the variables *β(t)* and *α(t)* from the system, we apply the Rouse-Hurwitz criterion to evaluate the stability of the system. The obtained necessary and sufficient conditions for its stable reversing motion of the road train can be represented by the inequalities [30]:

$$\begin{cases} \left(K_1 - \frac{l}{L}\right) \cdot \frac{K_2}{L} > K_3 > 0; \\ K_1 > \frac{l}{L}; \\ K_2 > 0. \end{cases} \quad (2)$$

Modelling of possible road train movement from different initial positions relative to the object is carried out and recommendations for the initial priority placement of the road train relative to the object are determined. Control methods and schemes of traffic control systems, as well as a method for determining the conditions for ensuring stable reversing movement of a semi-trailer combination with non-turning semi-trailer wheels have been developed [30].

Obviously, if we represent the functions in the form *β=f(t), α=f(t)* and *z=f(t),* it will be seen: the shorter the time of manoeuvring of the road train near the guide, the smaller will be the areas under each curve *β(t), α(t), z(t).* Thus, it is reasonable to choose the sum of areas of all functions *β(t), α(t), z(t)* as the optimality criterion for finding the coefficients K_1, K_2, K_3 i.e.

$$I = \int [A_1 \cdot |\beta(t)| + A_2 \cdot |\alpha(t)| + A_3 \cdot |z(t)|]dt, \quad (3)$$

where: I - quality criterion of the system; $A_1 \ldots A_3$ - proportionality coefficients, allowing to take into account the influence of each of the three parameters *β(t), α(t), z(t).*

If we assume that we do not care about the position of the tractor in relation to the semi-trailer in the final result, the coefficient A_1 in criterion (2) is not decisive.

Therefore, we set the coefficient $A_1 = 0.1$. The accuracy of the semi-trailer's position on the longitudinal collision line depends on two parameters: *α(t)* and *z(t).* Let's determine in what ratio linear and angular errors can be located, because it should depend on the length of the semi-trailer base and will be different for different road trains. For the example under consideration, let us take into account the lengths of the bases of the tractor and semi-trailer:

$$\frac{A_2}{A_3} = \frac{2}{L}. \quad (4)$$

Knowing the ratio of coefficients A_1, A, A_3, we can proceed to the solution of the problem, the essence of which is to find the minimum of criterion (3) when varying K_1, K_2, K_3 within the limits given by the conditions of stable movement of the road train [30].

To solve the problem, we will use the Gauss-Seidel method of optimisation (gradient descent). It is known that the gradient of the function is called the vector giving the direction of the maximum change in the function of this change:

$$y = I(K_1, K_2, K_3).$$

The essence of the gradient descent method is as follows: around the initial point defining the quality criterion in the space of varying parameters, a number of preliminary experiments and calculations are carried out, planned in such a way that each parameter K_1,... K_3, takes two values. After simple calculations, the components of the gradient vector equal to approximately measures of the "slope" of the function along the coordinate axes are determined.

Then one step is performed along the gradient direction at the found point.

The gradient is determined again and another step is performed. The procedure is repeated until the extremum is reached, where the gradient components become equal to zero or the difference of criteria calculated at different points is less than or equal to a predetermined value ε, which determines the accuracy of finding the minimum. The gradient vector of the target function has the following form:

$$\overline{I(K_1, K_2, K_3)} = \frac{\partial I}{\partial K_1} e_1 + \frac{\partial I}{\partial K_2} e_2 + \frac{\partial I}{\partial K_3} e_3, (5)$$

where $e_1 \ldots e_3$ are independent unit vectors directed along the corresponding coordinate axes, and the partial derivatives are calculated at the point under consideration.

Its inverse vector indicates the direction of descent. The components of the gradient vector are approximated in the neighbourhood of the point under consideration:

$$\frac{\partial I}{\partial K_i} = \frac{I(K_{1i} \ldots K_{3i}, K_{ii} + \Delta, \ldots, K_{Ni}) - F(K_{1i}, K_{2i}, \ldots, K_{Ni})}{\Delta}, (6)$$

where Δ is a small displacement in the K_i direction.

The unit gradient vector can be represented as:

$$\overline{I(K_1, K_2, K_3)} = U_1 \cdot e_1 + U_2 \cdot e_2 + U_3 \cdot e_3,$$
where

$$U_i = \frac{\frac{\partial I}{\partial K_i}}{\sum_{j=1}^{N}\left[\left(\frac{\partial I}{\partial K_j}\right)\right]^{\frac{1}{2}}}. \quad (7)$$

Then we find the position of the new point according to

$$K_{i+1} = K_i + \Delta(K_i) \cdot U_i. \quad (8)$$

Having determined the strategy of choosing the order of solving the problem, let's draw up its algorithm (Figure 2).

The solution of the system of nonlinear equations by the numerical Runge-Kutt method is denoted by RKGS.

The conducted researches have revealed possible areas of optimal coefficients $K_1...K_3$. Therefore, we can start the solution by setting one of the possible values of $K_1...K_3$, taking the initial step as follows

Let us calculate the total area under the curves for the given initial values of $K_1...K_3$, denoting it as $M_2(P)$.

Let us change the value of K_1 by the step size ΔK_1 and determine the total area for each case, denoting it as $N_1(P)$. Then we can determine the gradient in K_1 according to (5):

$$GR_1 = \frac{M_2(P) - N_1(P)}{\Delta K_1}. \quad (9)$$

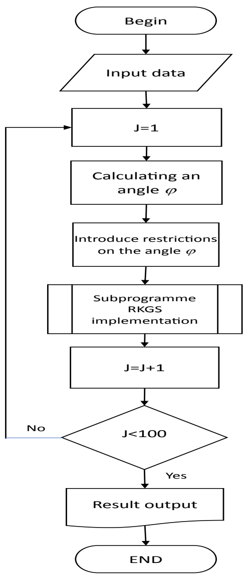

Fig. 2: Generalised algorithm for investigating the mathematical model (1). Source: created by the authors

Returning the original value of K_1, we do the same operations for K_2, K_3 and determine GR_2 and GR_3.

$$GR_2 = \frac{M_2(P) - N_2(P)}{\Delta K_2} \quad (10);$$

$$GR_3 = \frac{M_2(P) - N_3(P)}{\Delta K_3} \quad (11);$$

Having determined the gradients of the vector along each of the three directions, we find the step sizes ΔK_i, according to (7).

$$\Delta K_1 = \frac{GR_1}{\sqrt{GR_1^2 + GR_2^2 + GR_3^2}}; \quad (12)$$

$$\Delta K_2 = \frac{GR_2}{\sqrt{GR_1^2 + GR_2^2 + GR_3^2}}; \quad (13)$$

$$\Delta K_3 = \frac{GR_3}{\sqrt{GR_1^2 + GR_2^2 + GR_3^2}}. \quad (14)$$

Now, with the help of (8) we can determine the new values of K_{i+1}.

However, since the values of $K_1...K_3$ cannot go beyond the limits that ensure the stability of the straight-line motion of the road train and specified in (2), it is necessary to introduce a penalty function to form a new composite function:

$$P(K_i) = I(K_i) + \sum_{j=1}^{N} \varphi\left[G_j(K_i)\right], \quad (15)$$

where $I(K_i)$ – bounded function under consideration; $P(K_i)$ – composite unbounded function; $\varphi\left[G_j(K_i)\right]$ – penalty function that takes into account the constraints given by the inequalities.

Taking advantage of (15) in the case of output $K_1...K_3$ out of bounds, they are assigned a penalty value, which leads to an infinitely large value of the target function and allows us to arrive at another point more suitable for further research:

$$K_1 = K_1 + \Omega \cdot \left(1 + \frac{K_2}{l}\right);$$
$$K_2 = K_2 + \Omega \cdot \left[1 + \frac{1}{l} \cdot \left(K_1 - \frac{l}{L}\right)\right]; \quad (16)$$
$$K_3 = K_3 + \Omega,$$

where Ω is the value of the penalty.

Calculations are carried out until the minimum is found and the condition is fulfilled:

$$|M_2(P-1)| - |M_2(P)| \le \varepsilon. \quad (17)$$

As a result of the research, graphical dependences were obtained for different initial positions of the road train relative to the collision reference line at different values of $K_1...K_3$:

$\beta = f(t)$ (Figure 3);

$\alpha = f(t)$ (Figure 4);
$z = f(t)$ (Figure 5);
$\varphi = \varphi(t)$ (Figure 6).

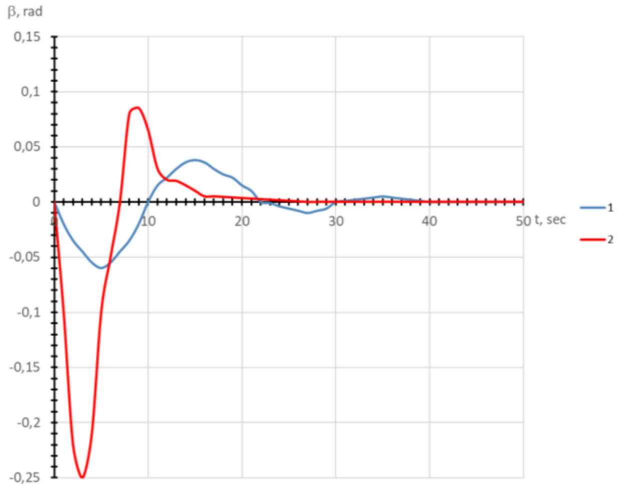

Fig. 3: Graph of the function $\beta = f(t)$ at the values of the coefficients. Source: created by the authors:
1. K_1=1.63; K_2=3.695; K_3=0.255;
2. K_1=3.9; K_2=21.9; K_3=2.69

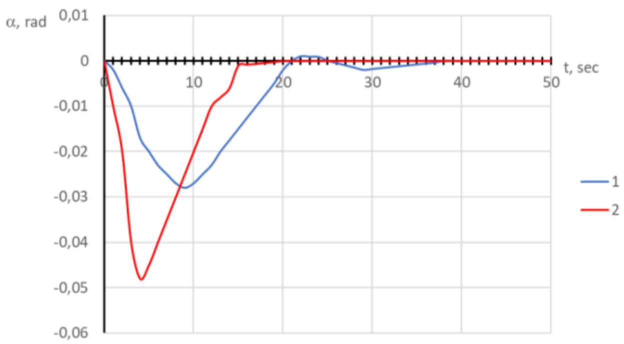

Fig. 4: Graph of the function $\alpha = f(t)$ at the values of the coefficients. Source: created by the authors:
1. K_1=1.63; K_2=3.695; K_3=0.255;
2. K_1=3.9; K_2=21.9; K_3=2.69

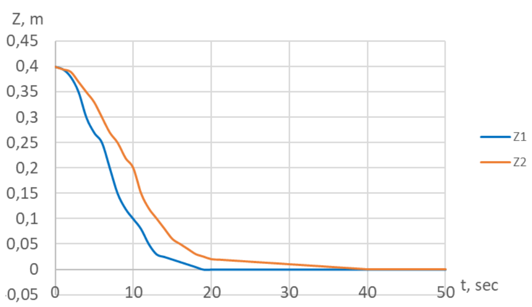

Fig. 5: Graph of the function $z = f(t)$ at the values of the coefficients. Source: created by the authors:
1. K_1=1.63; K_2=3.695; K_3=0.255;
2. K_1=3.9; K_2=21.9; K_3=2.69

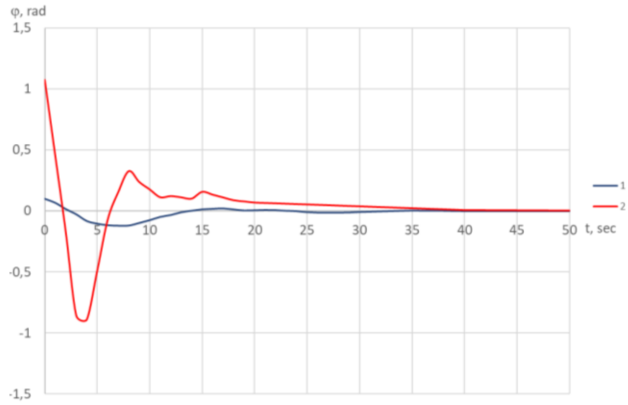

Fig. 6: Graph of the function $\varphi = \varphi(t)$ at the values of coefficients. Source: created by the authors:
1. K_1=1.63; K_2=3.695; K_3=0.255;
2. K_1=3.9; K_2=21.9; K_3=2.69

Studies have shown (Figure 3, Figure 4, Figure 5 and Figure 6) that the road train manoeuvres along the reference collision line in such a way as to reduce the angle of folding of the links β(*t*), the angle of inclination of the semi-trailer axis to the collision line α*(t)* and the displacement *z(t)* to zero value.

At K_1=1.63, K_2=3.695, K_3=0.255 and the initial position of the road train *β=0, α=0, z=0.4* m, manoeuvring continues for about 40 sec, and further the semi-trailer truck moves steadily along the collision line. Consequently, by fulfilling the conditions (2), it is possible to ensure the stable reversing movement of the road train along the collision reference line at different initial positions.

Creation of an automated or automatic system for controlling the movement of a road train, implementing the control law (1) under condition (2), makes it possible to significantly reduce manoeuvring time, especially in poor visibility conditions (at night), when the width of the semi-trailer exceeds the width of the tractor.

By setting the optimal values of $K_1 \dots K_3$ for the control system, it is possible to significantly reduce the manoeuvring area of the combination.

4 Conclusion

The questions of the possibility to improve the operational properties of a semi-trailer truck combination with non-rotating wheels of a semi-trailer, which is an unstable system when reversing in the infrastructure of the "Smart City" system, by introducing feedback of motion control with the help of an automated data processing system are considered.

As the results of research have shown, the use of such systems can significantly improve their manoeuvrability and directional stability in the Smart City system. The results of using the previously developed method of determining the conditions for ensuring the stable movement of the road train in reverse with non-rotating wheels of the semi-trailer allowed solving the problems of algorithmization of manoeuvring control described by a mathematical model of differential nonlinear equations.

1. On the basis of the conducted research, using the mathematical model (1) and using the necessary and sufficient conditions of stable reversing motion of the road train (2), the optimisation parameters included in the wheel turning control law are determined.

2. The additive optimisation criterion has been substantiated and developed, which makes it possible to evaluate the quality of the automated control system operation by minimising the area and manoeuvring time when reversing to the object.

3 Based on the method of numerical integration of the system of differential nonlinear equations with high accuracy, a generalised algorithm for the study of the mathematical model (1) has been developed.

4. An algorithm for determining the optimal parameters of the motion control system of a semi-trailer truck with non-rotating wheels is developed, which is used to develop a methodology for optimising the parameters.

5. An improved method of optimising the parameters of the tractor-trailer reversing motion control systems is proposed.

6. The presented graphical results confirm that the application of the developed method will reduce the area and time of manoeuvring vehicles in reverse when performing transport tasks in the infrastructure of the system “Smart City”.

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