

Integral-Equation Approach with Analytical Closed-Form ARL for DMEWMA Chart in ARMA Process: Enhancing Process-Change Detection

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Abstract: - This research establishes a definitive closed-form equation for the average run length (ARL) of the double modified exponentially weighted moving average (DMEWMA) control chart employed for the ARMA(p,q) process. The proposed formula is derived analytically and validated through comparison with the numerical integral-equation (NIE) method. To assess the agreement between the two methods, we examine the percentage of accuracy. To evaluate the monitoring capabilities of the DMEWMA chart, additional performance metrics—SDRL and MRL—are computed across various chart design configurations. We use RMI, AEQL, and PCI to examine how well the DMEWMA chart detects mean-process shifts when applied to the modified EWMA (MEWMA) and the EWMA charts. A real-world example of monthly unemployment-rate production-index data from Bangkok and southern Thailand demonstrates the effectiveness of the explicit ARL-based DMEWMA chart in detecting changes in process means. The findings demonstrate that the DMEWMA chart outperforms the MEWMA and EWMA charts in identifying process shifts.

Key-Words: - Autoregressive and Moving Average, Average Run Length, Double Modified Exponentially Weighted Moving Average, zero-state, process- shift detection, Numerical Integral equation.

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1 Introduction

In the present day, the Statistical Process control chart is more widely used than other tools for the control and detection of deviations in the production process. This is due to its high efficiency and ability to plainly display the results when the production process changes. When there is a significant movement in the process mean, the Shewhart control chart performs better than other chart types at identifying changes considerably, as demonstrated by the previous study, [1], [2], [3], [4]. In contrast, the Exponentially Weighted Moving Average control chart (EWMA) control chart is superior at identifying minor deviations in the mean, [3], [5], [6]. Subsequently, the Double Exponentially Weighted Moving Average control chart (DEWMA) was created by [7], which demonstrated superior efficacy in comparison to the original EWMA chart in subsequent investigations. A modified exponentially weighted moving average control chart (MEWMA) has been suggested as a method for the efficient identification of minor

alterations in the process by [8], [9]. A double modified exponentially weighted moving average control chart (DMEWMA) was recently developed in a study, [10]. This chart extends and incorporates the features of MEWMA to further enhance the ability to detect process abnormalities. These advancements have resulted in the continuous evolution of statistical control charts to accommodate the growing complexity of manufacturing processes and to facilitate the timely identification of abnormalities in both large and minor average changes.

It is common for observations to be normally distributed and independent. However, the characteristics of the application might appear non-normal in reality. For example, the number of defects is best suited to a Poisson distribution, while temporal data is best suited to an exponential distribution when the observations are correlated across process variables. This may lead to an elevated rate of false alarms. For instance, economic data, [11], health data, [12], and water toxicity data, [13], are all examples of stationary time series with

exponentially distributed errors, [14], [15]. Nevertheless, time series may be either stationary or non-stationary. Consequently, the objective of this research is to investigate the quality control aspects of non-stationary time series with exponentially distributed errors in which observations are correlated. Control charts are investigated using the Average Run Length (ARL), which is separated into two categories: ARL_1 (in-control ARL) when the process is under control and ARL_2 (out-of-control ARL) when the process is not under control. In general, the average run length (ARL) is a metric that is employed to evaluate the efficacy of control charts. Prior investigators have formulated numerous techniques to ascertain average run length. A widely utilized technique is Monte Carlo simulation, which is constrained by notably extended processing durations. Furthermore, the Markov chain method, [16], is a widely used approach for approximating the ARL value; however, it falls lacking in terms of convergence properties. In the numerical integral equations (NIE) technique, the ARL is determined by employing exact mathematical calculations. These calculations include the Gaussian rule, the trapezoidal rule, and Simpson's rule, among others. The fact remains, however, that there is no fundamental convergence theory, [17], [18], [19], [20]. Researchers have been motivated to investigate advanced mathematics to develop explicit formulas for calculating the ARL as a result of the limitations of the aforementioned methods. Several researchers have investigated this method and discovered that these explicit formulas are highly efficient and accurate, necessitating substantially less computing time than conventional methods. Explicit formulas for CUSUM control charts under correlated processes have been developed in earlier studies, [21], [22]. Subsequently, the studies, [23], [24], [25], focused on deriving and validating explicit formulas for the average run length (ARL) of EWMA control charts under autocorrelated processes, comparing their accuracy with that obtained from numerical integral equation methods. Subsequently, researchers extended this work by deriving explicit formulas for Extended EWMA, Modified EWMA, and DMEWMA control charts under various forms of data correlation, [26], [27], [28].

The author carried out research investigating the average run length formula for DMEWMA control charts for ARMA model data, which no other researchers have explicitly explored to this point. Moreover, the numerical integral equation approach was utilized as a reference for the proposed ARL of at the level of process mean variation. Furthermore,

the performance of DMEWMA, MEWMA, and EWMA control charts was additionally explored.

2 Materials and Methods

2.1 Process

In an ARMA(p,q) model, the sequences of Y_t can be expressed as:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q} \quad (1)$$

where ϕ_0 is the process mean, ϕ_i is the coefficient of the AR model ($-1 < \phi_i < 1$), θ_i is the coefficient of the MA model ($-1 < \theta_i < 1$) and $\varepsilon_t \sim \text{Exp}(\beta)$.

2.2 Control Charts

2.2.1 The Exponentially Weighted Moving Average Control Chart

The EWMA is a control chart that is appropriate for calculating moving averages of both historical and current data. It is capable of identifying process mean values that are less than 1.5σ . The EWMA control chart was initiated by [5]. The statistics relating to the EWMA control chart are listed below:

$$Z_t = \lambda Y_t + (1 - \lambda) Z_{t-1} \quad (2)$$

where λ is an exponential smoothing parameter, ($0 < \lambda \leq 1$), Y_t is a sequence of ARMA(p,q) model with exponential white noise.

The EWMA control chart's UCL and LCL are delineated as follows:

$$\mu \pm V_z \sigma \sqrt{\frac{\lambda}{2 - \lambda}}, \quad (3)$$

where the target mean is denoted by μ , σ is the standard deviation of the process, and V_z is the width of the control limit.

The stopping time (τ_b) is indicated as:

$$\tau_b = \{t \geq 0 : Z_t < a \text{ or } Z_t > b\} \quad (4)$$

where τ_b is the stopping time, and a and b are LCL and UCL, respectively.

2.2.2 The Modified Exponentially Weighted Moving Average Control Chart

A MEWMA control chart was incorporated in the study, [9]. The MEWMA control chart improves the EWMA chart's ability to detect differences in

processes. The MEWMA control chart can be represented as follows:

$$M_t = (1 - \lambda_1)M_{t-1} + \lambda_1 Y_t + r_1(Y_t - Y_{t-1}) \quad (5)$$

where M_0 is initial values and Y_t is a sequence of ARMA(p,q) model, r_1 is suitable constant. The upper and lower control limits of the MEWMA control charts are given by:

$$UCL / LCL = \mu_0 \pm V_M \sigma \sqrt{\frac{\lambda}{2 - \lambda} + \frac{2r_1(\lambda + r_1)}{2 - \lambda}} \quad (6)$$

where μ_0 and σ represent the mean and standard deviation of the process and V_M is the width of the appropriate control limit.

The stopping time of the MEWMA control chart (τ_d) is denoted as:

$$\tau_d = \{t \geq 0 : M_t < c \text{ or } M_t > d\} \quad (7)$$

where c and d are LCL and UCL, respectively.

2.2.3 The Double Modified Exponentially Weighted Moving Average Control Chart

The research, [10], created a DMEWMA control chart capable of identifying modifications in the process in a short amount of time. One way to define the statistics of the DMEWMA is as presented below.

$$D_t = (1 - \lambda_2)D_{t-1} + \lambda_2 M_t + r_2(M_t - M_{t-1}), \quad (8)$$

where $M_t = (1 - \lambda_1)M_{t-1} + \lambda_1 Y_t + r_1(Y_t - Y_{t-1})$ is the MEWMA statistic, λ_1 and λ_2 are exponential smoothing parameters ($0 < \lambda_1, \lambda_2 \leq 1$), r_1 and r_2 are appropriate constants, and Y_t is a sequence of ARMA(p,q) model at $t = 1, 2, 3, \dots$

A typical expression for the lower and upper control limits can be found as outlined below:

$$UCL / LCL = \mu_0 \pm V_D \sigma \sqrt{F_\sigma} \quad (9)$$

where V_D contains appropriate width of control limits and F_σ is a standard deviation with $r = r_1 = r_2$, $\lambda = \lambda_1 = \lambda_2$ and $\theta = (1 - \lambda)^2$ outlined as

$$F_\sigma = \frac{(r + \lambda)^4 + 4\lambda^2(r + \lambda)^2(r + \lambda - 1)^2}{(1 - \theta)^2} + \frac{4\lambda^2(r + \lambda)^2(r + \lambda - 1)^2(1 - \lambda)}{(1 - \theta)^2} - \frac{4\lambda^3(r + \lambda)(r + \lambda - 1)^3(1 - \lambda)}{(1 - \theta)^2} + \lambda^4(r + \lambda - 1)^4 \frac{(1 + \theta)}{(1 - \theta)^3} \quad (10)$$

The stopping time (τ_h) is denoted as:

$$\tau_h = \{t \geq 0 : D_t < g \text{ or } D_t > h\} \quad (11)$$

where τ_h is the stopping time, and g and h are LCL and UCL, respectively.

3 Approaches to Estimating the Average Run Length in Control Charts

3.1 The Exact Solution of ARL on the DMEWMA Control Chart for ARMA(p,q) Process

The time series data that are analyzed with the ARMA(p,q) model can be expressed as in Equation (1).

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q} \quad (12)$$

Therefore, the DMEWMA statistic for ARMA(p,q) model is given by:

$$D_t = (1 - \lambda_2)D_{t-1} + [(1 - \lambda_1)\lambda_2 + (1 - \lambda_1)r_2 - r_2]M_{t-1} + (\lambda_1\lambda_2 + r_1\lambda_2 + r_2\lambda_1 + r_1r_2)(\phi_0 + \varepsilon_t) + (\lambda_1\lambda_2 + r_1\lambda_2 + r_2\lambda_1 + r_1r_2) \sum_{i=1}^p \phi_i Y_{t-p} - (r_1\lambda_2 + r_1r_2)Y_{t-p} \quad (13)$$

In because the interval of D_t in the control process is $[g, h]$, and the initial value $D_0 = v$, we can identify

$$\psi = (\lambda_1\lambda_2 + r_1\lambda_2 + r_2\lambda_1 + r_1r_2)\phi_0 + (\lambda_1\lambda_2 + r_1\lambda_2 + r_2\lambda_1 + r_1r_2) \sum_{l=1}^p \phi_l A_{l-L} - (r_1\lambda_2 + r_1r_2)Y_0 + [(1 - \lambda_1)\lambda_2 + (1 - \lambda_1)r_2 - r_2]^* [(1 - \lambda_1)M_{-1} + \lambda_1 Y_0 + r_1(Y_0 - Y_{-1})], \quad (14)$$

then $p \leq (1 - \lambda_2)v + (\lambda_1\lambda_2 + r_1\lambda_2 + r_2\lambda_1 + r_1r_2)\varepsilon_1 + \psi \leq q$. (15)

After that, the control limit is transferred to the exponential residual term ε_t , as follows:

$$\frac{g - (1 - \lambda_2)v - \psi}{(\lambda_1\lambda_2 + r_1\lambda_2 + r_2\lambda_1 + r_1r_2)} \leq \varepsilon_1 \leq \frac{h - (1 - \lambda_2)v - \psi}{(\lambda_1\lambda_2 + r_1\lambda_2 + r_2\lambda_1 + r_1r_2)} \quad (16)$$

such that

$$P(LCL \leq \varepsilon_1 \leq UCL) = \int_{\frac{g - (1 - \lambda_2)v - \psi}{(\lambda_1\lambda_2 + r_1\lambda_2 + r_2\lambda_1 + r_1r_2)}}^{\frac{h - (1 - \lambda_2)v - \psi}{(\lambda_1\lambda_2 + r_1\lambda_2 + r_2\lambda_1 + r_1r_2)}} f(\varepsilon_1) d\varepsilon_1 \quad (17)$$

The ARL can be addressed through the application of the Fredholm integral equation, the ARL referred $\vartheta(v)$ can be expressed as

$$\begin{aligned}\vartheta(v) &= 1 + \frac{\frac{h-(1-\lambda_2)v-\psi}{(\lambda_1\lambda_2+r_1\lambda_2+r_2\lambda_1+r_1r_2)}}{\frac{g-(1-\lambda_2)v-\psi}{(\lambda_1\lambda_2+r_1\lambda_2+r_2\lambda_1+r_1r_2)}} \left[\frac{(1-\lambda)v+\psi}{(\lambda_1\lambda_2+r_1\lambda_2+r_2\lambda_1+r_1r_2)} f(x)dx \right. \\ &\quad \left. + \frac{r_2\lambda_1+r_1r_2}{(\lambda_1\lambda_2+r_1\lambda_2+r_2\lambda_1+r_1r_2)} x \right] \\ &= 1 + \frac{1}{(\lambda_1\lambda_2+r_1\lambda_2+r_2\lambda_1+r_1r_2)} * \\ &\quad \int_g^h \vartheta(z) f \left[\frac{z-(1-\lambda_2)v-\psi}{(\lambda_1\lambda_2+r_1\lambda_2+r_2\lambda_1+r_1r_2)} \right] dz.\end{aligned}\quad (18)$$

Using an exponential function, the explicit formula of ARL regarding the ARMA(p,q) model upon DMEWMA can be acquired by calculating the integral equation in Equation (18), as illustrated in the following:

$$\begin{aligned}\vartheta(v) &= 1 + \frac{e^{\frac{(1-\lambda_2)v+\psi}{\beta(\lambda_1\lambda_2+r_1\lambda_2+r_2\lambda_1+r_1r_2)}}}{\beta(\lambda_1\lambda_2+r_1\lambda_2+r_2\lambda_1+r_1r_2)} * \\ &\quad \int_g^h e^{\frac{-z}{\beta(\lambda_1\lambda_2+r_1\lambda_2+r_2\lambda_1+r_1r_2)}} \vartheta(z) dz.\end{aligned}\quad (19)$$

If $N = \int_g^h e^{\frac{-z}{\beta(\lambda_1\lambda_2+r_1\lambda_2+r_2\lambda_1+r_1r_2)}} \vartheta(z) dz$, consequently

$$\vartheta(v) = 1 + \frac{e^{\frac{(1-\lambda_2)v+\psi}{\beta(\lambda_1\lambda_2+r_1\lambda_2+r_2\lambda_1+r_1r_2)}}}{\beta(\lambda_1\lambda_2+r_1\lambda_2+r_2\lambda_1+r_1r_2)} * N. \quad (20)$$

Let $\eta = (\lambda_1\lambda_2 + r_1\lambda_2 + r_2\lambda_1 + r_1r_2)$, then

$$\vartheta(v) = 1 + \frac{e^{\frac{(1-\lambda_2)v+\psi}{\beta\eta}}}{\beta\eta} * N. \quad (21)$$

Moreover, N can be solved as follows:

$$\begin{aligned}N &= \int_g^h e^{\frac{-z}{\beta\eta}} \left(1 + \frac{e^{\frac{(1-\lambda_2)z+\psi}{\beta\eta}}}{\beta\eta} * N \right) dz \\ &= -\beta\eta(e^{\frac{-h}{\beta\eta}} - e^{\frac{-g}{\beta\eta}}) - \frac{e^{\frac{\psi}{\beta\eta}}}{\lambda_2} (e^{\frac{-h\lambda_2}{\beta\eta}} - e^{\frac{-g\lambda_2}{\beta\eta}}) N\end{aligned}\quad (22)$$

Afterwards,

$$\vartheta(v) = 1 - \left[\frac{e^{\frac{(1-\lambda_2)v+\psi}{\beta\eta}} (e^{\frac{-h}{\beta\eta}} - e^{\frac{-g}{\beta\eta}})}{1 + \frac{e^{\frac{\psi}{\beta\eta}}}{\lambda_2} (e^{\frac{-h\lambda_2}{\beta\eta}} - e^{\frac{-g\lambda_2}{\beta\eta}})} \right]. \quad (23)$$

In Equation 11, a will equal zero for one-sided DMEWMA;

hence, the explicit formula corresponding to ARL is as follows:

$$\vartheta(v) = 1 - \left[\frac{e^{\frac{(1-\lambda_2)v+\psi}{\beta\eta}} (e^{\frac{-h}{\beta\eta}} - 1)}{1 + \frac{e^{\frac{\psi}{\beta\eta}}}{\lambda_2} (e^{\frac{-h\lambda_2}{\beta\eta}} - 1)} \right]. \quad (24)$$

The explicit formula determining the ARL of the DMEWMA control chart on the ARMA(p,q) model is represented by $\vartheta(v)$ in Equations (23) and (24), respectively.

3.1.1 On the Existence and Uniqueness of Explicit Formulas

We demonstrate that the integral equation presented in Equation (18) has a solution, and that this solution is the only one of its kind in existence. First, we will outline:

$$T(\vartheta(v)) = 1 + \frac{e^{\frac{(1-\lambda_2)v+\psi}{\beta\eta}}}{\beta\eta} \int_g^h e^{\frac{-z}{\beta\eta}} \vartheta(z) dz. \quad (25)$$

Theorem 1. (Banach's fixed-point theorem)

Let $T[g, h]$ be the set comprising all of the continuous functions on a complete metric space (X, d) , and presume that $T: X \rightarrow X$ is a contraction mapping with contraction constant $0 \leq s < 1$; i.e.,

$$\|T(\vartheta_1) - T(\vartheta_2)\| \leq s \|\vartheta_1 - \vartheta_2\| \quad \forall \vartheta_1, \vartheta_2 \in X \quad (26)$$

Subsequently, $\psi(\cdot) \in X$ is unique at $T(\vartheta(v)) = \vartheta(v)$; i.e., it has a unique fixed point in X .

Proof: To demonstrate that T , as described in Equation (11), constitutes a contraction mapping for $\vartheta_1, \vartheta_2 \in T[g, h]$, we employ the inequality

$$\|T(\vartheta_1) - T(\vartheta_2)\| \leq s \|\vartheta_1 - \vartheta_2\|$$

$\forall \vartheta_1, \vartheta_2 \in T(g, h)$ with $0 \leq s < 1$.

(27)

$$\begin{aligned} \|T(\mathcal{G}_1) - T(\mathcal{G}_2)\|_{\infty} &= \sup_{v \in [g, h]} \left| F(v) \int_g^h (\mathcal{G}_1(z) - \mathcal{G}_2(z)) e^{\frac{-z}{\beta\eta}} dz \right| \\ &\leq \sup_{v \in [g, h]} \left\| \mathcal{G}_1 - \mathcal{G}_2 \right\|_{\infty} F(v) \left(e^{\frac{-g}{\beta\eta}} - e^{\frac{-h}{\beta\eta}} \right) \\ &= \left\| \mathcal{G}_1 - \mathcal{G}_2 \right\|_{\infty} \left| e^{\frac{-g}{\beta\eta}} - e^{\frac{-h}{\beta\eta}} \right| \sup_{v \in [g, h]} |F(v)| \leq s \left\| \mathcal{G}_1 - \mathcal{G}_2 \right\|_{\infty} \end{aligned} \quad (28)$$

where

$$s = \left| e^{\frac{-g}{\beta\eta}} - e^{\frac{-h}{\beta\eta}} \right| \sup_{v \in [g, h]} |F(v)| \quad (29)$$

$$\text{and } F(v) = \frac{e^{\frac{(1-\lambda_2)v+\psi}{\beta\eta}}}{\beta\eta}; \quad 0 \leq s < 1. \quad (30)$$

Consequently, as validated by Banach's fixed-point theorem, the solution is both existent and unique.

3.2 ARL Approximation of the DMEWMA Control Chart for ARMA(p,q) using the NIE Method

It is a well-known approach that may be utilized to validate the ARL by the use of the explicit formula. The estimated ARL is derived by utilizing the NIE method, which is derived by applying the composite midpoint rule, which is designated by $\mathcal{G}(v)$. The result of ARL achieved by this approach is extremely similar to that obtained by the proposed method, and it also obtained the shortest CPU time. The k linear equation system can be used to determine $\mathcal{G}(v)$ on the DMEWMA control chart.

This method is applied to the interval $[g, h]$, where the length of k is divided into intervals evenly, i.e.,

$h_j = \frac{h-g}{k}$ with the middle point of the j^{th} interval

$z_j = (j - \frac{1}{2})h_j + g$. From Equation (19), $\mathcal{G}_N(v)$ can be defined as:

$$\mathcal{G}_N(v) \approx 1 + \frac{1}{\eta} \sum_{j=1}^k h_j \cdot \mathcal{G}(z_j) f\left[\frac{z_j - (1-\lambda_2)v - \psi}{\eta}\right] \quad (31)$$

The process of establishing the threshold value

1. Locate the in-control ARL_0 value set at 370 and establish the control limit.
2. Choose the exponential smoothing parameters λ_1 and λ_2 , and the constant r_1 and r_2
3. Calculate the in-control ARL by applying Equation 23, MRL, and SDRL. Modify the control

limit value as required to accomplish the desired ARL_0 .

4. Calculate the in-control ARL by applying Equation 29.

Assessing the out-of-control ARL

1. Choose the exponential smoothing parameters λ_1 and λ_2 , and the constant r_1 and r_2
2. Utilize Equations 23 and 29 to calculate the ARL for the DMEWMA statistic.
3. Determine the ARL, MRL, and SDRL for various process phases.
4. Contrast the ARL values from (2).
5. Assess the efficacy of the DMEWMA control chart relative to the MEWMA and EWMA control charts.

This investigation contrasts the results of ARL_0 and ARL_1 by employing explicit formulas and the NIE procedure of ARMA(p,q) in conjunction with the DMEWMA control chart. The ARL's precision has been tested against the accuracy percentage, which may be determined by:

$$\%Acc = 100 - \left| \frac{\mathcal{G}(v) - \mathcal{G}_N(v)}{\mathcal{G}(v)} \right| \times 100\%. \quad (32)$$

The standard deviation of the run length (SDRL) and the median run length (MRL) are two additional statistical measures that are utilized in order to evaluate the performance of control charts, [29]. When calculating the SDRL and MRL values for a controlled process, the following formulas are utilized:

$$ARL_0 = \frac{1}{\rho_0}, SDRL_0 = \sqrt{\frac{1-\rho_0}{\rho_0^2}}, MRL_0 = \frac{\log(0.5)}{\log(1-\rho_0)}, \quad (33)$$

where ρ_0 represents a type I error. The ARL_0 value was determined to be 370 throughout the length of this investigation, and the $SDRL_0$, MRL_0 , values can be obtained by applying Equation 33.

To calculate $SDRL_1$, MRL_1 for an out-of-control situation, substitute ρ_0 with ρ_1 where ρ_1 represents the type II error. In order to monitor changes in the process mean, the control chart that is most useful will display the values that are the lowest for ARL_1 , $SDRL_1$, and MRL_1 . Additionally, the relative mean index (RMI) is utilized to assess the performance effectiveness of the MEWMA control chart in comparison to the standard EWMA control chart, [30]. This statistic indicates that the control chart

exhibits rapid and reliable efficacy in identifying shifts when the RMI is at its minimum. RMI is defined as

$$RMI(c) = \frac{1}{n} \sum_{i=1}^n \left[\frac{ARL_i(c) - ARL_i(m)}{ARL_i(m)} \right] \quad (34)$$

where $ARL_i(c)$ represents the ARL of the control chart for the shift size of row i , and $ARL_i(m)$ is the ARL that is the shortest across all control charts for the shift size of row i .

In addition, performance metrics can be employed to evaluate the effectiveness of a control chart across an interval of changes ($\delta_{\min} \leq \delta \leq \delta_{\max}$). Additionally, the average extra quadratic loss (AEQL) may indicate the average additional loss that is the result of an out-of-control condition. The average difference between the desired values and the observed values during out-of-control periods could be determined. The following is the calculation of AEQL: [31].

$$AEQL = \frac{1}{\Delta} \sum_{\delta_i=\delta_{\min}}^{\delta_{\max}} (\delta_i^2 \times ARL(\delta_i)) \quad (35)$$

where δ represents the change in the process, and Δ represents the sum of the number of divisions from $\delta_{\min}$ to $\delta_{\max}$. In this study, $\Delta = 7$ is determined from $\delta_{\min} = 0.004$ to $\delta_{\max} = 1.00$. The optimal performance is achieved by the control chart with the lowest AEQL values. The Performance Comparison Index (PCI) serves as a measure to assess the effectiveness of different control charts. The PCI is the proportion of the AEQL to identify the most efficient monitoring chart, indicated by the minimal AEQL. The equation for the PCI is presented as follows:

$$PCI = \frac{AEQL}{AEQL_{lowest}} \quad (36)$$

4 Numerical Results

A comparison of the ARL using explicit formulas and the NIE approach for the ARMA(2,1) process on the one-sided DMEWMA control chart presented in Table 1 (Appendix) for the various choices of ϕ at the change levels of 0.004, 0.008, ..., 1.0 with $r_1 = 0.5$, $r_2 = 1.5$, $\lambda_1 = 0.05$, $\lambda_2 = 0.05$, 0.1, $\phi_0 = 1.5$, $\theta_1 = 0.1$, $ARL_0 = 370$. The analysis revealed that, across all levels of change, the ARL values obtained from the explicit formula and the NIE method were nearly identical. When expressed in terms of accuracy, the percentage reached 100% at every

level. Furthermore, increasing λ_2 from 0.05 to 0.1 produced results that remained consistently close between the two approaches. In addition, the computation time showed that the explicit formula required less than one second to process. Table 2 (Appendix) displays the results of an ARL comparison using explicit formulas and the NIE approach for the ARMA(1,2) process implemented on a two-sided DMEWMA control chart for a variety of choices of θ with $g = 0.1$, $r_1 = 0.5$, $r_2 = 1.5$, $\lambda_1 = 0.05$, $\lambda_2 = 0.05$, 0.1, $\phi_0 = 1.5$, $\phi_1 = 0.1$.

The analysis showed that, for the ARMA(1,2) model applied to the two-sided DMEWMA control chart, the ARL values obtained using the explicit formula and the NIE method were remarkably close, with an accuracy rate of 100%. The results remained consistent when λ_2 was increased from 0.05 to 0.1 and were in full agreement with those presented in Table 1 (Appendix).

The DMEWMA control chart is contrasted with the MEWMA and EWMA control charts, with the findings presented in Table 3 (Appendix). This analysis is conducted following the revelation that the explicit formula of ARL for the DMEWMA control chart is precise. The ARL comparison for the ARMA(1,3) process is shown in Table 3 (Appendix), which includes control charts for one-sided and two-sided EWMA, MEWMA, and DMEWMA. The studied models were set with $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\phi_0 = 0.5$, $\phi_1 = 0.1$, $\theta_1 = 0.1$, $\theta_2 = 0.2$, $\theta_3 = 0.1$, $ARL_0 = 370$.

The research findings indicated that for a one-sided control chart with $g = 0$, DMEWMA control chart exhibited optimal performance with r_1 , $r_2 = 4$, demonstrating the lowest RMI and AEQL values, including a PCI value of 1. The research results indicated that DMEWMA control chart exhibited optimal performance when r_1 , r_2 were both equal to 4 in the analysis of a two-sided control chart. The results for the RMI, AEQL, and PCI values were the same as those for the one-sided control chart. Subsequently, the ARMA(2,2) model was implemented in Table 4 (Appendix) to verify the efficacy of the control chart. The ARL comparison for the ARMA(2,2) process with $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\phi_0 = 0.5$, $\phi_1 = 0.2$, $\phi_2 = 0.3$, $\theta_1 = 0.1$, $\theta_2 = 0.2$ and $ARL_0 = 370$ is provided in Table 4 (Appendix) on both one-sided and two-sided EWMA, MEWMA, and DMEWMA control charts. Additionally, the ARL_0 value of 370 is mentioned in order to demonstrate the outcome of the comparison of the performance of control charts. In accordance with

the findings in Table 3 (Appendix), the research results indicate that the DMEWMA control chart is the most effective in detecting changes at the r_1 , r_2 levels of 4, as well as in terms of the RMI, AEQL, and PCI values. The ARL comparison for the ARMA(2,2) process with $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\phi_0 = 0.5$, $\phi_1 = 0.2$, $\phi_2 = 0.3$, $\theta_1 = 0.1$, $\theta_2 = 0.2$ and $ARL_0 = 370$ is provided in Table 4 (Appendix) on both one-sided and two-sided EWMA, MEWMA, and DMEWMA control charts. Additionally, the ARL_0 value of 370 is mentioned in order to demonstrate the outcome of the comparison of the performance of control charts.

4.1 Application

For the real-data application, the researchers focused on regional unemployment rates. The first dataset examined the unemployment rate in Bangkok, while the second analyzed the unemployment rate in the Southern region. Both datasets consisted of monthly observations spanning from January 2012 to December 2021. The study evaluated the performance of the DMEWMA control chart in comparison with the MEWMA and EWMA control charts using the explicit ARL formula, as shown in Appendix in Table 5 and Table 6. The values of λ_1 and λ_2 are 0.1 and 0.2, respectively. The coefficient parameters for the ARMA(1,1) model are determined through maximum likelihood estimation.

In the first dataset, the coefficient parameters for ARMA(1,1) are determined through the maximum likelihood estimation; $\hat{\phi}_1 = 0.983$, $\hat{\theta}_1 = 0.459$. The exponential parameter of this data set is 0.2827. The following can be illustrated by applying the parameters of this forecasting model: $\hat{Y}_t = 0.983Y_{t-1} - 0.459\varepsilon_{t-1}$. In the second dataset, the coefficient parameters for ARMA(1,1) are also determined using the maximum likelihood estimation; $\hat{\phi}_1 = 0.996$, $\hat{\theta}_1 = 0.621$. This data set has an exponential parameter of 0.3335. The following can be illustrated by applying the parameters of this forecasting model: $\hat{Y}_t = 0.996Y_{t-1} - 0.621\varepsilon_{t-1}$, as seen in Table 7 (Appendix). In Table 5 (Appendix), the ARL comparison for the ARMA(1,1) process of unemployment rate in Bangkok was presented on one-sided and two-sided EWMA, MEWMA, and DMEWMA control charts with $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\phi_1 = 0.983$, $\theta_1 = 0.459$, $\beta = 0.2827$ and $ARL_0 = 370$. Additionally, the ARL comparison for the ARMA(1,1) process of unemployment rate in the Southern region is also shown in Table 6

(Appendix) on one-sided and two-sided EWMA, MEWMA, and DMEWMA control charts with $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\phi_1 = 0.996$, $\theta_1 = 0.621$, $\beta = 0.3335$ and $ARL_0 = 370$. The results of the analysis in Table 5 and Table 6 (Appendix) indicate that both one- and two-sided control charts yield comparable outcomes. The DMEWMA control chart is the most effective in detecting changes when $r_1 = 4$ as demonstrated by the lowest ARL value and the lowest RMI and PCI values of 1. The comparisons of the RMI, AEQL, and PCI values for each control chart presented in Appendix in Table 5 and Table 6 can be more clearly visualized in Appendix in Figure 1 and Figure 2, respectively. Furthermore, Figure 3 (Appendix) illustrates the shift-detection performance of each control chart. It can be observed that the DMEWMA control chart provides the fastest detection, identifying the process shift as early as the second observation. Consequently, it can be concluded that the simulation results in Table 3 and Table 4 (Appendix) are in accordance with the results that were applied to the actual data in Table 5 and Table 6 (Appendix).

5 Conclusion

This study used the explicit ARL calculation algorithm to figure out the ARMA process on the DMEWMA control chart. The formula cut down on computing time by a lot, showing that it was quite accurate and efficient. We compared the explicit formula and the numerical integral equation (NIE) method using the percentage of accuracy (%Acc) criterion for ARL outcomes. The reliability of the explicit formula was validated by the lack of substantial discrepancies between the two ways. The results also show that the explicit formula substantially reduced computing time compared to the NIE technique. The results show that the DMEWMA control chart outperforms the other types in detecting changes in the process, as compared to the performance of DMEWMA, MEWMA, and EWMA control charts. The lowest values for RMI, AEQL, and PCI show that this is the best way to find changes in the process mean.

The study corroborates that the simulation validation results align with real data applications, hence reinforcing the conclusion that the DMEWMA control chart operates at peak efficiency. Subsequent studies may focus on formulating a precise equation for ARL values within the DMEWMA control chart to aid in the innovation of novel control charts or to explore its

potential utility in other models. For future work, deriving explicit formulas for the variance and full distribution of the run length would be valuable, as the current study only provides SDRL and MRL using approximations from the ARL.

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Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work, the author(s) utilized artificial intelligence (AI) technologies solely for the purpose of linguistic refinement and grammatical correction. Following the application of these tools, the author(s) comprehensively reviewed, verified, and edited the resulting text. The author(s) maintain full accountability and responsibility for the accuracy, integrity, and final content of the publication.

References:

- [1] W. A. Shewhart, *Economic control of quality of manufactured product*, D. Van Nostrand Company, Inc., New York, 1923, [Online]. <https://archive.org/details/in.ernet.dli.2015.150272/mode/2up> (Accessed Date: January 10, 2026).
- [2] J.M. Lucas, M.S. Saccucci, Exponentially weighted moving average control schemes: properties and enhancements, *Technometrics*, Vol. 32, No. 1, 1990, pp.1-29. <https://doi.org/10.2307/1269841>.
- [3] C. M. Borror, D. C. Montgomery, G. C. Runger, Robustness of the EWMA control chart to Non-normality. *Journal of Quality Technology*, Vol.31, 1999, pp. 309-316. <https://doi.org/10.1080/00224065.1999.11979929>.
- [4] Knoth, S. (2007). Accurate ARL calculation for EWMA control charts monitoring normal mean and variance simultaneously, *Sequential Analysis*, 26, 2007, pp.251-263. <https://doi.org/10.1080/07474940701404823>.
- [5] S. W. Roberts, Control chart tests based on geometric moving average, *Technometrics*, Vol 1, No. 3, 1959, pp.239-250. <https://doi.org/10.2307/1266443>.
- [6] C. W. Champ, S. E. Rigdon, A comparison of the Markov chain and the integral equation approaches for evaluating the run length distribution of quality control charts. *Communications in statistics. Simulation and computation*, Vol. 20, No. 1, 1991, pp.191-204. <https://doi.org/10.1080/03610919108812948>.
- [7] S.E. Shamma, R.W. Amin, A.K. Shamma, A. K., A double exponentially weighted moving average control procedure with variable sampling intervals, *Communications in Statistics - Simulation and Computation*, Vol. 20, No. 2-3, 1991, pp. 511–528.
- [8] A. K. Patel, J. Divecha, Modified exponentially weighted moving average (EWMA) control chart for an analytical process data, *Journal of Chemical Engineering and Materials Science*, Vol. 2, No. 1, 2011, pp.12-20, [Online]. <https://academicjournals.org/journal/JCEMS/article-full-text-pdf/466796E1469> (Accessed Date: January 10, 2026).
- [9] N. Khan, M. Alsam, CH. Jun, Design of a control chart using a MEWMA statistics, *Quality and Reliability Engineering International*, Vol. 33, No. 5, 2017, pp.1095-1104. <https://doi.org/10.1002/qre.2102>.
- [10] V. Alevizakos, K. Chatterjee, C. Koukouvinos, Modified EWMA and DEWMA control charts for process monitoring. *Communications in Statistics-Theory and Methods*, Vol. 51, No.21, 2021, pp.7390-7412. <https://doi.org/10.1080/03610926.2021.1872642>.
- [11] C.C. Torng, PH. Lee, NY. Liao, An economic-statistical design of double sampling $\bar{X}$ control chart, *International Journal of Production Economics*, Vol. 120, No. 2, 2009, pp.495-500. <https://doi.org/10.1016/j.ijpe.2009.03.013>.
- [12] P.S. Rao,Ch. Ratnam, Health monitoring of welded structures using statistical process control, *Mechanical Systems and Signal Processing*, Vol. 27, 2012, pp. 683-695. <https://doi.org/10.1016/j.ymssp.2011.09.023>.
- [13] E.M. Smeti,D.E. Koronakis,S.K. Golfinopoulos, Control charts for the toxicity of finished water—Modeling the structure of toxicity, *Water Research*, Vol. 41, No. 12, 2007, pp. 2679-2689. <https://doi.org/10.1016/j.watres.2007.02.036>.
- [14] M. Ibazizen, H. Fellag, Bayesian estimation of an AR (1) process with exponential white noise, *Statistics*, Vol. 37, No. 5, 2003, pp.365–372. <https://doi.org/10.1080/0233188031000078042>.

- [15] G. N. Farah, B. Lindner, Exponentially distributed noise—its correlation function and its effect on nonlinear dynamic, *Journal of Physics A: Mathematical and Theoretical*, Vol.54, 2021, 035003.
<https://doi.org/10.1088/1751-8121/abd2fd>
- [16] D. Brook, D. A. Evans, An approach to the probability distribution of Cusum run length, *Biometrika*, Vol. 59, No. 3, 1972, pp.539-549. <https://doi.org/10.1093/biomet/59.3.539>.
- [17] P. Paichit, W. Peerajit, The average run length for continuous distribution process mean shift detection on a MEWMA control chart, *Asia-Pacific Journal of Science and Technology*, Vol. 27, No. 6, 2022, APST-27-06-21.
<https://doi.org/10.14456/apst.2022.103>
- [18] W. Peerajit, Approximating the ARL to Monitor Small Shifts in the Mean of an AR Fractionally Integrated with an exogenous variable Process Running on an EWMA Control Chart, *WSEAS Transactions on Systems*, Vol.23, 2024, 176-187.
- [19] P. Paichit, W. Peerajit, Development of a Numerical Approach to Evaluate the Performance of the CUSUM Control Chart for a Long Memory SARFIMA Model, *WSEAS Transactions on Systems and Control*, Vol. 20, 2025, pp. 172-184.
- [20] P. Wilasinee, Gauss-Legendre Numerical Integrations for Average Run Length Running on EWMA Control Chart with Fractionally Integrated MAX Process, *WSEAS Transactions on Mathematics*, Vol. 23, 2024, pp. 579 – 590.
- [21] K. Petcharat, The effectiveness of CUSUM control chart for trend stationary seasonal autocorrelated data, *Thailand Statistician*, Vol 20, No. 2, 2022, pp. 475-488, [Online].
<https://ph02.tci-thaijo.org/index.php/thaistat/article/view/246353>, (Accessed Date: January 15, 2026).
- [22] S. Phanyaem, Explicit formulas and numerical integral equation of ARL for SARX (P,r)_L model based on CUSUM chart, *Mathematics and Statistics*, Vol. 10, No. 1, 2022, pp. 88-99, [Online].
https://www.hrpub.org/journals/article_info.php?aid=11654 (Accessed Date: January 15, 2026).
- [23] W. Suriyakat, K. Petcharat, Exact run length computation on EWMA control chart for stationary moving average process with exogenous variables. *Mathematics and Statistics*, Vol.10, No.3, 2022, pp. 624-635, [Online].
https://www.hrpub.org/journals/article_info.php?aid=12172 (Accessed Date: January 15, 2026).
- [24] S. Phanyaem, Precise Average Run Length of an Exponentially Weighted Moving Average Control Chart for Time Series Model, *Thailand Statistician*, Vol 22, No. 4, 2024, pp. 909-925, [Online].
<https://ph02.tci-thaijo.org/index.php/thaistat/article/view/256080> (Accessed Date: January 15, 2026).
- [25] D. Polyeam, S. Phanyaem, Application of EWMA Control Chart for Analyzing Changes in SAR(P)L Model with Quadratic Trend, *Mathematics and Statistics*, Vol. 13, No. 3, 2025, pp. 163-174, [Online].
https://www.hrpub.org/journals/article_info.php?aid=15192 (Accessed Date: January 15, 2026).
- [26] T. Muangngam, Y. Areepong, S. Sukparungsee, Analytic Solution to ARL Integral Equation of Extended EWMA Control Chart for Seasonal ARX Model, *WSEAS Transactions on Mathematics*, Vol. 24, 2025, pp. 462 – 476.
- [27] K. Sittikorn, Y. Areepong, S. Sukparungsee, R. Sunthornwat, The Accurate Results of Average Run Length on Modified EWMA Control Chart for the First-Order Moving Average Process with Exogenous Variables Models, *Thailand Statistician*, Vol. 22, No.1, 2024, pp. 63-75, [Online].
<https://ph02.tci-thaijo.org/index.php/thaistat/article/view/252223> (Accessed Date: January 15, 2026).
- [28] J. Neammai, S. Sukparungsee, and Y. Areepong, Explicit Analytical Form for the Average Run Length of Double-Modified Exponentially Weighted Moving Average Control Charts Through the MA(q) Process and Applications, *Symmetry*, Vol.17, No.2, 2025, Article number 238.
<https://doi.org/10.3390/sym17020238>.
- [29] A. Fonseca, PH. Ferreira, DC. Nascimento, R. Fiaccone, CU. Correa, AG. Piña, F. Louzada, Water Particles Monitoring in the Atacama Desert: SPC approach Based on proportional data, *Axioms*, Vol.10, No.3, 2021, pp. 154.
<https://doi.org/10.3390/axioms10030154>.
- [30] A. Tang, P. Castagliola, J. Sun, X. Hu, Optimal design of the adaptive EWMA chart for the mean based on median run length and expected median run length. *Quality*

Technology & Quantitative Management,
Vol. 16, No. 4, 2018, pp. 439-458.
<https://doi.org/10.1080/16843703.2018.1460908>.

- [31] V. Alevizakos, K. Chatterjee, C. Koukouvinos, The triple exponentially weighted moving average control chart, *Quality Technology & Quantitative Management*, Vol. 18, No. 3, 2021, pp.326-354.
<https://doi.org/10.1080/16843703.2020.1809063>.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Saowanit Sukparungsee has implemented the methodology and simulation.
- Yupaporn Areepong has organized the writing-original draft, conceptualization, and validation.

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Conflicts of Interest

The authors declare no conflict of interest.

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APPENDIX

Table 1. ARL comparison using explicit formulas and NIE method for ARMA(2,1) process on one-sided DMEWMA control char for different choices of ϕ with $r_1 = 1.5$, $r_2 = 2.5$, $\lambda_1 = 0.05$, $\phi_0 = 1.5$, $\theta_1 = 0.1$, $ARL_0 = 370$

λ_2	Coefficients of process			Methods	Shift size (δ)						
	ϕ_1	ϕ_2	h		0.004	0.008	0.01	0.04	0.08	0.1	1.0
0.05	0.1	0.1	2.165558	Explicit	110.002	64.9205	53.9611	15.7897	8.51519	7.02498	1.5823
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	110.002	64.9205	53.9611	15.7897	8.51519	7.02498	1.5823
				CPU _{NIE}	(1.688)	(1.672)	(1.687)	(1.672)	(1.672)	(1.672)	(1.672)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	0.2	0.2	1.95679	Explicit	107.080	62.8938	52.2164	15.2227	8.20769	6.77227	1.54439
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	107.080	62.8938	52.2164	15.2227	8.20769	6.77227	1.54439
				CPU _{NIE}	(1.640)	(1.657)	(1.672)	(1.659)	(1.672)	(1.641)	(1.641)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.1	0.1	2.653525	Explicit	116.607	69.5210	57.9270	17.0850	9.21760	7.60206	1.66895
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	116.607	69.5210	57.9270	17.0850	9.21760	7.60206	1.66895
				CPU _{NIE}	(1.687)	(1.672)	(1.656)	(1.656)	(1.687)	(1.657)	(1.703)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	0.2	0.2	2.39697	Explicit	113.223	67.1392	55.8691	16.4082	8.85033	7.30032	1.62367
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	113.223	67.1392	55.8691	16.4082	8.85033	7.30032	1.62367
				CPU _{NIE}	(1.875)	(1.875)	(1.875)	(1.875)	(1.813)	(1.829)	(1.875)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00
0.10	0.1	0.1	2.211325	Explicit	109.845	64.7920	53.8475	15.7525	8.49672	7.01049	1.58189
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	109.845	64.7920	53.8475	15.7525	8.49672	7.01049	1.58189
				CPU _{NIE}	(1.890)	(1.844)	(1.766)	(1.750)	(1.797)	(1.860)	(1.843)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.1	0.1	2.71814	Explicit	116.779	69.6272	58.0163	17.1123	9.23245	7.61432	1.67100
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	116.779	69.6272	58.0163	17.1123	9.23245	7.61432	1.67100
				CPU _{NIE}	(1.844)	(2.047)	(1.859)	(1.813)	(1.843)	(1.844)	(1.891)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.1	0.1	2.71814	Explicit	116.779	69.6272	58.0163	17.1123	9.23245	7.61432	1.671
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	116.779	69.6272	58.0163	17.1123	9.23245	7.61432	1.671
				CPU _{NIE}	(1.859)	(1.828)	(1.953)	(1.875)	(1.891)	(1.875)	(1.859)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.1	0.2	3.34608	Explicit	125.007	75.5109	63.1195	18.8075	10.1497	8.36639	1.78002
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	125.007	75.5109	63.1195	18.8075	10.1497	8.36639	1.78002
				CPU _{NIE}	(1.843)	(1.813)	(1.860)	(1.828)	(1.813)	(1.828)	(1.828)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Note: The numerical results in parentheses are computational times in seconds

Source: created by the authors.

Table 2. ARL comparison using explicit formulas and the NIE method for ARMA(1,2) process on two-sided DMEWMA control chart for different choices of θ with $g = 0.1$, $r_1 = 1.5$, $r_2 = 2.5$, $\lambda_1 = 0.05$
 $\phi_0 = 1.5, \phi_1 = 0.1, ARL_0 = 370$.

λ_2	Coefficients of process			Methods	Shift size (δ)						
	θ_1	θ_2	h		0.004	0.008	0.01	0.04	0.08	0.1	1.0
0.05	0.2	0.2	3.35818	Explicit	122.464	73.6927	61.5449	18.3051	9.89287	8.16148	1.76537
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	122.464	73.6927	61.5449	18.3051	9.89287	8.16148	1.76537
				CPU _{NIE}	(1.828)	(1.859)	(1.843)	(1.859)	(1.844)	(1.843)	(1.860)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00
		0.3	3.70908	Explicit	126.899	76.9107	64.3456	19.2456	10.4022	8.57909	1.82599
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	126.899	76.9107	64.3456	19.2456	10.4022	8.57909	1.82599
				CPU _{NIE}	(1.844)	(1.860)	(1.828)	(1.875)	(1.860)	(1.843)	(1.844)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.2	0.2	2.26849	Explicit	108.277	63.7216	52.9314	15.4747	8.35798	6.9008	1.5759
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	108.277	63.7216	52.9314	15.4747	8.35798	6.9008	1.5759
				CPU _{NIE}	(1.844)	(1.860)	(1.859)	(1.828)	(1.891)	(1.828)	(1.875)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00
		0.3	2.500209	Explicit	111.435	65.8966	54.8013	16.0803	8.68626	7.17059	1.61682
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	111.435	65.8966	54.8013	16.0803	8.68626	7.17059	1.61682
				CPU _{NIE}	(1.703)	(1.671)	(1.703)	(1.672)	(1.672)	(1.719)	(1.656)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00
0.10	0.2	0.2	3.454925	Explicit	123.223	74.2592	62.0401	18.4701	9.97985	8.23185	1.77291
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	123.223	74.2592	62.0401	18.4701	9.97985	8.23185	1.77291
				CPU _{NIE}	(1.687)	(1.687)	(1.672)	(1.687)	(1.688)	(1.656)	(1.688)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00
		0.3	3.82487	Explicit	128.023	77.7435	65.0732	19.4879	10.5297	8.68212	1.83672
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	123.223	77.7435	65.0731	19.4879	10.5297	8.68212	1.83672
				CPU _{NIE}	(1.687)	(1.703)	(1.672)	(1.688)	(1.656)	(1.703)	(1.703)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.2	0.2	2.3171	Explicit	108.174	63.6704	52.8910	15.4664	8.35477	6.89857	1.57647
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	108.174	63.6703	52.8910	15.4664	8.35477	6.89857	1.57647
				CPU _{NIE}	(1.703)	(1.687)	(1.688)	(1.687)	(1.688)	(1.672)	(1.719)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00
		0.3	2.55769	Explicit	111.520	65.9662	54.8631	16.1025	8.69862	7.18084	1.61853
				CPU _{Exp}	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
				NIE	111.520	65.9662	54.8631	16.1025	8.69862	7.18084	1.61853
				CPU _{NIE}	(1.703)	(1.704)	(1.656)	(1.703)	(1.687)	(1.672)	(1.687)
				%Acc	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Note: The numerical results in parentheses are computational times in seconds

Source: created by the authors.

Table 3. ARL comparison for the ARMA(1,3) process on one-sided and two-sided EWMA, MEWMA, and DMEWMA control charts with $\lambda_1 = 0.1, \lambda_2 = 0.2, \phi_0 = 0.5, \phi_1 = 0.1, \theta_1 = 0.1, \theta_2 = 0.2, \theta_3 = 0.1$ and $ARL_0 = 370$.

g	δ	Control Chart	DMEWMA				MEWMA			EWMA
			$r_1 = 2.0, r_2 = 0.5$	$r_1 = 2.0, r_2 = 2.0$	$r_1 = 4, r_2 = 2$	$r_1 = 4, r_2 = 4$	$r_1 = 0.5$	$r_1 = 2.0$	$r_1 = 4$	$r_1 = 0$
0	0.000	ARL ₀	$h =$				$d =$			$b =$
			$h = 2.922075$	9.70705	$h = 19.32$	$h = 36.875$	$d = 1.24456$	$d = 4.9988$	$d = 10.0234$	0.00363
			370.481	370.125	370.008	370.565	370.395	370.200	370.256	370.161
	0.004	SDRL ₀	369.981	369.625	369.508	370.065	369.895	369.700	369.756	369.661
			MRL ₀	256.451	256.204	256.123	256.509	256.392	256.256	256.229
			ARL ₁	244.103	226.042	224.265	220.653	310.284	284.533	284.097
	0.008	SDRL ₁	243.602	225.541	223.764	220.152	309.784	284.033	283.597	354.904
			MRL ₁	168.852	156.334	155.102	152.598	214.726	196.876	196.574
			ARL ₁	181.870	162.626	160.800	157.031	266.456	230.265	229.440
	0.01	SDRL ₁	181.369	162.125	160.299	156.530	265.956	229.764	228.939	340.843
			MRL ₁	125.716	112.377	111.111	108.499	184.346	159.261	158.689
			ARL ₁	161.266	142.599	140.844	137.227	248.715	209.974	208.995
	0.04	SDRL ₁	160.765	142.098	140.343	136.726	248.214	209.473	208.494	334.060
			MRL ₁	111.434	98.4951	97.2786	94.7715	172.049	145.196	144.517
			ARL ₁	59.3498	49.9336	49.0459	47.3479	121.353	87.2226	85.4590
	0.08	SDRL ₁	58.8477	49.4311	48.5433	46.8452	120.852	86.7212	84.9575	249.322
			MRL ₁	40.7906	34.2636	33.6483	32.4713	83.7684	60.1109	58.8884
			ARL ₁	31.9612	26.7103	26.1888	25.2705	69.3188	46.7003	45.0551
	0.10	SDRL ₁	31.4572	26.2055	25.6839	24.7655	68.8170	46.1976	44.5523	172.944
			MRL ₁	21.8054	18.1654	17.8039	17.1673	47.7007	32.0224	30.8819
			ARL ₁	25.9258	21.6828	21.2511	20.5159	56.3724	37.4021	35.8816
	1.00	SDRL ₁	25.4209	21.1769	20.7451	20.0097	55.8702	36.8987	35.3781	145.429
			MRL ₁	17.6215	14.6801	14.3808	13.8711	38.7268	25.5770	24.5230
			ARL ₁	3.17062	2.95161	2.92540	2.88469	4.52776	3.57633	3.43651
	RMI	AEQL	2.62340	2.40008	2.37330	2.33168	3.99660	3.03543	2.89363	2.27157
			PCI	1.82930	1.67550	1.65707	1.62843	2.77743	2.11344	2.01561
				0.19193	0.04459	0.02999	0.00297	1.08164	0.58068	0.54635
	RMI	AEQL	0.53729	0.49251	0.48742	0.47923	0.82517	0.63273	0.60865	0.83657
			PCI	1.12115	1.02770	1.01709	1.00000	1.72185	1.32028	1.27004
										1.74564
0.1	0.000	ARL ₀	$h =$				$d =$			$b =$
			$h = 3.07195$	9.85985	19.47425	37.0288	$d = 1.36904$	5.12622	10.15155	0.10953
			370.948	370.132	370.099	370.198	370.525	370.402	370.252	370.716
	0.004	SDRL ₀	370.448	369.632	369.599	369.698	370.025	369.902	369.752	370.216
			MRL ₀	256.775	256.209	256.186	256.255	256.482	256.396	256.292
			ARL ₁	243.574	226.078	224.382	220.566	306.018	284.316	284.242
	0.008	SDRL ₁	243.073	225.577	223.881	220.065	305.518	283.816	283.742	356.792
			MRL ₁	168.486	156.358	155.183	152.538	211.769	196.726	196.675
			ARL ₁	181.164	162.655	160.899	157.007	260.128	229.862	229.600
	0.01	SDRL ₁	180.663	162.154	160.398	156.506	259.628	229.361	229.099	343.951
			MRL ₁	125.226	112.397	111.180	108.482	179.960	158.981	158.800
			ARL ₁	160.548	142.624	140.934	137.215	241.819	209.519	209.146
	0.04	SDRL ₁	160.047	142.123	140.433	136.714	241.318	209.018	208.645	337.740
			MRL ₁	110.936	98.5124	97.3410	94.7632	167.269	144.881	144.622
			ARL ₁	58.9053	49.9272	49.0721	47.3537	114.704	86.6862	85.4201
	0.08	SDRL ₁	58.4032	49.4247	48.5695	46.8510	114.203	86.1847	84.9186	259.035
			MRL ₁	40.4825	34.2592	33.6664	32.4753	79.1597	59.7391	58.8614
			ARL ₁	31.6909	26.6961	26.1957	25.2718	64.8696	46.2762	44.9511
	0.10	SDRL ₁	31.1869	26.1913	25.6908	24.7668	64.3677	45.7735	44.4483	185.887
			MRL ₁	21.6180	18.1555	17.8087	17.1682	44.6167	31.7284	30.8099
			ARL ₁	25.7012	21.6677	21.2543	20.5160	52.6531	37.0252	35.7727
	1.00	SDRL ₁	25.1962	21.1618	20.7483	20.0098	52.1507	36.5218	35.2692	159.339
			MRL ₁	17.4658	14.6696	14.3830	13.8711	36.1487	25.3158	24.4475
			ARL ₁	3.16235	2.95031	2.92498	2.88445	4.36903	3.54655	3.42262
	RMI	AEQL	2.61498	2.39875	2.37287	2.33144	3.83659	3.00524	2.87953	3.41912
			PCI	1.82350	1.67458	1.65677	1.62826	2.66681	2.09261	2.00589
				0.18218	0.04144	0.02744	0	0.98240	0.56800	0.54115
	RMI	AEQL	0.46850	0.43075	0.42646	0.41930	0.69250	0.54898	0.53061	0.90233
			PCI	1.11733	1.02731	1.01707	1.00000	1.65155	1.30928	1.26546
										2.15197

Source: created by the authors.

Table 4. ARL comparison for the ARMA(2,2) process on one-sided and two-sided EWMA, MEWMA, and DMEWMA control charts with $\lambda_1 = 0.1, \lambda_2 = 0.2, \phi_0 = 0.5, \phi_1 = 0.2, \phi_2 = 0.3, \theta_1 = 0.1, \theta_2 = 0.2$ and $ARL_0 = 370$.

g	δ	Control Chart	DMEWMA				MEWMA			EWMA
			$r_1=2.0,$ $r_2=0.5$	$r_1=2.0,$ $r_2=2.0$	$r_1=4,$ $r_2=2$	$r_1=4,$ $r_2=4$	$r_1=0.5$	$r_1=2.0$	$r_1=4$	$r_1=0$
			$h = 1.8767$	$h = 6.20975$	$h = 12.34145$	$h = 23.55575$	$d = 0.7223$	$d = 2.88588$	$d = 5.77948$	$b = 0.00219$
0	0.000	ARL ₀	370.754	370.372	370.966	370.186	370.262	370.384	370.307	370.760
		SDRL ₀	370.254	369.872	370.466	369.686	369.762	369.884	369.807	370.260
		MRL ₀	256.640	256.376	256.787	256.247	256.299	256.384	256.331	256.645
	0.004	ARL ₁	184.258	155.021	149.246	145.389	257.299	174.535	157.926	355.231
		SDRL ₁	183.757	154.520	148.745	144.888	256.799	174.034	157.425	354.731
		MRL ₁	127.371	107.105	103.102	100.429	177.999	120.632	109.119	245.881
	0.008	ARL ₁	122.753	98.3428	93.7621	90.8216	196.867	114.396	100.679	340.465
		SDRL ₁	122.252	97.8415	93.2608	90.3202	196.366	113.895	100.178	339.965
		MRL ₁	84.7388	67.8189	64.6437	62.6055	136.111	78.9462	69.4382	235.646
	0.01	ARL ₁	105.241	83.2282	79.1575	76.5669	176.094	97.6458	85.3146	333.354
		SDRL ₁	104.740	82.7267	78.6559	76.0653	175.593	97.1445	84.8131	332.854
		MRL ₁	72.6004	57.3421	54.5205	52.7248	121.712	67.3357	58.7883	230.717
	0.04	ARL ₁	33.8829	25.7893	24.3741	23.4936	67.2405	31.0357	26.5493	245.211
		SDRL ₁	33.3792	25.2844	23.8689	22.9882	66.7386	30.5316	26.0445	244.710
		MRL ₁	23.1375	17.5269	16.5458	15.9354	46.2601	21.1638	18.0538	169.621
	0.08	ARL ₁	18.0968	13.9022	13.1748	12.7198	36.1462	16.6405	14.3127	167.107
		SDRL ₁	17.5897	13.3929	12.6649	12.2096	35.6427	16.1328	13.8036	166.606
		MRL ₁	12.1939	9.2854	8.7809	8.46539	24.7064	11.1842	9.5701	115.483
	0.10	ARL ₁	14.7638	11.4248	10.8461	10.4825	29.1885	13.6149	11.7593	139.376
		SDRL ₁	14.2550	10.9134	10.3340	9.96997	28.6841	13.1054	11.2482	138.875
		MRL ₁	9.88286	7.56720	7.16578	6.91355	19.8833	9.08615	7.79922	96.2611
	1.00	ARL ₁	2.33510	2.16535	2.13656	2.11267	2.97469	2.30880	2.20821	2.41307
		SDRL ₁	1.76567	1.58852	1.55831	1.53320	2.42365	1.73832	1.63340	1.84657
		MRL ₁	1.23987	1.11877	1.09816	1.08103	1.69172	1.22115	1.14942	1.29527
	RMI		0.33887	0.07737	0.03028	0	1.30479	0.25089	0.10448	5.93699
	AEQL		0.38201	0.34670	0.34066	0.33604	0.51997	0.37443	0.35391	0.76135
	PCI		1.13681	1.03173	1.01376	1.00000	1.54735	1.11423	1.05318	2.26565
0.1	0.000	ARL ₀	$h = 2.13851$	$h = 6.41046$	$h = 12.5278$	$h = 23.4984$	$d = 0.8360$	$d = 3.00098$	$d = 5.89492$	$b = 0.10568$
		SDRL ₀	370.486	370.013	370.026	370.768	370.673	370.440	370.436	370.607
		MRL ₀	369.986	369.513	369.526	370.268	370.173	369.940	369.936	370.107
	0.004	ARL ₁	256.455	256.127	256.136	256.650	256.584	256.423	256.420	256.538
		SDRL ₁	186.822	154.894	148.961	144.460	246.902	171.114	156.233	356.373
		MRL ₁	186.321	154.393	148.460	143.959	246.401	170.613	155.732	355.873
	0.008	ARL ₁	129.148	107.017	102.905	99.7851	170.793	118.260	107.946	246.672
		SDRL ₁	125.072	98.2702	93.6003	90.0728	184.881	111.499	99.3143	342.790
		MRL ₁	124.571	97.7689	93.0990	89.5714	184.380	110.998	98.8130	342.290
	0.01	ARL ₁	86.3463	67.7685	64.5316	62.0865	127.803	76.9381	68.4923	237.257
		SDRL ₁	107.380	83.1696	79.0253	75.8999	164.189	95.0250	84.0968	336.232
		MRL ₁	106.879	82.6681	78.5237	75.3982	163.688	94.5237	83.5953	335.732
	0.04	ARL ₁	74.0830	57.3015	54.4289	52.2625	113.460	65.5191	57.9442	232.712
		SDRL ₁	34.7842	25.7818	24.3426	23.2571	60.6459	30.0596	26.1178	253.844
		MRL ₁	34.2806	25.2769	23.8374	22.7516	60.1438	29.5554	25.6129	253.344
	0.08	ARL ₁	23.7623	17.5217	16.5240	15.7715	41.6890	20.4872	17.7547	175.604
		SDRL ₁	18.6123	13.9048	13.1619	12.5947	32.4745	16.1328	14.0894	178.695
		MRL ₁	18.1054	13.3955	12.6520	12.0844	31.9706	15.6248	13.5802	178.194
	0.10	ARL ₁	12.5513	9.28719	8.77200	8.37863	22.1612	10.8321	9.41520	123.515
		SDRL ₁	15.1922	11.4293	10.8369	10.3815	26.2467	13.2102	11.5811	151.343
		MRL ₁	14.6837	10.9179	10.3248	9.86884	25.7418	12.7004	11.0698	150.842
	1.00	ARL ₁	10.1799	7.57033	7.15940	6.84348	17.8440	8.80549	7.67562	104.556
		SDRL ₁	2.40558	2.17325	2.13918	2.10298	2.86445	2.28750	2.19812	3.25579
		MRL ₁	1.83881	1.59680	1.56106	1.52301	2.31098	1.71615	1.62284	2.71005
	RMI		0.33466	0.07574	0.03304	0	1.00016	0.20100	0.08682	5.61646
	AEQL		0.30600	0.27054	0.26522	0.26003	0.38489	0.28781	0.27368	0.70892
	PCI		1.17679	1.04042	1.01997	1.00000	1.48016	1.10684	1.05251	2.72629

Source: created by the authors.

Table 5. Real data ARL comparison for the ARMA(1,1) process on one-sided and two-sided EWMA, MEWMA, and DMEWMA control charts with $\lambda_1 = 0.1, \lambda_2 = 0.2, \phi_1 = 0.983, \theta_1 = 0.459, \beta = 0.2827$ and $ARL_0 = 370$.

g	δ	Control Chart	DMEWMA				MEWMA			EWMA
			$r_1=2.0,$ $r_2=0.5$	$r_1=2.0,$ $r_2=2.0$	$r_1=4,$ $r_2=2$	$r_1=4,$ $r_2=4$	$r_1=0.5$	$r_1=2.0$	$r_1=4$	$r_1=0$
0	0.000	ARL ₀	$h = 0.838$ 370.158	$h = 2.030$ 370.856	$h = 4.009$ 370.855	$h = 6.933$ 370.628	$d = 0.00281$ 370.777	$d = 1.11084$ 370.819	$d = 2.62415$ 370.533	$b = 0.000028$ 370.457
		SDRL ₀	369.658	370.356	370.355	370.128	370.277	370.319	370.033	369.957
		MRL ₀	256.227	256.711	256.710	256.553	256.656	256.685	256.487	256.434
	0.004	ARL ₁	252.157	100.597	81.6648	65.9889	286.096	196.546	183.594	305.612
		SDRL ₁	251.657	100.096	81.1633	65.4870	285.596	196.045	183.093	305.112
		MRL ₁	174.435	69.3814	56.2584	45.3926	197.960	135.888	126.911	211.487
	0.008	ARL ₁	188.692	58.3256	46.2437	36.6968	230.623	132.258	120.198	253.438
		SDRL ₁	188.191	57.8234	45.7410	36.1933	230.122	131.757	119.697	252.938
		MRL ₁	130.444	40.0807	31.7059	25.0881	159.509	91.3272	82.9678	175.323
	0.01	ARL ₁	166.880	48.2363	38.0929	30.1490	207.428	113.285	102.041	231.231
		SDRL ₁	166.379	47.7337	37.5896	29.6448	206.927	112.784	101.540	230.730
		MRL ₁	115.325	33.0871	26.0559	20.5492	143.431	78.1761	70.3823	159.930
	0.04	ARL ₁	54.5745	13.7012	10.9994	8.86281	59.5305	33.4377	28.7326	67.0151
		SDRL ₁	54.0722	13.1917	10.4875	8.34785	59.0284	32.9339	28.2282	66.5132
		MRL ₁	37.4805	9.145997	7.27212	5.78974	40.9158	22.8289	19.5673	46.1039
	0.08	ARL ₁	25.2162	7.28071	6.06416	5.04555	19.0634	15.9881	13.6883	18.0113
		SDRL ₁	24.7111	6.76225	5.54165	4.51797	18.5567	15.4800	13.1788	17.5042
		MRL ₁	17.1296	4.69150	3.84638	3.13799	12.8641	10.7318	9.13705	12.1346
	0.10	ARL ₁	19.2345	5.98480	5.06391	4.26979	12.2781	12.4855	10.7481	10.5617
		SDRL ₁	18.7278	5.46196	4.53644	3.73648	11.7675	10.2359	10.0493	10.0493
		MRL ₁	12.9827	3.79122	3.15076	2.59762	8.15905	8.30289	7.09780	6.96849
	1.00	ARL ₁	1.78847	1.45139	1.42760	1.27510	1.02652	1.65326	1.68322	1.00370
		SDRL ₁	1.18750	0.80941	0.78131	0.71819	0.16499	1.03923	1.07238	0.06094
		MRL ₁	0.84631	0.59348	0.57496	0.53356	0.18959	0.74650	0.76876	0.12371
	RMI		3.56291	0.50720	0.25893	0.03863	3.55623	2.12185	1.79876	3.83026
	AEQL		0.32319	0.22713	0.22039	0.19581	0.20095	0.27955	0.27787	0.19658
	PCI		1.65050	1.15994	1.12551	1.00000	1.02623	1.42766	1.41908	1.00391
0.1	0.000	ARL ₀	$h = 0.99129$ 370.9000	$h = 2.167575$ 370.7690	$h = 4.14688$ 370.4030	$h = 7.06599$ 370.8820	$d = 0.11725$ 370.4520	$d = 1.23276$ 370.5530	$d = 2.750325$ 370.9070	$b = 0.100982$ 370.3390
		SDRL ₀	370.3997	370.2687	369.9027	370.3817	369.9517	370.0527	370.4067	369.8387
		MRL ₀	256.7416	256.6508	256.3971	256.7291	256.4310	256.5010	256.7464	256.3527
	0.004	ARL ₁	252.9110	98.0210	80.5056	65.4196	293.1840	186.7760	181.0090	320.9960
		SDRL ₁	252.4105	97.5197	80.0040	64.9177	292.6836	186.2753	180.5083	320.4956
		MRL ₁	174.9577	67.5958	55.4549	44.9979	202.8729	129.1164	125.1190	222.1507
	0.008	ARL ₁	189.1530	56.6497	45.5341	36.3584	237.6160	123.5740	117.8470	279.2870
		SDRL ₁	188.6523	56.1475	45.0313	35.8549	237.1155	123.0730	117.3459	278.7866
		MRL ₁	130.7640	38.9190	31.2140	24.8535	164.3560	85.3079	81.3382	193.2402
	0.01	ARL ₁	167.2030	46.8248	37.5030	29.8694	215.4580	105.3590	99.8742	260.8720
		SDRL ₁	166.7023	46.3221	36.9996	29.3651	214.9574	104.8578	99.3729	260.3715
		MRL ₁	115.5494	32.1087	25.6470	20.3553	148.9973	72.6822	68.8804	180.4759
	0.04	ARL ₁	54.2056	13.3331	10.8492	8.79244	69.8510	30.7450	27.9072	103.5250
		SDRL ₁	53.7033	12.8234	10.3371	8.27735	69.3492	30.2409	27.4026	103.0238
		MRL ₁	37.2248	8.89072	7.16793	5.74091	48.0696	20.9623	18.9951	71.4109
	0.08	ARL ₁	24.9145	7.12397	5.99901	5.01466	25.6903	14.8213	13.2965	38.2437
		SDRL ₁	24.4094	6.60507	5.47623	4.48689	25.1853	14.3126	12.7867	37.7404
		MRL ₁	16.9205	4.58265	3.80110	3.11649	17.4583	9.92273	8.86534	26.1604
	0.10	ARL ₁	18.9915	5.86980	5.01564	4.24676	17.4805	11.6930	10.4514	25.1948
		SDRL ₁	18.4847	5.34647	4.48787	3.71325	16.9731	11.1819	9.93883	24.6897
		MRL ₁	12.8142	3.71128	3.11717	2.58157	11.7666	7.75325	6.89198	17.1148
	1.00	ARL ₁	1.80996	1.45358	1.42866	1.37551	1.10277	1.64506	1.67928	1.05909
		SDRL ₁	1.21078	0.81198	0.78257	0.71869	0.33665	1.03013	1.06804	0.25016
		MRL ₁	0.86204	0.59517	0.57579	0.53389	0.29209	0.74039	0.76583	0.24017
	RMI		3.12257	0.41448	0.21234	0.03735	3.68190	1.69252	1.52800	5.08192
	AEQL		0.25321	0.17656	0.17139	0.16339	0.17724	0.21416	0.21489	0.19673
	PCI		1.54977	1.08064	1.04898	1.00000	1.08480	1.31076	1.31518	1.20404

Source: created by the authors.

Table 6. South ARL comparison for the ARMA(1,1) process on one-sided and two-sided EWMA, MEWMA, and DMEWMA control charts with $\lambda_1 = 0.1, \lambda_2 = 0.2, \phi_1 = 0.996, \theta_1 = 0.621, \beta = 0.3335$ and $ARL_0 = 370$.

g	δ	Control Chart	DMEWMA				MEWMA			EWMA
			$r_1 = 2.0,$ $r_2 = 0.5$	$r_1 = 2.0,$ $r_2 = 2.0$	$r_1 = 4,$ $r_2 = 2$	$r_1 = 4,$ $r_2 = 4$	$r_1 = 0.5$	$r_1 = 2.0$	$r_1 = 4$	$r_1 = 0$
			$h = 1.2072$	$h = 3.25745$	$h = 6.77613$	$h = 12.0174$	$d = 0.09612$	$d = 1.54089$	$d = 3.06809$	$b = 0.000465$
0	0.000	ARL ₀	370.019	370.733	370.674	370.458	370.418	370.065	370.923	370.505
		SDRL ₀	369.519	370.233	370.174	369.958	369.918	369.565	370.423	370.005
		MRL ₀	256.131	256.626	256.585	256.435	256.407	256.163	256.758	256.468
	0.004	ARL ₁	317.485	167.225	162.928	126.589	316.433	232.586	184.813	324.608
		SDRL ₁	316.985	166.724	162.427	126.088	315.933	232.085	184.312	324.108
		MRL ₁	219.717	115.565	112.586	87.39778	218.988	160.870	127.756	224.654
	0.008	ARL ₁	273.676	107.157	103.420	76.1973	273.408	167.555	121.727	285.254
		SDRL ₁	273.176	106.656	102.919	75.6956	272.908	167.054	121.226	284.754
		MRL ₁	189.351	73.9285	71.3381	52.4686	189.165	115.793	84.0277	197.376
	0.01	ARL ₁	254.634	90.6424	87.2087	63.5213	255.079	146.446	103.640	267.698
		SDRL ₁	254.134	90.1410	86.7073	63.0193	254.579	145.945	103.139	267.198
		MRL ₁	176.152	62.4813	60.1012	43.6821	176.460	101.162	71.4906	185.207
	0.04	ARL ₁	103.697	26.3056	24.8295	18.1627	109.909	46.3319	30.1534	111.845
		SDRL ₁	103.196	25.8008	24.3244	17.6556	109.408	45.8292	29.6492	111.344
		MRL ₁	71.5301	17.8848	16.8615	12.2396	75.8360	31.7670	20.5522	77.1780
	0.08	ARL ₁	46.5406	13.1138	12.3171	9.46455	49.7183	21.9986	14.6913	42.5403
		SDRL ₁	46.0379	12.6039	11.8065	8.95060	49.2158	21.4928	14.1825	42.0373
		MRL ₁	31.9117	8.73864	8.18610	6.20730	34.1144	14.8990	9.83259	29.1387
	0.10	ARL ₁	34.5625	10.4633	9.83318	7.71498	36.2570	17.0531	11.6089	28.1248
		SDRL ₁	34.0588	9.95075	9.31978	7.19763	35.7535	16.5455	11.0976	27.6203
		MRL ₁	23.6086	6.90023	6.46307	4.99303	24.7832	11.4702	7.69490	19.1460
	1.00	ARL ₁	2.19297	1.74779	1.74404	1.56140	1.46716	1.89583	1.78407	1.04454
		SDRL ₁	1.61745	1.14323	1.13914	0.93625	0.82789	1.30320	1.18272	0.21569
		MRL ₁	1.13853	0.81644	0.81368	0.67763	0.60568	0.92461	0.84309	0.21970
	RMI		2.90206	0.43110	0.37570	0.07069	2.93030	1.17748	0.58774	2.68862
	AEQL		0.43578	0.28529	0.28270	0.24878	0.33884	0.33005	0.29479	0.26103
	PCI		1.75167	1.14678	1.13635	1.00000	1.36201	1.32670	1.18496	1.04926
0.1	0.000		$h = 1.37719$	$h = 3.41279$	$h = 6.93467$	$h = 12.1707$	$d = 0.2081$	$d = 1.66673$	$d = 3.1939$	$b = 0.11084$
		ARL ₀	370.392	370.334	370.254	370.657	370.915	370.630	370.317	370.248
		SDRL ₀	369.892	369.834	369.754	370.157	370.415	370.130	369.817	369.748
	0.004	MRL ₀	256.389	256.349	256.294	256.573	256.752	256.554	256.337	256.290
		ARL ₁	343.546	168.145	164.203	126.669	304.558	227.932	182.176	336.665
		SDRL ₁	343.046	167.644	163.702	126.168	304.058	227.431	181.675	336.165
	0.008	MRL ₁	237.781	116.202	113.470	87.4532	210.757	157.644	125.928	233.012
		ARL ₁	313.543	107.877	104.417	76.2364	255.388	162.505	119.448	306.776
		SDRL ₁	313.043	107.376	103.916	75.7347	254.888	162.004	118.947	306.276
	0.01	MRL ₁	216.985	74.4275	72.0292	52.4957	176.675	112.293	82.4480	212.294
		ARL ₁	298.185	91.2619	88.0738	63.5497	235.359	141.546	101.563	293.068
		SDRL ₁	297.685	90.7605	87.5724	63.0477	234.858	141.045	101.062	292.568
	0.04	MRL ₁	206.339	62.9107	60.7009	43.7018	162.792	97.7652	70.0510	202.792
		ARL ₁	129.664	26.4035	25.0101	18.1564	93.3450	44.0614	29.3782	156.140
		SDRL ₁	129.163	25.8987	24.5050	17.6493	92.8437	43.5585	28.8739	155.639
	0.08	MRL ₁	89.5292	17.9527	16.9867	12.2352	64.3546	30.1931	20.0148	107.881
		ARL ₁	54.3820	13.1213	12.3659	9.45697	41.6325	20.8934	14.3149	77.2982
		SDRL ₁	53.8797	12.6114	11.8554	8.94300	41.1295	20.3873	13.8058	76.7966
	0.10	MRL ₁	37.3471	8.74384	8.21994	6.20204	28.5095	14.1328	9.5716	53.2317
		ARL ₁	39.2930	10.4604	9.86221	7.70818	30.4785	16.2155	11.3204	57.0104
		SDRL ₁	38.7898	9.94784	9.34885	7.19082	29.9743	15.7075	10.8088	56.5082
	1.00	MRL ₁	26.8878	6.89822	6.48322	4.98831	20.7776	10.8895	7.49479	39.1690
		ARL ₁	2.23145	1.75251	1.74668	1.66241	1.46851	1.88445	1.77885	1.47239
		SDRL ₁	1.65768	1.14838	1.14202	1.04938	0.82946	1.29101	1.17705	0.83399
		MRL ₁	1.16601	0.81991	0.81563	0.75331	0.60672	0.91635	0.83926	0.60972
	RMI		3.00327	0.32134	0.27573	0.01650	2.11942	0.89767	0.42686	3.68320
	AEQL		0.35947	0.22245	0.22029	0.20470	0.24821	0.25322	0.22793	0.31571
	PCI		1.75608	1.08670	1.07613	1.00000	1.21251	1.23703	1.11348	1.54226

Source: created by the authors.

Table 7. The ARMA model parameters and the model fit statistics

Model		Coefficient	t	Sig	Model fit	
					RMS E	Normalized BIC
Unemployment rate in Bangkok ARMA(1,1)	AR(1) ($\hat{\phi}_1$)	0.983	55.835	0.000	0.360	-1.964
	MA(1) ($\hat{\theta}_1$)	0.459	5.202	0.000		
	Residual of Application		Residual ARMA(1,1) model			
	Exponential parameter		0.2827			
	One-sample Kolmogorov-Smirnov test		1.137			
	p-value		0.151			
Unemployment rate in Southern region ARMA(1,1)	AR(1) ($\hat{\phi}_1$)	0.996	130.220	0.000	0.392	-1.792
	MA(1) ($\hat{\theta}_1$)	0.621	7.661	0.041		
	Residual of Application		Residual ARMA(1,1) model			
	Exponential parameter		0.3335			
	One-sample Kolmogorov-Smirnov test		0.730			
	p-value		0.662			

Source: created by the authors.

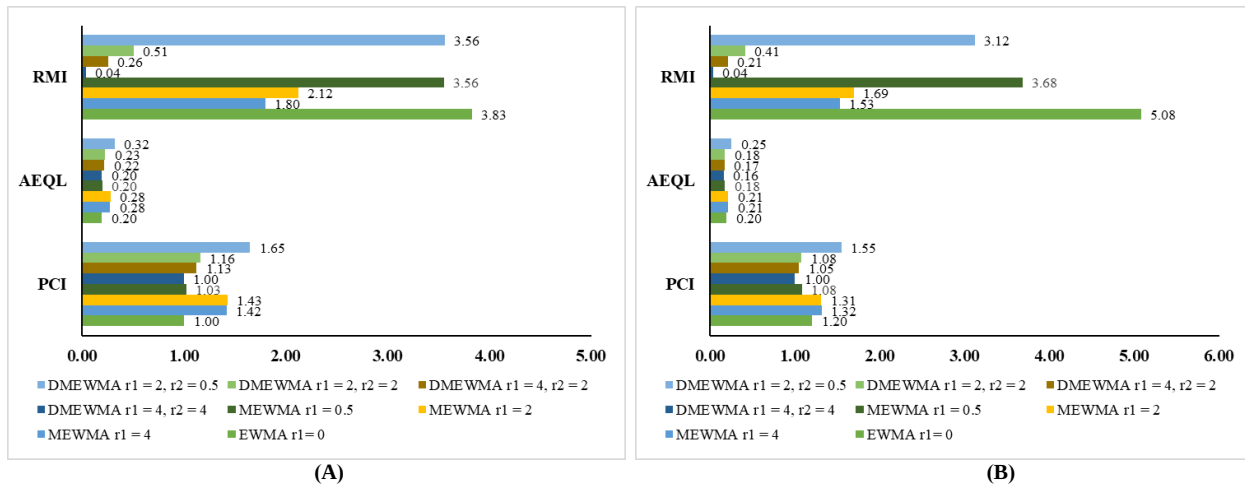


Fig. 1: The RMI, AEQL and PCI values on various charts for the first set of actual data when (A) one-sided and (B) two-sided.

Source: created by the authors.

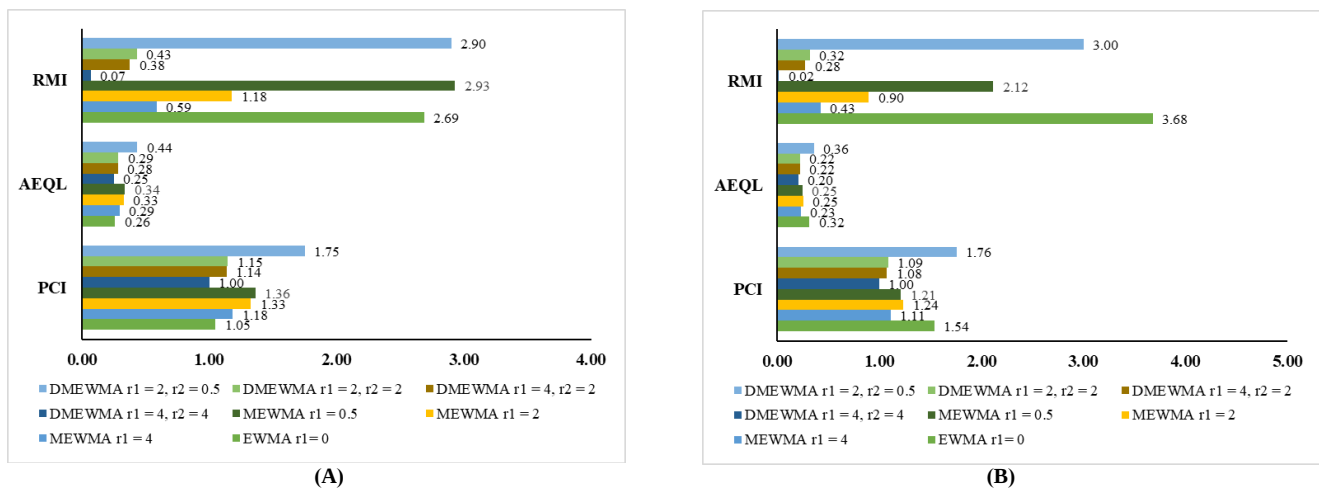


Fig. 2: The RMI, AEQL and PCI values on various charts for the second set of actual data when (A) one-sided and (B) two-sided.

Source: created by the authors.

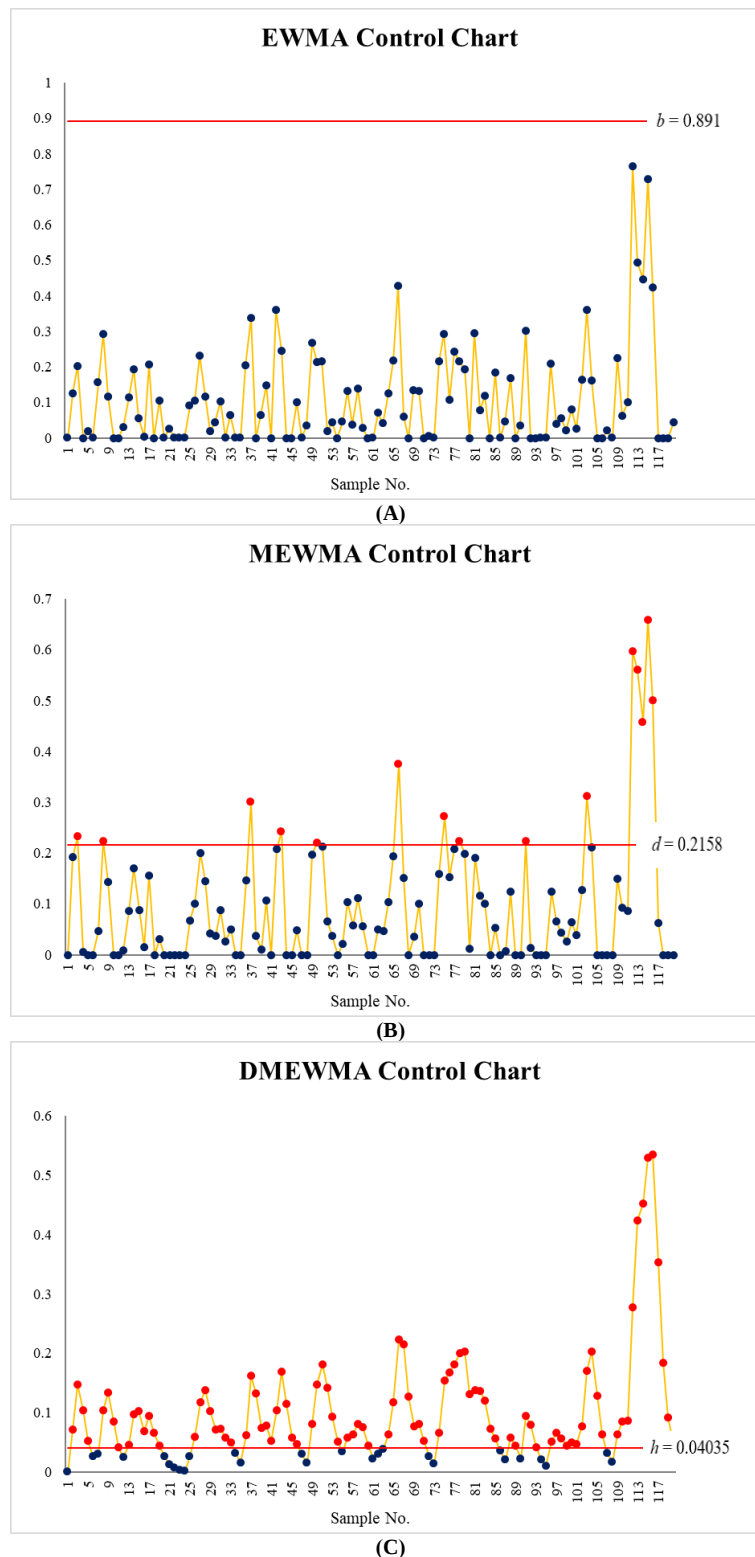


Fig. 3: The first real data with $\lambda_1 = 0.1, \lambda_2 = 0.5, \phi_1 = 0.983, \theta_1 = 0.459$, and $\beta = 0.2827$ when (A) EWMA, (B) MEWMA, and (C) DMEWMA control charts

Source: created by the authors.