

Convergence analysis of nonlinear model predictive control based on fractional Hammerstein model

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Abstract: This study develops a predictive control strategy based on Fractional Hammerstein Models, applicable to both Single-Input Single-Output and Multiple-Input Multiple-Output systems. The proposed predictive control solution is obtained by minimizing a cost function based on the squared prediction error. We have used deterministic optimization methods based on gradient techniques and simplex theory. A convex optimization framework is obtained from the initial non-convex problem by restricting the control space through tightened constraints around the optimal control value that minimizes energy consumption. Indeed, the optimization problem is theoretically studied and the algorithms are analysed and compared in tracking performances.

Key- Words: - fractional order system, Hammerstein model, optimization problem, polynomial structure, nonlinear models, predictive control

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1 Introduction

Over the past decades, Nonlinear Model Predictive Control (NMPC) has emerged as an effective control strategy for nonlinear systems. Nevertheless, its practical implementation remains constrained by the limitations and inaccuracies of available mathematical models. To improve the applicability of NMPC, fractional-order nonlinear models are considered in this work. Among the different approaches used for nonlinear system representation, Fractional Hammerstein Models (FHM) provide a suitable and powerful modeling framework. Accordingly, this study focuses on developing an NMPC scheme based on FHMs. The control of nonlinear dynamical systems exhibiting fractional-order behavior remains a challenging research problem, particularly with the growing development and application of fractional calculus.

Based on the existing literature, nonlinear fractional models can be broadly divided into several main classes, including Volterra series models, [1], neural network-based models, [2], and block-structured models. The Hammerstein model is one of the commonly considered structured models, combining a static nonlinear block with a subsequent linear dynamic fractional-order block, [3].

The FHM have been widely used to represent nonlinear dynamical systems, with applications including thermal and mechanical processes, [4],[5]. Their structure combines a static nonlinear component with linear dynamics, providing an effective framework for nonlinear model-

ing and identification. Consequently, FHMs have been investigated for various modeling and control applications. FHMs have been widely studied due to their ability to represent nonlinear dynamic processes, with applications including thermal and mechanical systems, [5]. The Hammerstein structure separates the static nonlinearity from the linear dynamics, allowing established linear-system methods to be applied. Several control strategies have been developed for integer-order Hammerstein systems, including PID control, [6] and sliding-mode control, [7].

To the best of our knowledge, few studies have addressed the control of FHM, particularly fractional predictive control. These models require advanced mathematical tools, while the control law of fractional systems depends on the history of inputs and outputs, increasing the computational burden. Therefore, a time-efficient optimization approach is required for fast dynamic systems and limited computational resources. Moreover, the static nonlinearity of the model may lead the NMPC algorithm to a non-convex optimization problem. Unlike linear models, no analytical optimal control law is generally available for nonlinear systems, requiring numerical optimization at each iteration.

Optimization methods for such problems are generally classified as deterministic or stochastic. Deterministic approaches include the steepest descent method, [8], sequential quadratic programming (SQP), [9], and interior-point methods, [10]. Stochastic methods include Genetic Algorithms (GA), [11], and Particle Swarm Op-

timization (PSO), [12].

This study develops a predictive control approach for nonlinear fractional-order systems using a Hammerstein model with polynomial nonlinearities. The static nonlinear characteristic is analyzed to assess the convexity and number of solutions for different nonlinearity degrees. Two deterministic optimization methods, namely the Nelder-Mead (NM) and Trust-Region (TR) algorithms, are considered. Restricting the control variables around the energy-efficient optimal solution converts the problem into a convex formulation. The method requiring the shortest computation time is therefore preferred. Several Single-Input Single-Output (SISO) and Multiple-Input Multiple-Output (MIMO) models are simulated using MATLAB.

The remainder of this paper is organized as follows. Section I presents the approximation method used to characterize the linear part of the FHM. Section II describes the fractional predictive control algorithm. The proposed approach is evaluated through convex and non-convex optimization problems using SISO and MIMO models, followed by the conclusion.

2 Fractional Hammerstein structure

At each sampling instant k the input $u(k)$ is first processed by the static nonlinear function $f(\cdot)$ and then applied to a linear system described by the fractional-order transfer function $H(s)$. The intermediate signal $\tilde{y}(k)$, generally assumed to be unmeasurable, results from the nonlinear transformation of $u(k)$ through $f(\cdot)$. The nonlinear function $f(\cdot)$ is represented using a polynomial form, given by:

$$f(u) = \tilde{y}(k) = c_1 u(k) + c_2 u^2(k) + \dots + c_{n_s} u^{n_s}(k) \quad (1)$$

where n_s is the degree of the polynomial function.

The discrete-time transfer function $H(z)$ is obtained by applying the Z-transform to the continuous-time fractional-order model $H(s)$ given in (2).

$$H(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \dots + b_0}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \dots + a_0} \quad (2)$$

where $a_0, \dots, a_n, b_0, \dots, b_m$ are constant coefficients, whereas $\alpha_0, \dots, \alpha_n$ and $\beta_0, \dots, \beta_m$ denote the fractional powers. The parameters $a_0, \dots, a_n, b_0, \dots, b_m, \alpha_0, \dots, \alpha_n, \beta_0, \dots, \beta_m$, and $c_1, \dots, c_{n_s}$ define the fractional Hammerstein model and can be identified through various algorithms.

For numerical implementation, different sampling techniques can be employed to discretize the continuous transfer function. In this study, the Grünwald-Letnikov (G-L) method is selected. The differential equation describing the relation between the input $\tilde{y}(t)$ and output $y(t)$ of the SISO fractional-order system is obtained from equation (2) as follows:

$$\sum_{l=0}^L a_l D_t^{\alpha_{a_l}} y(t) = \sum_{m=0}^M b_m D_t^{\alpha_{b_m}} \tilde{y}(t) \quad (3)$$

where $D_t^{\alpha_{a_l}}$ is the operator of the fractional derivative, $(a_l, b_m) \in R^2$, $(\alpha_{a_l}, \alpha_{b_m}) \in R^2$, L and M denote, respectively, the numbers of derivatives applied to the output and input. The variable $y(t)$ represents the output, while $\tilde{y}(k)$ denotes the input.

For the MIMO case, the differential equation describing the relationship between the outputs y_i , $i \in [1, \dots, N_o]$, and the input functions $\tilde{y}_j$, $j \in [1, \dots, N_i]$, where N_o and N_i denote the respective numbers of outputs and inputs, can be written as follows:

$$\sum_{l_i=0}^{L_i} a_{l_i} D_t^{\alpha_{a_{l_i}}} y_i(t) = \sum_{j=1}^{N_i} \sum_{m_{ij}=0}^{M_{ij}} b_{m_{ij}} D_t^{\alpha_{b_{m_{ij}}}} \tilde{y}_j(t) \quad (4)$$

where $(a_{l_i}, b_{m_{ij}}) \in R^2$, $(\alpha_{a_{l_i}}, \alpha_{b_{m_{ij}}}) \in R^2$. M_{ij} denotes the number of fractional derivatives associated with the inputs, while L_{ij} represents the number of fractional derivatives applied to the outputs.

The fractional order derivative can be obtained using G-L approximation as represented in, [13].

3 Nonlinear fractional predictive control

Based on the model used to predict the system response over a finite horizon, the predictive control law is determined. At each sampling instant, a quadratic cost function is minimized to obtain an optimal control sequence. Only the first control value of this sequence is applied to the fractional model. The procedure is then repeated using the updated control and output values. The predicted output over the prediction horizon N_p can be written in matrix form as follows:

$$\hat{Y} = AY_p + B_p \tilde{Y}_p + B_f \tilde{Y}_f \quad (5)$$

where

$$\hat{Y} = [y(k+1), y(k+2), \dots, y(k+N_p)]^T \quad (6)$$

The future values of the internal signal are stored in the vector $\tilde{Y}_f$, defined as follows:

$$\tilde{Y}_f = \begin{bmatrix} \tilde{y}(k) \\ \tilde{y}(k+1) \\ \vdots \\ \tilde{y}(k+(N_p-1)) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n_s} c_i u^i(k) \\ \sum_{i=1}^{n_s} c_i u^i(k+1) \\ \vdots \\ \sum_{i=1}^{n_s} c_i u^i(k+(N_p-1)) \end{bmatrix} \quad (7)$$

The matrix A defines the relationship between the vectors $\hat{Y}$ and Y_p :

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{j1} & a_{jN_p} & \cdots & a_{N_p N} \end{bmatrix} \quad (8)$$

The matrix B_p establishes the relationship between the vectors $\hat{Y}$ and $\tilde{Y}_p$:

$$B_p = \begin{bmatrix} bp_{11} & bp_{12} & \cdots & bp_{1N} \\ bp_{21} & bp_{22} & \cdots & bp_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ bp_{N_p 1} & bp_{N_p 2} & \cdots & bp_{N_p N} \end{bmatrix} \quad (9)$$

The matrix B_f defines the relationship between the vectors $\hat{Y}$ and $\tilde{y}_f$ and is given by:

$$B_f = \begin{bmatrix} bf_{11} & 0 & \cdots & 0 \\ bf_{21} & bf_{22} & \vdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ bf_{N_p 1} & bf_{N_p 2} & \cdots & bf_{N_p N_p} \end{bmatrix} \quad (10)$$

The cost function J is numerically minimized in order to estimate the predictive best control solution. It consists of two terms: the first accounts for the deviations between the future reference signals and predicted outputs, while the second penalizes the future control increments. It can be written in matrix form as follows:

$$J = \frac{1}{2} [Y_r(k) - \hat{Y}(k)]^T Q [Y_r(k) - \hat{Y}(k)] + \Delta \bar{U}(k)^T R \Delta \bar{U}(k) \quad (11)$$

submitted to: $lb < u(k) < ub$

where lb and ub denote the lower and upper bounds of the control, respectively, and $Y_r(k)$ represents the set-point vector defined as follows:

$$Y_r(k) = [y_r(k+1), y_r(k+2), \dots, y_r(k+N_p)]^T \quad (12)$$

where $y_r(k)$ denotes the set-point, while $\Delta \bar{U}(k)$ represents the control increment vector defined by:

$$\Delta \bar{U}(k) = U(k) - U(k-1) \quad (13)$$

$$\Delta \bar{U}(k) = [\Delta U(k), \Delta U(k+1), \dots, \Delta U(k+(N_c-1))]^T \quad (14)$$

$$U(k) = [u(k), u(k+1), \dots, u(k+N_p-1)]^T \quad (15)$$

N_c denotes the control horizon, while Q is the weighting matrix associated with the prediction error and R is the weighting matrix for the control increments, used to reduce the control energy.

4 Stability proof

Let the criterion $J(k)$, defined in equation (11), be considered as a Lyapunov function. If the monotonous decreasing of this objective function, with the sample index k , is guaranteed, we can say that the control algorithm attains asymptotic stability, [14], [15]. Besides, if the initialization is well chosen in convex part, the NM method, differently from all other nonlinear optimization methods, guarantees the monotonous decreasing of the objective function. Indeed, for the same set-point value, it furnishes a better point at each sample iteration k .

5 Simulation results

The proposed predictive control approach for fractional-order Hammerstein models is determined through MATLAB simulations. Several numerical examples reported in the literature are considered to assess the effectiveness of the control law and identify the main factors affecting set-point tracking. The Trust-Region (TR) and Nelder-Mead (NM) methods are employed to handle both convex and non-convex nonlinear optimization problems.

5.1 Convex optimization problem

5.1.1 SISO model 1

The model (16) represents a SISO system in which the nonlinear function is represented by a polynomial structure with odd degree. The FHM of the following model is detailed in the reference, [16]:

$$\begin{cases} \tilde{y}(k) = 1.6174u^3(k) \\ H(s) = \frac{1.7397s^{0.4988} + 2.1601}{0.9852s^{1.4964} + 1.7439s^{0.9976} + 3.2185s^{0.4988} + 1} \end{cases} \quad (16)$$

where $y(k)$ denotes the output, $u(k)$ represents the control input, and $\tilde{y}(k) = f(u)$ is the non-measurable nonlinear signal.

The NMPC is implemented using the following set of parameters: $N_p = 5$; $N_c = 1$; $N = 100$;

$h = 0.1$; $-2.5 \leq u(k) \leq 2.5$; $R = 0.1I$; $Q = I$; where I is the identity matrix.

Figure 1 shows the evolution of the static non-linear characteristic and the corresponding static criterion J over the different iterations k .

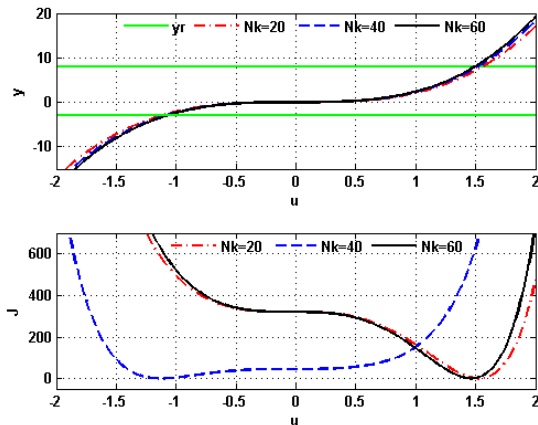


Figure 1: Static nonlinear characteristic of model (16) and corresponding objective function J for $N_p = 5$. Source: created by the authors.

The curves presented in Fig.1 show that the shape of the nonlinear characteristic varies with the iteration number k , which is a distinctive feature of fractional-order systems. In such systems, the output computed at iteration k depends not only on the current measurements but also on the measurements obtained during the preceding $(k - 1)$ iterations.

Furthermore, the static nonlinear characteristic shows that each desired set-point is associated with a unique control value, as indicated by the criterion curve. Consequently, for this example, determining the optimal control $u^*(k)$ that minimizes the criterion J leads to a convex optimization problem.

The choice of the prediction horizon N_p :

The prediction horizon N_p appears in equation (11) and therefore affects both the shape of the objective function and the system's temporal response. The value of N_p that provides the best performance will be determined below based on the simulation results.

Figures 2 and 3 present the system's temporal responses obtained using the TR and NM methods, respectively, for different prediction horizons N_p .

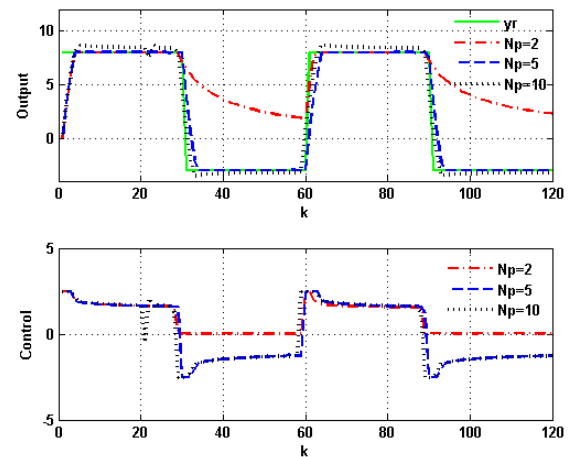


Figure 2: The temporal responses for $N_p = 2$, $N_p = 5$ and $N_p = 10$ using TR method. Source: created by the authors.

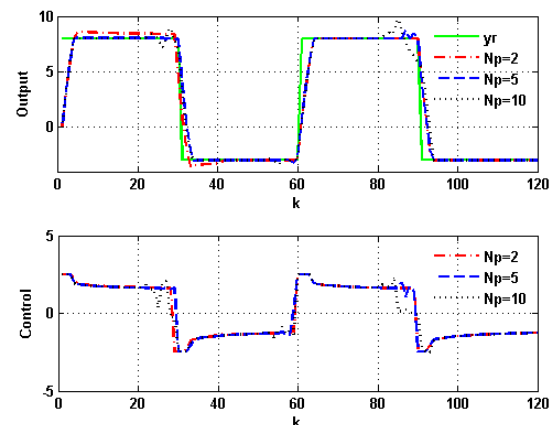


Figure 3: The temporal responses for $N_p = 2$, $N_p = 5$ and $N_p = 10$ using NM method. Source: created by the authors.

Figure 2 shows that the determination of the optimal control minimizing the criterion J is influenced by the prediction horizon N_p . For a short horizon ($N_p = 2$), the TR method fails to ensure proper tracking of the set-point change. Indeed, the TR algorithm is trapped in a larger palliate, as shown in Fig. 4. For higher prediction horizon values ($N_p = 10$), a non-zero tracking error appears. As for the NM method, the temporal results are presented in Fig. 3. Figure 3 shows that the NM method achieves successful conver-

gence for all values of N_p . However, a transient overshoot appears for a higher prediction horizon ($N_p = 10$). Moreover, if it is high ($N_p = 10$) the performances are degraded for both methods. This is explained by Fig.4 .

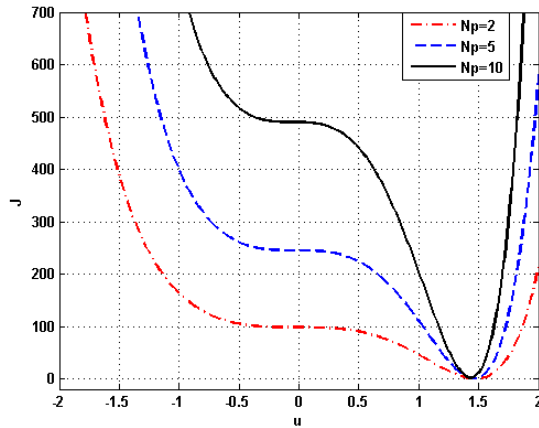


Figure 4: The objective function J for $N_p = 2$, $N_p = 5$, $N_p = 10$ and $y_r = 8$. Source: created by the authors.

Fig.4 shows that for $N_p = 2$ the criterion curve presents a wide palliate which can cause the divergence of the search algorithm for the two methods NM and TR because for the first method the simplex degenerates and for the second one the gradient is close to zero. Moreover, the width of this palliate decreases if N_p increases, which explains the improvement in the responses. However, the high value of N_p can involve overshoot. Thus, we should choose a prediction horizon that guarantees the best performances.

Table 1 reports the control signal variance and the Root Mean Squared Error (RMSE) for different prediction horizons N_p , evaluated using the following expressions, respectively:

$$\sigma_u = \frac{1}{N} \sum_{i=1}^N u(i)^2 \quad (17)$$

$$e = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_r(i) - y(i))^2} \quad (18)$$

The control signal variance and the mean squared error reported in Table 1 correspond to the simulation results presented in Fig. 3. The results indicate that ($N_p = 5$) provides the best performance, yielding the lowest control signal

Table 1: The control signal variance and the RMSE. Source: created by the authors.

N_p	2	5	7	10
σ_u	0.3498	0.3465	0.3510	0.3714
e	1.4924	1.4818	1.5517	1.6804

variance and mean squared error, as defined by (17) and (18), respectively.

The choice of the initial conditions:

In the following, NM and TR algorithms will start from zero initial conditions $u_0 = 0$. The time responses obtained are plotted in Fig.5. Regarding

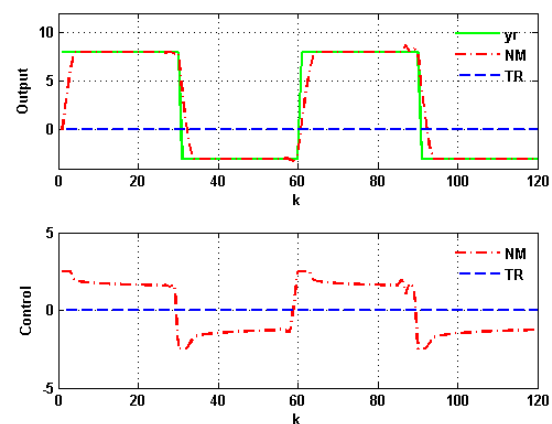


Figure 5: The temporal responses using TR and NM methods starting from $u_0 = 0$ for $N_p = 5$. Source: created by the authors.

this figure, we can conclude that the system response, using TR, depends strongly on the initial conditions. However, when using NM approach, the output reaches the set-point with all considered initializations. This is explained by the fact that, for TR, the optimal solution search is based on the gradient calculation. Therefore, the algorithm can diverge or give a non-optimal solution, when the gradient is null which is the case of palliate zone in the criterion created around the inflection point of the characteristic especially for lower prediction horizon $N_p = 2$ which is illustrated by Fig.1 and Fig.4. Whereas, for NM approach, the search algorithm uses the simplex theory, which does not block in the critical points with null derivative.

The temporal response of the system to the predictive control law is depicted by Fig.6. The NM and TR algorithms part from the initial con-

ditions $u_0 = 0.2$.

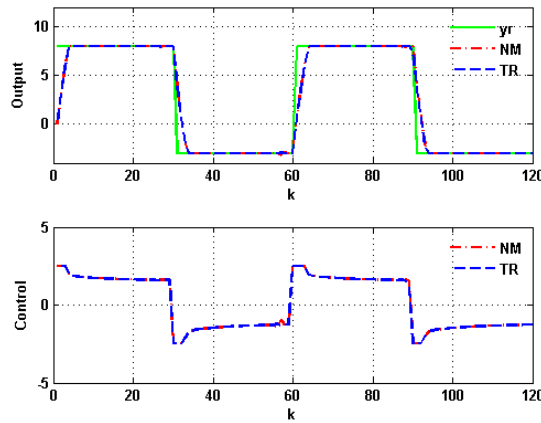


Figure 6: The temporal responses using NM and TR algorithms starting from $u_0 = 0.2$ for $N_p = 5$. Source: created by the authors.

Fig.6 confirms the solution unicity of control value. Indeed, both methods produce the same temporal responses, since the optimization problem is convex and the static characteristic has a unique solution despite its nonlinear form.

5.1.2 MIMO model 1

To illustrate the previous interpretations, let consider the following MIMO model. The estimation of Hammerstein parameters is studied in, [17]:

$$Y(s) = G(s)\tilde{Y}(s) \quad (19)$$

Where

$$G(s) = \begin{bmatrix} \frac{3.976}{5.127s^{0.299}+1} & \frac{2.959}{3.120s^{0.299}+1} \\ \frac{3.996}{6.118s^{0.299}+1} & \frac{4.970}{2.017s^{0.299}+1} \end{bmatrix} \quad (20)$$

$$Y(s) = \begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} \text{ and } \tilde{Y}(s) = \begin{bmatrix} \tilde{y}_1(s) \\ \tilde{y}_2(s) \end{bmatrix}$$

where y_1, y_2 are the outputs, $\tilde{y}_1, \tilde{y}_2$ are the non-linear signals.

The same example as that considered in [17] is adopted, while the last terms of the nonlinear function are omitted. As a result, the nonlinear characteristic has an odd degree and can be expressed as follows:

$$\begin{cases} \tilde{y}_1(k) = u_1(k) + 0.501u_1^2(k) + 0.289u_1^3(k) \\ \tilde{y}_2(k) = u_2(k) + 0.4u_2^2(k) + 0.2u_2^3(k) \end{cases} \quad (21)$$

Where u_1, u_2 are the controls. $-2.5 \leq u_1 \leq 2.5$ and $-2 \leq u_2 \leq 1$.

Minimizing the objective function yields a unique solution, confirming the convexity of the optimization problem. This behavior is illustrated in Fig.7, which shows the static criterion curve with a single minimum corresponding to each set-point.

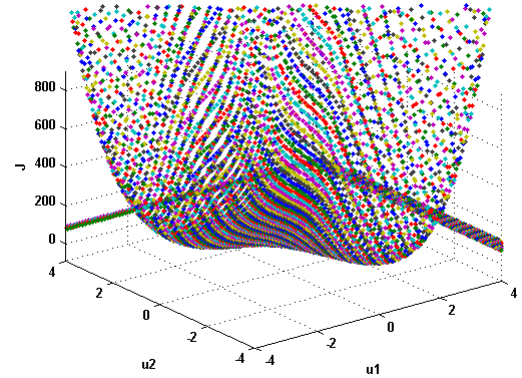


Figure 7: Criterion J of model (19) for the set-points $y_{r1} = 4$ and $y_{r2} = 2$ at iteration $k = 80$. Source: created by the authors.

Figures 8 and 9 show, respectively, the temporal responses and predictive control signals of system (19) obtained with the nonlinear function (21).

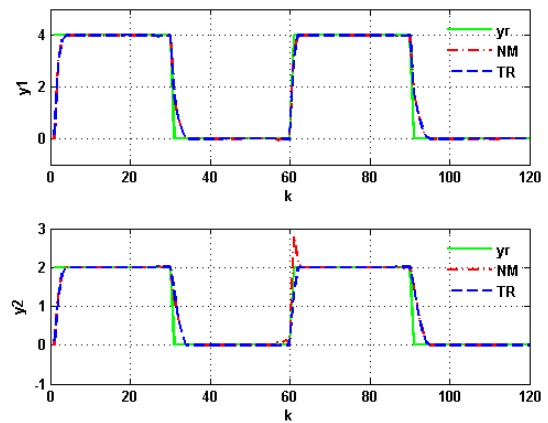


Figure 8: The step responses of model (19) with $-2.5 \leq u_1 \leq 2.5$ and $-2 \leq u_2 \leq 1$ for $N_p = 5$. Source: created by the authors.

Figure 8 shows that both outputs successfully

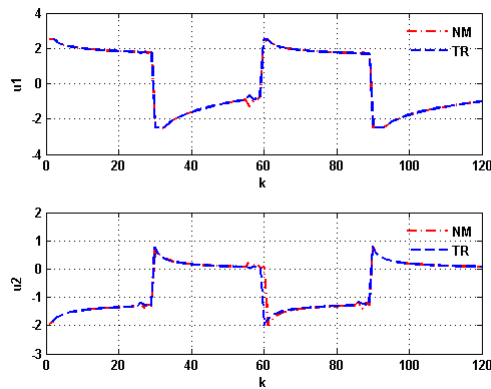


Figure 9: The predictive controls of model (19) with $-2.5 \leq u_1 \leq 2.5$ and $-2 \leq u_2 \leq 1$. Source: created by the authors.

track their respective set-points with the TR and NM algorithms. This behavior is attributed to the convexity of the optimization problem and the fact that the set-points are sufficiently far from the critical points of the static nonlinear characteristics.

5.2 Non-convex optimization problem

5.2.1 SISO model 2

The following system is described by a SISO nonlinear model represented by a polynomial structure with peer degree. The FHM of the following model is studied in, [18]. It is expressed by equation (22):

$$\begin{cases} \tilde{y}(k) = 0.9587u(k) + 2.0028u^2(k) \\ H(s) = \frac{1}{1+1.9992s^{0.5}} \end{cases} \quad (22)$$

where $y(k)$ denotes the output, $u(k)$ represents the control input, and $\tilde{y}(k)$ is the nonlinear, non-measurable signal.

In the following analysis, the constraint $-5 \leq u(k) \leq 5$ is imposed, resulting in a non-convex optimization problem. The simulations are performed using the following parameter values: $R = 0.1I$, $Q = I$, $N_p = 5$, $N_c = 1$, $N = 120$, and $h = 0.1$. Figure 10 presents the static nonlinear characteristic and the objective function J obtained at iterations $k = 20$, $k = 50$, and $k = 70$. As illustrated in the figure, the resulting curve exhibits a parabolic shape due to the even degree of the static nonlinear function. The optimization problem is therefore non-convex, with two optimal control values $u(k)$ associated with each

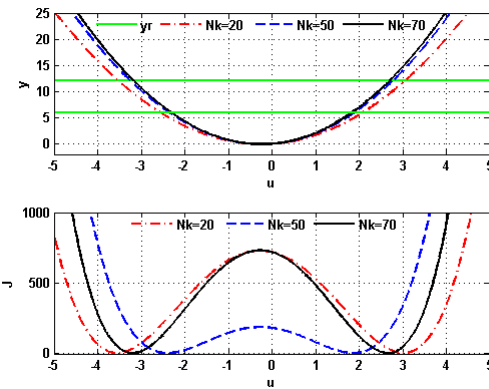


Figure 10: The objective function J and static nonlinear characteristic of model (22). Source: created by the authors.

set-point value y on the characteristic curve. The minimum located at $(-0.24, 0)$ corresponds to a critical point at which the derivative of the objective function vanishes.

Figure 11 presents the temporal responses obtained using the NM and TR methods to solve the constrained optimization problem.

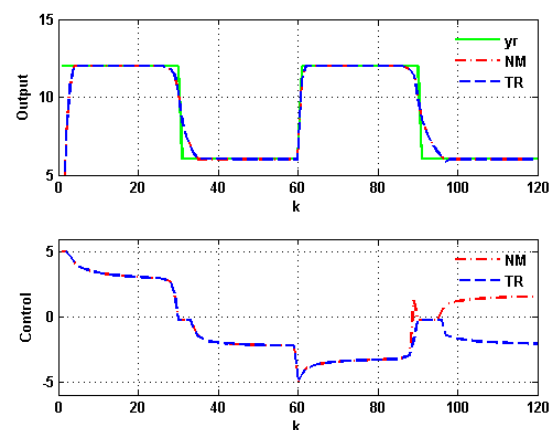


Figure 11: The temporal responses of model (22) when $-5 \leq u(k) \leq 5$ for $N_p = 5$. Source: created by the authors.

Both the NM and TR algorithms converge toward the optimal solution and provide improved performance. However, the control trajectory shows that the algorithms alternate between two solutions, resulting in significant variations in the control input and, consequently, increased energy

consumption.

To ensure convergence toward a single solution associated with minimum energy consumption, the control domain is restricted by imposing tighter constraints.

Figures 12 and 13 present, respectively, the step responses obtained with the NM and TR algorithms after tightening the constraints of model (22) to enforce convergence toward a unique solution that minimizes energy consumption.

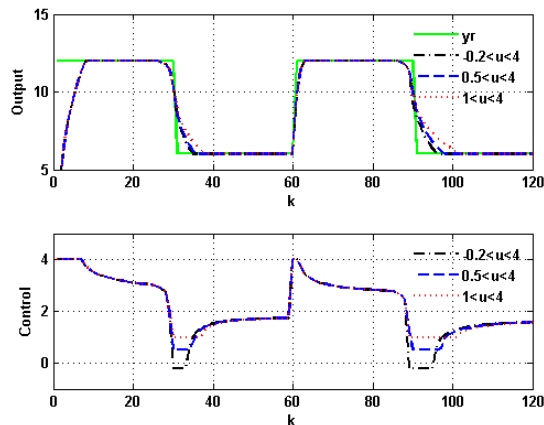


Figure 12: The step responses using NM algorithm for $N_p = 5$. Source: created by the authors.

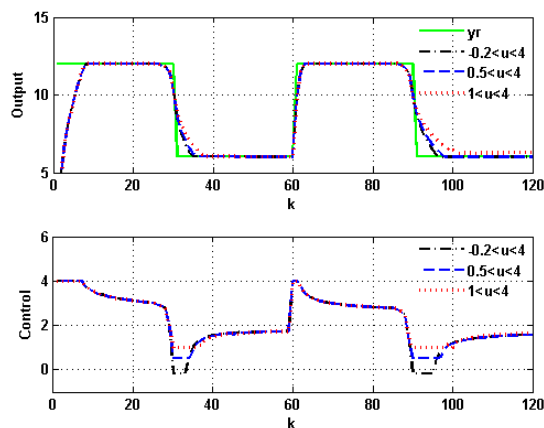


Figure 13: The step responses using TR algorithm for $N_p = 5$. Source: created by the authors.

From Fig.12 and Fig.13, it can be observed that, when the optimization problem is convex,

both methods, based on gradient and simplex strategies, are able to determine the optimal control. However, the constraints imposed on the control input can affect the optimization process and consequently reduce the system response speed.

The following section investigates the system behavior for set-points located near the critical point

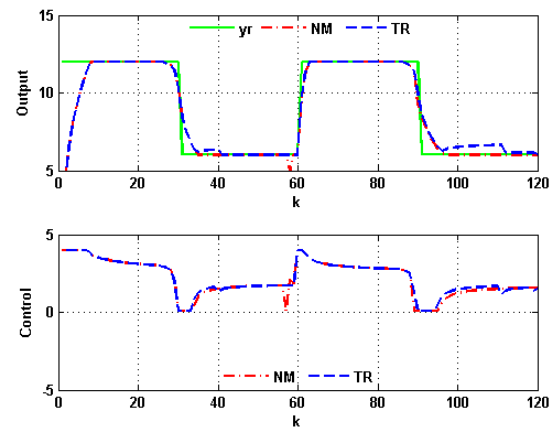


Figure 14: The step response when the set-point y_r sways between 4 and 1 for $1 \leq u(k) \leq 4$. Source: created by the authors.

The simulations shown in Fig.14 reveal that both optimization methods fail to reach the set-point within the specified time when the reference is low ($y_r = 1$). This behavior can be attributed to the small gradient encountered by the TR method and the limited simplex size generated by the NM algorithm. Fig.15 illustrates this analyse. Indeed, if $y_r = 1$ the criterion curve presents a low value of the gradient between the two minima of the function J .

The curve shown in Fig.15 illustrates the slow convergence of the TR and NM optimization methods when the set-point is located close to the minimum of the nonlinear characteristic presented in Fig.10, the gradient becomes close to zero. Similarly, for the NM method, the simplex degenerates.

5.2.2 MIMO model 2

Let consider the MIMO model studied in section 4 using a polynomial structure with peer degree, in order to study non-convex optimization

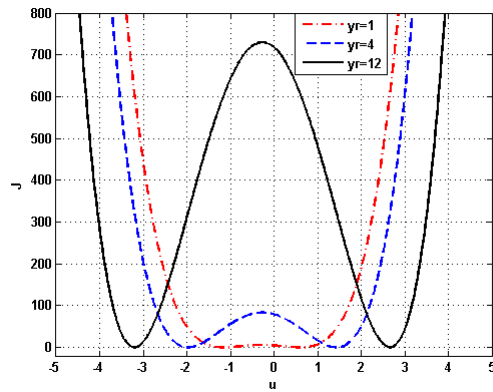


Figure 15: The objective function J for different set-points y_r . Source: created by the authors.

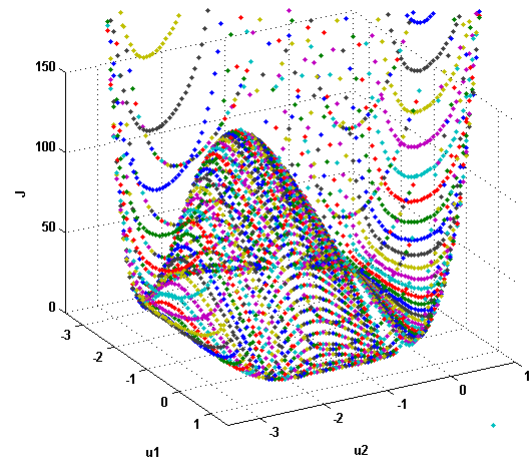


Figure 16: The criterion J for model (23) for the chosen set-points $y_{r1} = 2$ and $y_{r2} = 1$. Source: created by the authors.

problems for MIMO systems:

$$\begin{cases} \tilde{y}_1(k) = u_1(k) + 0.501u_1^2(k) + 0.289u_1^3(k) + 0.095u_1^4(k) \\ \tilde{y}_2(k) = u_2(k) + 0.4u_2^2(k) + 0.2u_2^3(k) + 0.1u_2^4(k) \end{cases} \quad (23)$$

where y_1, y_2 are the outputs, $\tilde{y}_1, \tilde{y}_2$ are the non-linear signals and u_1, u_2 are the controls.

The above parameters are chosen because they give better temporal behaviours and a clear interpretation of simulation results: $N_p = 5$; $N_c = 1$; $R = 0.5I$; $Q = I$; $N = 120$; $h = 0.1$.

In this example, the objective function curves have critical points, where the gradient is null, which can cause difficulty in seeking the optimal controls. This is illustrated by the criterion curve represented by Fig.16.

The criterion curve shows that for each set-point, it corresponds four pairs (u_1, u_2) of control values. In the following, the MIMO model is simulated by choosing set-points far from the critical point of the static nonlinear characteristic, in order to avoid the trapping of the optimization algorithm.

Fig.17 shows the temporal responses of the fractional MIMO model to the predictive controls. The constraints imposed to the system are: $-5 \leq u_1 \leq 2$ and $-3 \leq u_2 \leq 1$.

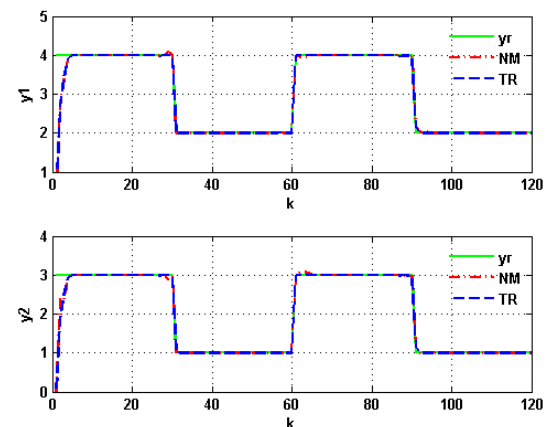


Figure 17: The temporal responses of model (23) with $-5 \leq u_1 \leq 2$ and $-3 \leq u_2 \leq 1$. Source: created by the authors.

It's clear from Fig.17 that when the set-points are far from the minimum of the nonlinearity curves, although the optimization problem is non-convex, both NM and TR methods give the same results in terms of tracking performances. Moreover, there is a clear difference on the predictive control curves traced in Fig.18.

The control curves plotted in Fig.18 show, on the one hand, the anticipatory effect of the predictive controls. On the other hand, it is shown the nonlinear aspect of the system because two

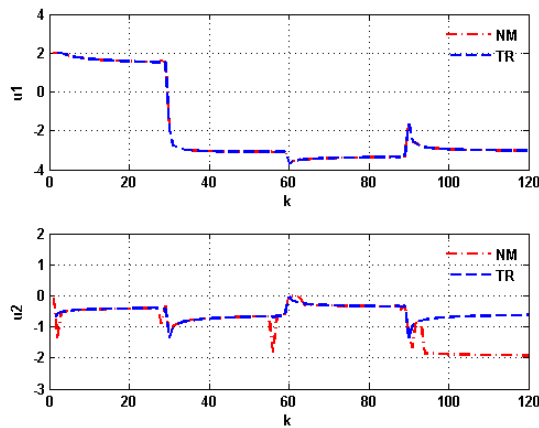


Figure 18: The predictive controls applied to model (23) with $-5 \leq u_1 \leq 2$ and $-3 \leq u_2 \leq 1$. Source: created by the authors.

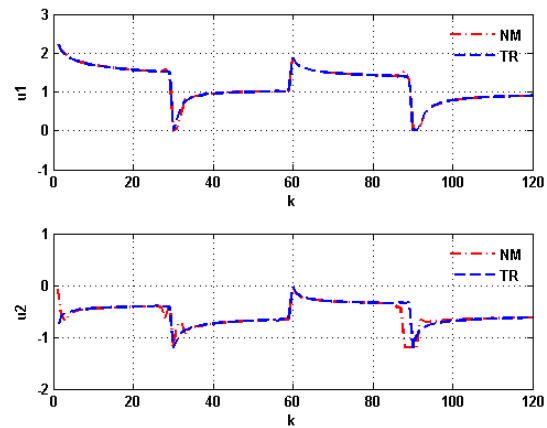


Figure 20: The predictive controls of model (23) with $0 \leq u_1 \leq 2.5$ and $-1.2 \leq u_2 \leq 0.5$. Source: created by the authors.

different controls values (specially u_2) generate the same outputs.

In the following we will consider other constraints to the system in order to improve energy consumption: $0 \leq u_1 \leq 2.5$, $-2 \leq u_2 \leq 0.5$, $|\Delta u_1| \leq 1.7$ and $|\Delta u_2| \leq 1.7$, where $\Delta u_1 = u_1(k) - u_1(k-1)$ and $\Delta u_2 = u_2(k) - u_2(k-1)$. The outputs and controls are plotted, respectively, in Fig.19 and Fig.20.

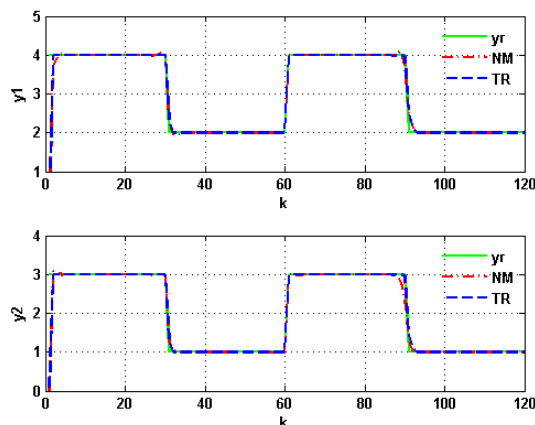


Figure 19: The temporal responses of model (23) with $0 \leq u_1 \leq 2.5$ and $-1.2 \leq u_2 \leq 0.5$. Source: created by the authors.

The two Fig.19 and Fig.20 show that reducing the variation intervals of the controls u_1 and u_2 improves the energy consumption which is very

important when controlling physical systems.

6 Conclusion

In this study, a predictive control strategy was developed for nonlinear systems described by Fractional Hammerstein Models (FHM). The TR and NM deterministic optimization methods were employed to solve both convex and non-convex optimization problems. Different nonlinear characteristics and transfer-function orders were considered to evaluate and compare the effectiveness of the two approaches. We have demonstrated that the efficiency of TR strongly depends on the choice of the initial conditions. Whereas, NM approach, based on simplex strategy is slightly dependent on initial conditions. Besides, the performances of the two methods depend on the prediction horizon. Furthermore, for SISO models, the NM method provides faster and more accurate results than the TR approach when the set-point is located far from the critical point, where the derivative of the nonlinear characteristic is zero. Moreover, when the set-point is close to that point, where the gradient is low for TR and the simplex degenerates for NM methods, both methods produce noticeable delays in the temporal responses. The best control solution is that consuming the least energy.

This work could be further extended by developing an optimization strategy that accounts for the cumulative nature of fractional systems and the performance degradation occurring near the global minimum.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

The authors wrote, reviewed and edited the content as needed and they have not utilised artificial intelligence (AI) tools. The authors take full responsibility for the content of the publication.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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