

Robust control of a parametrically uncertain cascade speed control system for a DC motor

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Abstract: - This paper is devoted to the development of a cascade control system ensuring high static and dynamic characteristics of an electric drive. The relevance of the work is determined by the need to solve problems that take into account the inaccuracy of knowledge about the controlled objects and external disturbances acting on them. The system is divided into loops with the main and auxiliary controllers, allowing for the independent and sequential adjustment of their parameters. Proportional-integral-derivative (PID) control is used to control the current in the inner loop and to control the speed in the outer loop, ensuring the robust properties of the system. The novelty of the robust control synthesis lies in the fact that several controllers ensure the stability of closed subordinate loops and the entire system as a whole. The model was investigated in the Matlab/Simulink environment using the m -synthesis program.

Key-Words: - robust control, DC motor, uncertain dynamic models, cascade control, PID-controller, m -synthesis, Matlab.

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1 Introduction

Currently, the cascade control principle, or the subordinate control principle [1], [2], is widely used in the development of electric drive control systems. This principle ensures high static and dynamic characteristics of the electric drive. The main

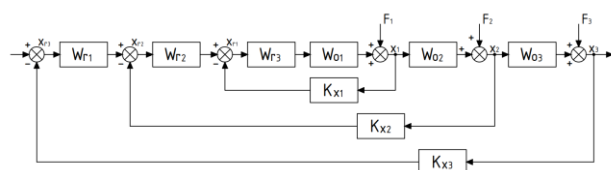


Fig. 1: Simplified structural diagram of the cascade control system

Source: created by the authors

The control object is represented as several series-connected links $W_{o1}(s)$, $W_{o2}(s)$, $W_{o3}(s)$, etc., with intermediate coordinates x_1 , x_2 , x_3 , ... and the controlled (output) coordinate X [7], [8]. Physically existing coordinates are used as the specified coordinates, such as current, voltage, EMF, magnetic flux, torque, speed, position, etc. A separate circuit is organized to control each coordinate, each with its own feedback and its own controller. The coordinate sensors are represented by links with transfer coefficients K_{x1} , K_{x2} , and K_{x3} , and the transfer

application areas of the subordinate control principle include energy, machine tool manufacturing, medicine, robotics, conveyor electric drives, and other areas [3], [4], [5], [6]. This principle is as follows (Figure 1).

functions of the controllers are designated as W_{r1} , W_{r2} , and W_{r3} .

Closed control loops form a system in which there is an inner control loop consisting of the W_{r1} controller of the first link of the object W_{o1} and a feedback circuit along the x_1 coordinate, a first outer loop including the inner loop, the second link of the object W_{o2} , the W_{r2} controller and a feedback circuit along the x_2 coordinate, and a second outer loop including the first outer loop, the third link of the control object W_{o3} , the W_{r3} controller and feedback along the x_3 coordinate, which for the case under consideration is the output coordinate, that is $x_3 = X$. The output signal of the controller of each outer loop is the reference signal for the subsequent loop contained within it. Thus, each inner control loop is subordinate to the corresponding outer one.

Cascade control systems are characterized by superior control quality compared to single-loop systems for the following reasons:

- disturbances $F_1(s)$, $F_2(s)$ arriving at parts of the plant, located closer to the input, are pre-parsed in

internal control loops before affecting the output coordinate (controlled variable);

- the transfer function of the plant's components included in the closed loop is often of lower order than the transfer function of the entire plant;

- measurement of internal coordinates and the presence of controllers allow for their limitation in dynamic operating modes;

- the presence of internal loops reduces the impact of changes in plant parameters on the dynamic performance of the control system.

The practical advantage of dividing the system into loops with primary and secondary controllers is that their parameters can be adjusted independently and sequentially. This is accomplished as follows:

1. The first internal loop is adjusted to the optimum absolute value.

2. When transitioning to the outer loop, the transfer function of the closed inner loop is simplified by approximating it with a first-order aperiodic link.

The modern development of control theory is characterized by the formulation and solution of problems that take into account the uncertainty of knowledge about control objects and the external disturbances acting on them. The problems of controller synthesis and state estimation, taking into account uncertainty in the object model and the characteristics of input signals, are among the central problems in modern control theory. Their importance is primarily due to the fact that virtually every engineering problem of control system design involves uncertainty (or error) in the object model (the mathematical model of the object, obtained on the basis of theory or through identification, differs from the actual technical system) and in the knowledge of the class of input disturbances.

2 Problem Formulation

The fundamental and fundamentally new idea behind robust control synthesis is to use multiple controllers to ensure the stability of closed subordinate loops and the entire system, not only for the nominal (without taking model errors into account) object, but also for any object belonging to a set of "disturbed" objects defined by an uncertainty class. Approaches to studying robust systems are described in [2], [9], [10].

The most widely used DC motor speed control systems are those in which the speed is regulated by changing the motor armature voltage using a controlled electrical converter (generator, controlled thyristor, or transistor pulse-width converter) with a slave motor current control loop. The motor excitation winding is powered by an additional uncontrolled converter [11], [12].

Matching the nominal voltages of the power supply network, the controlled converters, and the motor is ensured by a power transformer. To improve the motor armature current switching conditions by limiting its rate of change, a choke is installed in the armature circuit. Smoothing the armature current pulsations also helps reduce vibration and noise in the motor.

3 Problem Solution

Let's consider an electric drive control system containing two control loops: an inner armature current loop and an outer speed loop.

The current loop includes the electric drive's power section with an armature current output, a negative armature current feedback circuit, and an armature current controller. At the input of the armature current controller, the armature current reference voltage and the feedback voltage received from the current sensor are compared.

The motor speed loop includes a closed current loop, a negative motor speed feedback circuit, and a speed controller. At the input of the speed controller, the speed reference voltage supplied from the reference device output and the motor speed feedback voltage received from the tach generator are compared. The speed controller can have different control laws depending on the accuracy requirements.

The block diagram of the electric drive is shown in Figure 2. It meets the following assumptions: the control characteristic of the controlled converter is linear; the motor armature circuit current is continuous; there is no motor armature response; The moment of inertia reduced to the motor shaft is constant; the inertia of the current and speed sensors is not taken into account due to its smallness.

To construct a mathematical model in the form of a structural diagram, we will select a DC motor with separate excitation with the following data:

- nominal voltage, current, torque, and speed of the motor $U_{nom} = 220 \text{ V}$, $I_{nom} = 21 \text{ A}$,

- $M_{nom} = 23.5 \text{ H} \cdot \text{m}$, $\omega_{nom} = 162 \text{ rad / s}$;

- maximum permissible motor current and torque $I_{max} = 63 \text{ A}$, $M_{max} = 70 \text{ N} \cdot \text{m}$;

- gear ratio and moment of inertia of the motor $K_d = 0.83 \text{ rad / (V} \cdot \text{s)}$, $J = 0.15 \text{ kg} \cdot \text{m}^2$;

- resistance and electromagnetic time constant $R_d = 1.8 \text{ Ohm}$, $T_e = 0.022 \text{ s}$;

- maximum EMF, gain, and time constant of the power converter $E_{pmax} = 300 \text{ V}$, $k_p = 25$, $T_p = 0.01 \text{ s}$;

- the transfer coefficients of the current and speed sensors are $K_{oi} = 0.16 \text{ Ohm}$, $K_{os} = 0.032 \text{ V} \cdot \text{s/rad}$.

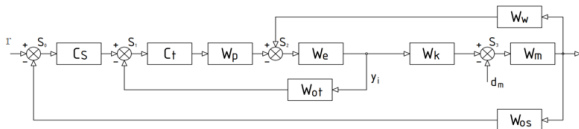


Fig. 2: Structural diagram of the DC motor speed control system with a slave armature current control loop

Source: created by the authors

The diagram shows the designations and transfer functions of the following system components:

Cs - speed controller,

Ct - current controller, Wos - speed sensor,

Wot - current sensor,

$Wp(s) = Kp / (Tp \cdot s + 1)$ - power semiconductor converter,

$We(s) = Ke / (Te \cdot s + 1)$ - motor armature circuit,

$Wk(s) = 1 / Kd$ - electromagnetic coupling of torque with armature current,

$Wm(s) = 1 / Js$ - mechanical part of the system,

dm - mechanical torque on the motor shaft, r - speed reference signal.

The block diagram corresponds to the following assumptions: the control characteristic of the controlled converter is linear; the motor armature circuit current is continuous; there is no motor armature reaction; the moment of inertia reduced to the motor shaft is constant; the inertia of the current and speed sensors is ignored due to its smallness. We neglect the influence of internal motor EMF feedback on the current loop due to its small impact. If it needs to be taken into account, using known rules for transforming structural diagrams, this circuit can be converted to a form in which the EMF feedback is moved outside the current loop.

3.1 Application of the m-synthesis program of the Matlab/Simulink environment for the system under study

In the system under study, the values of the link parameters have some uncertainty. We use the uncertain real parameters to represent these quantities in terms of their nominal or most probable value and the uncertainty range around this value using the ureal function:

$Kp = \text{ureal}('Kp', 25, 'Percentage', 30);$

$Tp = \text{ureal}('Tp', 0.01, 'Percentage', 40);$

$Ke = \text{ureal}('Ke', 0.555, 'Percentage', 30);$

$Te = \text{ureal}('Te', 0.022, 'Percentage', 40);$

$Km = \text{ureal}('Km', 1.2048, 'Percentage', 30);$

$Tm = \text{ureal}('Tm', 0.062, 'Percentage', 40);$

$Kd = \text{ureal}('Kd', 1.2048, 'Percentage', 20);$

$J = \text{ureal}('J', 0.15, 'Percentage', 20);$

$Kot = \text{ureal}('Kot', 0.16, 'Percentage', 10);$

$Kos = \text{ureal}('Kos', 0.032, 'Percentage', 10).$

The structural model of the armature current control loop contains three uncertain elements. For the design of the current regulator, it is necessary to simplify the uncertainty model while approximately preserving its overall variability. This can be accomplished using the ucover command. This command accepts an array of LTI responses |Pa| and a nominal response |Pn| and models the difference |Pa-Pn| as the multiplicative uncertainty in the dynamic system in percent (|ultidyn|).

To use |ucover|, first map the uncertain model |P| to a family of LTI % responses using the |usample| command. This command samples the uncertain elements in the %-uncertain system. It returns an array of LTI models, each representing one of the possible behaviors of the uncertain system. In this example, 60 sample values of |P| are generated, and a random number generator ensures repeatability of the Pa realizations. We then use the ucover function to cover all Pa realizations and reduce them to a simple uncertain model of the following form:

$$W_{sys} = P_r * (1 + W_t * \Delta),$$

where all uncertainty is concentrated in the "unmodeled dynamics" component (Delta(ultidynobject)). We select the nominal value of Pn as the center of the frequency response graphs and use a third-order shaping filter, Wt, to capture changes in the relative gap between Pa and Pcn as a function of frequency. After executing these commands, we find a stable approximation of the upper bound with minimum phase. The approximation process is graphically shown in Figure 3.

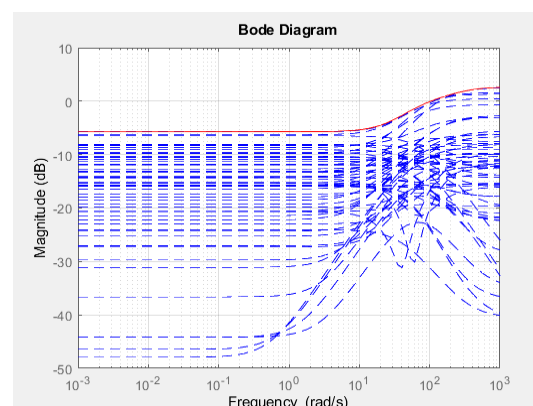


Fig. 3: Approximation of the frequency characteristics of a n object with a minimum phase

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The transfer function of the stable minimum-phase approximation of multiplicative uncertainty is $W_t(s) =$

$$(1.363s^3 + 296.7s^2 + 7683s + 4059) / (s^3 + 302.4s^2 + 1.47e04s + 7789).$$

3.2 Creating an open-loop current model.

Synthesis of a current controller

To design a robust controller using μ -synthesis for an uncertain setpoint W_{sys} , it is necessary to determine the open-loop current loop structure, select the target open-loop bandwidth $desBW$, and perform a sensitivity-minimizing calculation using a simplified uncertainty model. The current loop structure is shown in Figure 4.

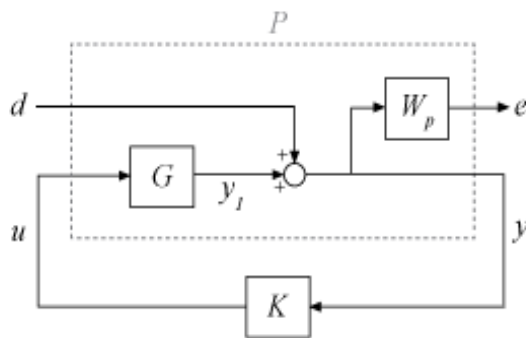


Fig. 4: Structural diagram of the armature current circuit

Source: created by the authors

The main signals are the disturbance signal d , the control signal u , and the output coordinate of the plant y . The W_{perf} filters reflect the frequency content of the disturbance signals, or, equivalently, the frequency ranges in which disturbances are observed and good noise rejection properties are required. The goal is to keep y close to zero, rejecting the disturbance d and minimizing the influence of noise. It is necessary to design a controller that maintains the gain from $[d]$ to y as small as possible. Given that

$$y = W_{perf} * 1/(1+PC) * d,$$

the transfer function of interest thus consists of performance- and noise-weighted versions of the sensitivity functions $1/(1+PC)$ and an additional sensitivity function $PC/(1+PC)$. We choose the performance weighting function W_{perf} as a first-order low-pass filter with a magnitude greater than 1 at frequencies below the desired closed-loop bandwidth:

$desBW = 6.6$ is the desired cutoff frequency for the closed-loop system;

$$W_{perf} = makeweight(300, desBW, 0.2).$$

The armature current controller can be designed using the automated μ syn command applied to an uncertain open-loop model, which is specified by

$$M: [K, CL_{perf}] = \mu\text{syn}(G, ny, nu).$$

This command synthesizes an unstructured, robust "black-box" controller for a system where the plant

contains some dynamic uncertainty. The controller also eliminates the effects of noise at the system output. The controller transfer function obtained from the calculation is usually of fairly high order. Therefore, the reduce command is used to attempt to reduce its order. A more efficient reduction in controller order can be achieved by using one of the μ -synthesis methods with a robust structured controller. For this, we'll use the same control structure, but restrict the controller structure to a state-space model. We will specify the controller order and the single-input, single-output PID structure using the command:

$$C0 = \text{tunablePID}('K', 'PID').$$

Controller synthesis is performed using the automated μ syn command, applied to an open-loop uncertain model, which is defined by

$$M: [CL, CL_{perf}] = \mu\text{syn}(CL0).$$

As a result of experimental calculations, a second-order PID controller of the following form was selected:

$$Cp(s) = (74.85s^2 + 6587s + 1.619e05) / (s^2 + 4.779e04s).$$

With $CL_{perf} = 6.5852e-01$, the plot shows that the best achieved robust performance is approximately 0.658. This result means that the closed-loop gain from d to e remains below 0.658, which is $1/0.658 = 1.518$ times the plant setpoint. Thus, the controller achieves robust performance targets for the full range of modeled uncertainty.

Robust performance can be investigated using analytical commands such as robgain and wcgainplot . For example, examine the worst-case gain of a closed-loop system:

$$wcg = \text{structwithfields}:$$

LowerBound: 3.0450e-01; UpperBound: 3.0511e-01; CriticalFrequency: 1.5323e+02.

To evaluate the behavior of the primary and secondary sensitivities, we present their frequency responses (Figure 5).

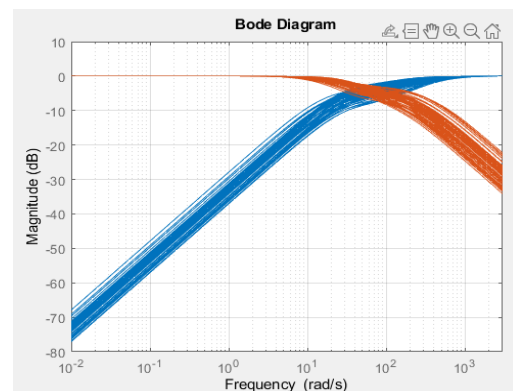


Fig. 5: Frequency characteristics of sensitivity and additional sensitivity

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Using the connect function, we determine the closed-loop transfer function of the armature current:

$F = \text{connect}(W_p, W_e, W_{ot}, C_p, \text{Sum1}, \text{Input}, \text{Output})$,

which we need for further calculations of the speed loop transfer function:

$V = \text{connect}(F, W_k, W_m, W_{os}, \text{Sum3}, \text{Inputdata}, \text{Outputdata})$.

We then calculate the nominal transfer function uV corresponding to V . To successfully design a speed controller, it is necessary to simplify the uncertainty model while approximately preserving its overall variability. This can be accomplished using the `ucover` command. The transfer function of the stable minimum-phase approximation of multiplicative uncertainty is:

$W_{1t}(s) = (2.231s^3 + 102.8s^2 + 714.7s + 1780) / (s^3 + 111.4s^2 + 1098s + 3434)$.

To design a robust speed loop controller for an uncertain V_{sys} setpoint, it is necessary to determine the open-loop speed controller structure, select the target closed-loop bandwidth $desBW$, and perform a sensitivity-minimizing calculation using a simplified uncertainty model. The loop structure is shown in Figure 4; it is similar to the current control loop structure.

We select the performance weighting function W_{perf} as a first-order low-pass filter with a magnitude greater than 1 at frequencies below the desired closed-loop bandwidth:

$W_{perf} = \text{makeweight}(300, desBW, 0.2)$,

where $desBW = 4.6$ is the desired cutoff frequency for the closed-loop speed controller.

We synthesize the speed PID controller similarly to the calculation presented above for the current controller. The resulting speed controller transfer function is written as

$C_s(s) = (1254s^2 + 4755s + 48.52) / (s^2 + 1.598e04s)$.

$CL_{perf} = 0.819$: the display shows that the best achieved stable efficiency is approximately 0.819. This result means that the gain from d to e remains below 0.819 to $1/0.819 = 1.22$, which is greater than the uncertainty specified in the plant.

$wcg = \text{struct with fields: LowerBound: } 6.2506e-01$;

$\text{UpperBound: } 6.2647e-01$;

$\text{CriticalFrequency: } 2.5046e-01$;

To summarize the study, we present the transient process in the speed loop for a single reference (Figure 6).

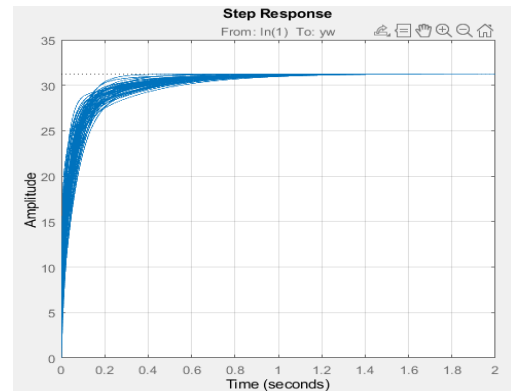


Fig. 6: Transient process of engine acceleration during starting

Source: created by the authors

From Figure 6, it is evident that the transient processes of all transfer functions of the studied uncertain model are aperiodic in the speed loop without overshoot for the adopted value of $desBW$.

4 Conclusion

The synthesis method discussed here preserved the well-known multi-loop structure of slave control systems, widely used in variable-speed DC and AC electric drives. This allowed for the arbitrary generation of loop quality indicators and the consideration of changes in the plant's parameters, which is an advantage. Robust system properties are achieved in this case through the use of the μ -synthesis technique, developed for cases with a fixed controller structure, such as a PID controller. Of course, the structure of robust systems with slave loops discussed here requires further research and the development of practical recommendations.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process The authors wrote, reviewed and edited the content as needed and they have not utilised artificial intelligence (AI) tools. The authors take full responsibility for the content of the publication.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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